UNIVERSITY OF CRETE
DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

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# THEORY OF MANIFOLDS
## (de Rham cohomology, characteristic classes, prequantization)

Lecturer: Konstantin Athanassopoulos

### I. De Rham cohomology

1. Differential forms
2. The exterior algebra of a smooth manifold
3. Orientable smooth manifolds
4. Integration on oriented manifolds
5. Homotopy invariance of de Rham cohomology
6. The degree of a smooth map and applications
7. The Mayer-Vietoris long exact sequence
8. Poincaré duality and applications
9. The Künneth formula for de Rham cohomology with compact supports
10. Generalised Mayer-Vietoris exact sequences
11. Presheaves, Čech cohomology and the Čech-de Rham theorem

### II. Vector bundles

1. Basic notions and examples
2. Whitney sums and inner products
3. The functors $K$ and $KO$
4. The topological classification of vector bundles
5. Constructions with vector bundles and their sections

### III. Geometry of characteristic classes

1. Connections and curvature on vector bundles
2. Induced connections
3. Invariant polynomials
4. Chern classes of complex vector bundles
5. The Pfaffian and the Euler class
6. The splitting principle for complex vector bundles
7. Pontryagin classes and applications

### IV. Prequantization

1. Topological classification of complex line bundles
2. Connections on complex line bundles
3. Hermitian connections
4. Integer cohomology classes and the theorem of Kostant

## Bibiography


1. K. Athanassopoulos, An introduction to smooth manifolds: de Rham cohomology and characteristic classes, Course Notes in electronic form in the website http://users.math.uoc.gr/ athanako/diff-manifolds-v2.pdf

2. R. Bott and L.W. Tu, Differential Forms in Algebraic Topology, Springer, 1982

3. B. Kostant, Quantization and unitary representations, Lectures in Modern Analysis and Applications III, Lecture Notes in Mathematics vol. 170, Springer, 1970

4. I. Madsen and J. Tornehave, From Calculus to Cohomology, Cambridge University Press, 1997

5. J. Milnor and J. Stasheff, Characteristic Classes, Princeton University Press, 1974

6. M. Postnikov, Lectures in Geometry: Semester III Smooth manifolds, Mir Publishers Moscow, 1987


## Evaluation of the students

The students will be evaluated from:
(a) obligatory exercises which they will be asked to solve throughout the semester,
(b) public presentations of supplementary subjects, and
(c) a final written examination.

The course will be taught on Tuesday and Thursday from 11:00 to 13:00 in room B 214.