# Skyrmionic Textures and Applications

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## Introduction

Skyrmions are named after the British nuclear physicist Tony Skyrme, who first proposed their existence in 1961 [2]. His idea was to model subatomic entities like protons and neutrons using convoluted twists in the quantum field that all particles possess .Chiral magnetic skyrmions are topological magnetic configurations that are stabilised in materials with the Dzyaloshinskii-Moriya interaction.Effective two dimensional textures described by a local magnetization direction $\mathbf{m}$ can be classified by the skyrmion number :

$$Q = \frac{1}{4\pi}\int \mathbf{m}\cdot(\partial_x\mathbf{m}\times\partial_y\mathbf{m}) \quad dxdy. \tag{1}$$

## Model

We assume a ferromagnetic material as a two-dimensional system lying on the xy-plane. The micromagnetic structure is described via the magnetization vector $\mathbf{m} = \mathbf{m}(x, y)$ with a fixed magnitude normalized to unity, $\mathbf{m}^2 = 1$. We will assume a ferromagnet with exchange interaction, a Dzyaloshinskii-Moriya (DM) interaction, and an anisotropy of the easy-axis type perpendicular to the film, governed by the normalized energy

$$E = \frac{1}{2}\int \partial_\mu\mathbf{m}\cdot\partial_\mu\mathbf{m}\, d^2x + \frac{1}{2}\int (1-m_3^2)\, d^2x + \lambda\int \hat{e}_\mu\cdot(\partial_\mu m\times m)\, d^2x. \tag{2}$$

A summation over repeated indices $\mu = 1, 2$ is assumed.Static magnetization fields are local minimizers of $\mathbf{E}$ satisfying the normalized Landau-Lifshitz equation

$$\mathbf{m}\times\mathbf{f} = 0 \quad , \quad \mathbf{f} = \partial_\mu\partial_\mu\mathbf{m} + m_3\hat{e}_3 - 2\lambda(\hat{e}_\mu\cdot(\partial_\mu\mathbf{m}\times\mathbf{m})) \tag{3}$$

is minus the variational gradient of $E = E(\mathbf{m})$ . We measure lengths in units of the domain wall width $\ell_w = \sqrt{A/K}$ where $A$ is the exchange and $K$ the anisotropy parameter. The equation contains a single parameter

$$\lambda = \frac{\ell_S}{\ell_w} = \frac{D}{2\sqrt{AK}}, \tag{4}$$

defined via an additional length scale of this model $\ell_S = D/(2K)$ , where $D$ is the DM parameter ( a parameter which differs from $\lambda$ only by a constant factor has been introduced).

## BP Skyrmion

we have the model that contains only the exchange interaction,our goal now is to find explicit solutions of the static Landau-Lifshitz equation .It is convenient to map the magnetization defined on the sphere, to a complex function $u = u(z)$, with $z = x + iy$.The stereographic projection of a point on the sphere $\mathbf{m} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ to the complex plane is given by

$$u = \frac{m_x + im_y}{1 + m_z} = \tan\frac{\theta}{2}e^{i\phi}. \tag{5}$$

The static Landau-Lifshitz equation in the new function $u$, becomes

$$0 = i\partial_t u = -\partial\bar{\partial}u + 2\frac{2\bar{u}}{1+|u|^2}\partial u\bar{\partial}u, \tag{6}$$

where $\partial = \partial/\partial_z = (\partial/\partial_x - i\partial/\partial_y)/2$ . One immediately observes that any harmonic function is a stationary solution [1].

$$\bar{\partial}u(z) = 0 \quad \texttt{or} \quad \partial u(z) = 0. \tag{7}$$

For example, a solutions of this form are

$$u = \alpha z,\ \alpha z^2,\ \frac{\alpha}{z},\ \frac{\alpha}{\bar{z}},\ \alpha\bar{z}, \qquad \alpha\in\mathbb{R}^+$$

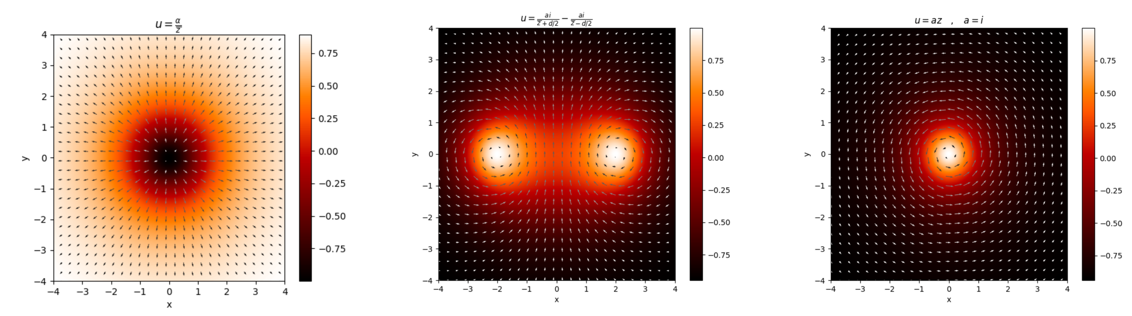

Fig. 1: The figure represent some BP-skyrmions , the vectors shows the projection of $\mathbf{m}$ on the plane, i.e., $(m_1, m_2)$.The 3rd component of the magnetization $m_3$ is represented by the color code as shown on the right of the figure.

## Axially symmetric skyrmionic textures

We can find the skyrmion numerically , within a model with exchange , anisotropy and DM interaction, a skyrmionic texture with Q = 1 , for the parameter value range $0 < \lambda < \frac{2}{\pi}$ . We Choose latice spaces $\delta x = \delta y = 0.1, 0.15, 0.2$ and for the numerical meshes was fixed to $200\times 200$ , $300\times 300$ sites

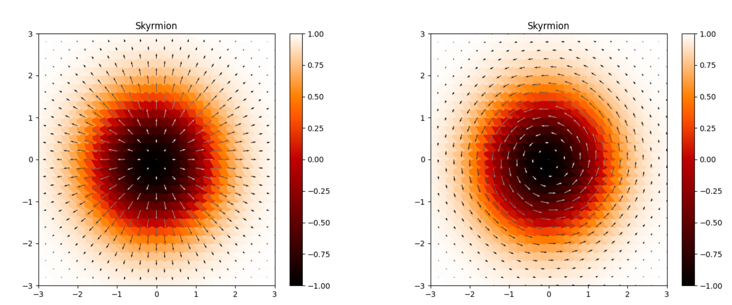

Fig. 2: Two Skyrmions for lattice space $\delta x = \delta y = 0.2$ and lattice size $30\times 30$ for value of DM parameter $\lambda = 0.5$ .

A magnetic skyrmionium is a topological quasi particle that is composed of a superposition of two magnetic skyrmions of opposite topological charge (A skyrmion with Q = 1 and an anti-skyrmion with Q = 1) adding up to zero total topological charge Q = 0 . An ansatz for a $Q = 0$ configuration is

$$u = z - \frac{a}{\bar{z}} \quad \texttt{or} \quad u = i(z - \frac{a}{\bar{z}}), \tag{8}$$

Skyrmionium isn't BP-Skyrmion, but we can find it numerically for the parameter value range $0.5 < \lambda < \frac{2}{\pi}$ and for initial condition with radius $\rho > 2.0$ . We Choose latice spaces $\delta x = \delta y = 0.1, 0.15, 0.2$ and for the numerical meshes was fixed to $200\times 200$ , $300\times 300$ sites

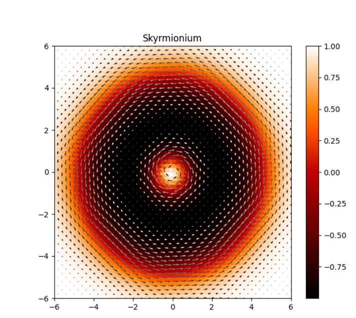

Fig. 3: A skyrmionium for lattice space $\delta x = \delta y = 0.2$ and lattice size $30\times 30$ for value of DM parameter $\lambda = 0.55$.

Another interesting result is the dependence of the skyrmion and skyrmionium radius on the parameter of the DM term.

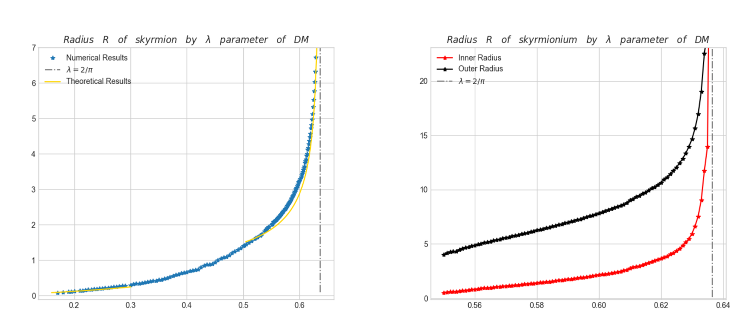

Fig. 4: The figure represent the radius of skyrmion(blue) and skyrmionium inner (red) and outer (black) by the parameter of DM term .

## Non-Axially symemetric skyrmionic textures

We study $Q = 0$ solitons that can be constructed as skyrmion-antiskyrmion pairs. This is an asymmetric configuration and its shape resembles that of a liquid droplet . An ansatz for a $Q = 0$ configuration is conveniently given in terms of the stereographic variable as

$$u = \frac{a}{x + i|y|} \quad \texttt{or} \quad u = \frac{a}{|y| - ix}, \tag{9}$$

We find numerically the droplet for range of the values of the dimensionless parameters $0.6125 \le \lambda < 2/\pi, k = 1.0$.We are looking for a non-axially symmetric solutions of this model with skyrmion number $Q_+ = 1/2$ and $Q_- = 1/2$ . An interesting object can be created if we assume that we have in a film a meron and an antimeron in proximity to each other. An ansatz given in terms of the stereographic variable

$$u = \frac{z - a}{z + a}, \tag{10}$$

Bimeron we can find it numerically for a model with easy-plane anisotropy

$$\mathbf{m}\times\mathbf{f} = 0 \quad , \quad \mathbf{f} = \partial_\mu\partial_\mu\mathbf{m} - k^2 m_3\hat{e}_3 - 2\lambda(\hat{e}_\mu\cdot(\partial_\mu\mathbf{m}\times\mathbf{m})), \tag{11}$$

and the parameter of DMI value range $0.25 < \lambda < 0.5$ .

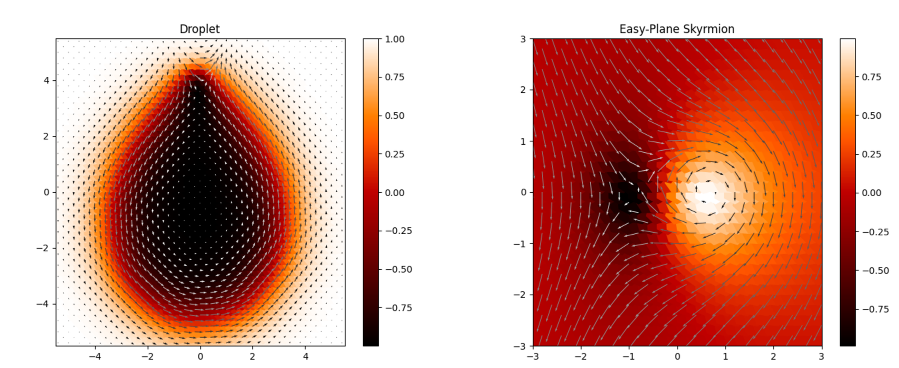

Fig. 5: The figure represent a Droplet(left) which find it numerically with parameter of $\lambda = 0.62$ and Bimeron(right) wich found numerically as a static solution for DM parameter $\lambda = 0.35$ and easy-plane anisotropy $k = 1.0$.

## Conclusions

We studied static solutions in 2-dimensions of micromagnetic phenomena in models on the sphere (heisenberg models). We considered and studied a model with exchange,anisotropy (easy-axis and easy-plane) and Dzyaloshinskii-Moriya (DM) interaction . Especially the Dzyaloshinskii-Moriya (DM) term gived to us the possibility to found and studied Skyrmionic textures solutions for realist models in micromagnetics. We started our computations for axially and non-axially symmetric configurations and we compared our numerical result with BP-Skyrmions . Furthermore we manage to find solutions with topological skyrmion number $Q = 0, \pm 1$ and energy boundaries for every single configuration .

## Acknowledgements

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## References

[1] A. A. Belavin and A. M. Polyakov. "Metastable States of 2-Dimensional Isotropic Ferromagnets". In: *JETP Lett.* 22 (1975), p. 245.

[2] T. H. R. Skyrme. In: *Nucl. Phys.* 31 (1962), p. 56.