Π.Α.Τ. $\begin{cases} y'(t) = f(t, y(t)) & a \le t \le b \\ y(a) = y_0 \checkmark \end{cases}$

- Άμεση Euler, Πεπλεγμένη Euler.

N, $h = \dfrac{b-a}{N}$, $t_n = a + nh$, $n = 0, 1, 2, \dots, N$

$$\int_{t_n}^{t_{n+1}} y'(t)\,dt = \int_{t_n}^{t_{n+1}} f(t, y(t))\,dt$$

$$y(t_{n+1}) - y(t_n) = \int_{t_n}^{t_{n+1}} f(t, y(t))\,dt$$

$\approx (t_{n+1} - t_n) \cdot f(t_n, y(t_n)) \Rightarrow$ Άμ. Euler

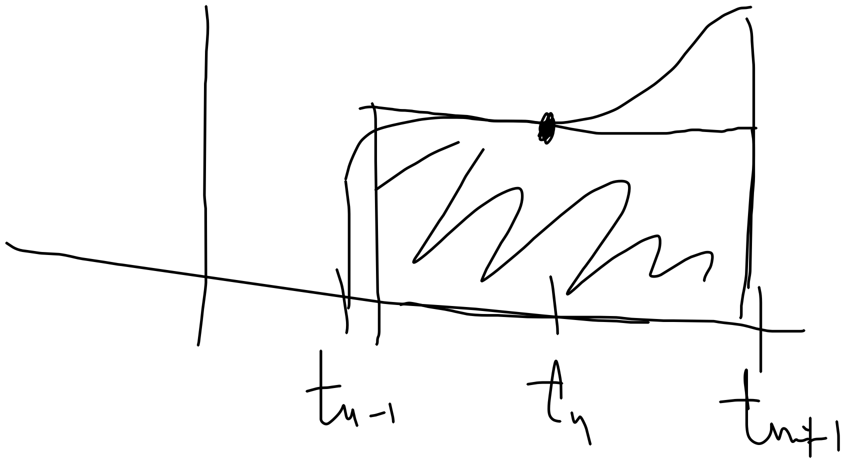
$\approx h\, f(t_{n+1}, y(t_{n+1})) \Rightarrow$ Πεπλ. Euler

$\approx \dfrac{h}{2}\left(f(t_n, y(t_n)) + f(t_{n+1}, y(t_{n+1}))\right) \Rightarrow$ Τραπέζιο

$[t_{n-1}, t_{n+1}]$

$$\int_{t_{n-1}}^{t_{n+1}} y'(t)\,dt = \int_{t_{n-1}}^{t_{n+1}} f(t, y(t))\,dt$$

$$y(t_{n+1}) - y(t_{n-1}) = \int_{t_{n-1}}^{t_{n+1}} f(t, y(t))\,dt \approx (t_{n+1} - t_{n-1}) f(t_n, y(t_n)) = \underline{\underline{2h\, f(t_n, y(t_n))}}$$



Μέθοδος του μέσου.

$y_0, y_1, \dots, y_N$.

$$y_{n+1} - y_{n-1} = 2h f(t_n, y_n), \quad n = 1, 2, 3, \dots, N-1.$$

y_0 ✓, y_1 (Αμ. Euler, Πεπ. Euler ή άλλη μέθοδο)

$$y_2 = y_0 + 2h f(t_1, y_1), \dots, y_N = \dots$$

Τοπικό σφάλμα διακριτοποίησης

$$\delta_n = y(t_{n+1}) - \left\{ y(t_{n-1}) + 2h \underbrace{f(t_n, y(t_n))}_{y'(t_n)} \right\}$$

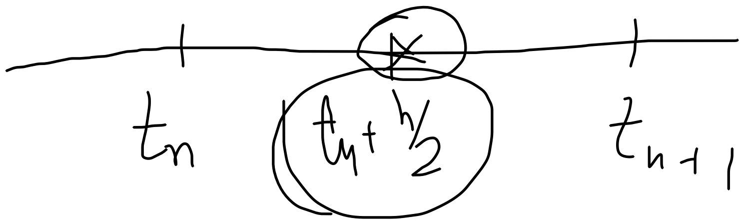
$$y(t_{n+1}) = y(t_{n-1}) + 2h\, y'(t_{n-1}) + \frac{(2h)^2}{2} y''(t_{n-1}) + \frac{(2h)^3}{3!} y'''(\xi_n), \quad \xi_n \in [t_{n-1}, t_{n+1}]$$

$$y'(t_n) = y'(t_{n-1}) + h\, y''(t_{n-1}) + \frac{h^2}{2} y'''(J_n), \qquad J_n \in [t_{n-1}, t_n]$$

$$\delta_n = \cancel{y(t_{n-1})} + \cancel{2h\, y'(t_{n-1})} + \cancel{2h^2 y''(t_{n-1})} + \frac{8}{6} h^3 y'''(\xi_n)$$

$$- \cancel{y(t_{n-1})} - 2h \left(\cancel{y'(t_{n-1})} + \cancel{h\, y''(t_{n-1})} + \frac{h^2}{2} y'''(J_n) \right) = h^3 \left(\frac{8}{6} y'''(\xi_n) - y'''(J_n) \right)$$

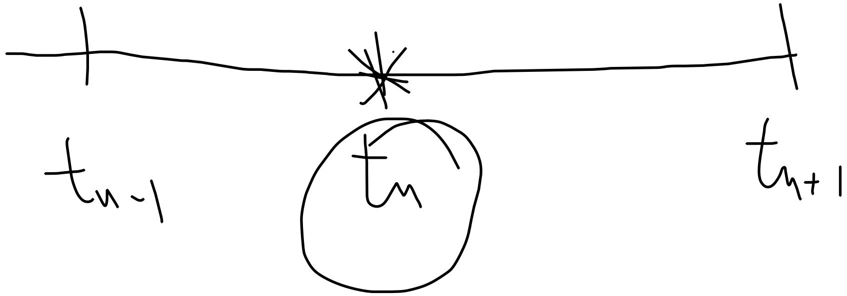
$$= O(h^3)$$



$$y_n \approx y(t_n)$$

$$y_{n+1} \approx y(t_{n+1})$$

$$? \approx y\left(t_n + \frac{h}{2}\right)$$



$$y(t_n) \approx y_n$$

$$[t_{n-1}, t_{n+1}] \longrightarrow y(t_{n+1}) - y(t_{n-1}) = \int_{t_{n-1}}^{t_{n+1}} f(t, y(t))\,dt$$

$$2h\, f(t_n, y(t_n))$$

$$\Downarrow$$

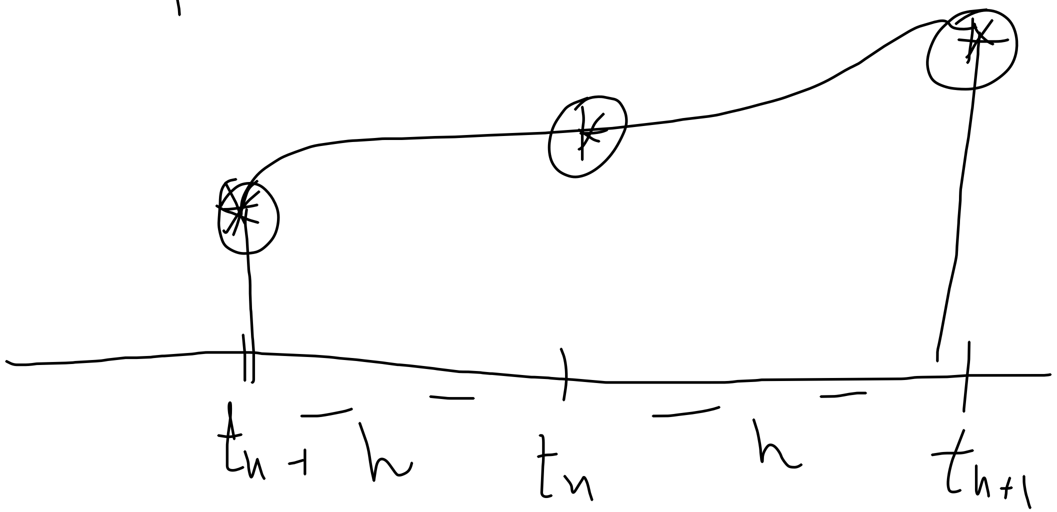
Μέθοδος του μέσου

Ολοκλήρωση τύπος Simpson

$$\frac{h}{3}\Big(f(t_{n-1}, y(t_{n-1})) + 4 f(t_n, y(t_n)) + f(t_{n+1}, y(t_{n+1})) \Big)$$

$$\Downarrow$$

Μέθοδος του Simpson.



Μέθοδος Simpson

$y_0, y_1, y_2, \ldots, y_N$

$$\underline{y_{n+1}} = y_{n-1} + \frac{h}{3}\left(f(t_{n-1}, y_{n-1}) + 4f(t_n, y_n) + f(t_{n+1}, \underline{y_{n+1}})\right)$$

Πεπλεγμένη μέθοδος

Να γνωρίζω y_0, $\underline{y_1}$

$$\begin{cases} y'(t) = f(t, y(t)) & a \leq t \leq b \\ y(a) = y_0 \end{cases}$$

h : βήμα διαμερισμού.

$$y(t+h) = y(t) + hy'(t) + \frac{h^2}{2} y''(t) + \frac{h^3}{3!} y'''(J), \quad J \in [t, t+h]$$

$$y'(t+h) = y'(t) + hy''(t) + \frac{h^2}{2} y'''(\xi), \quad \xi \in [t, t+h]$$

$$hy''(t) = y'(t+h) - y'(t) - \frac{h^2}{2} y'''(\xi)$$

$$y(t+h) = y(t) + hy'(t) + \frac{h}{2}\left(y'(t+h) - y'(t)\right) - \frac{h}{2} \cdot \frac{h^3}{2} y'''(\xi) + \frac{h^3}{3!} y'''(J)$$

$$y(t+h) = y(t) + \frac{h}{2}\left(y'(t+h) + y'(t)\right) + O(h^3)$$

$$\Downarrow$$

Τραπεζίου

$$y_{n+1} = y_n + \frac{h}{2}\left(f(t_n, y_n) + f(t_{n+1}, y_{n+1})\right)$$

$$y'(t-h) = y'(t) - hy''(t) + \frac{h^2}{2}y'''(\tilde{\jmath}) \quad , \quad \tilde{\jmath} \in [t-h, t]$$

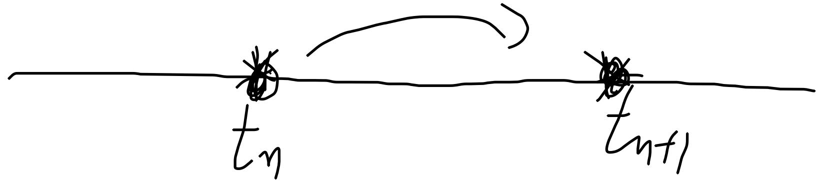
$$hy''(t) = y'(t) - y'(t-h) + \frac{h^2}{2}y'''(\tilde{\jmath})$$

$$y(t+h) = y(t) + hy'(t) + \frac{h}{2}\left(y'(t) - y'(t-h)\right) + \frac{h}{2}\frac{h^2}{2}y'''(\tilde{\jmath}) + \frac{h^3}{3!}y'''(\jmath)$$

$$= y(t) + \frac{3}{2}hy'(t) - \frac{h}{2}y'(t-h) + O(h^3)$$

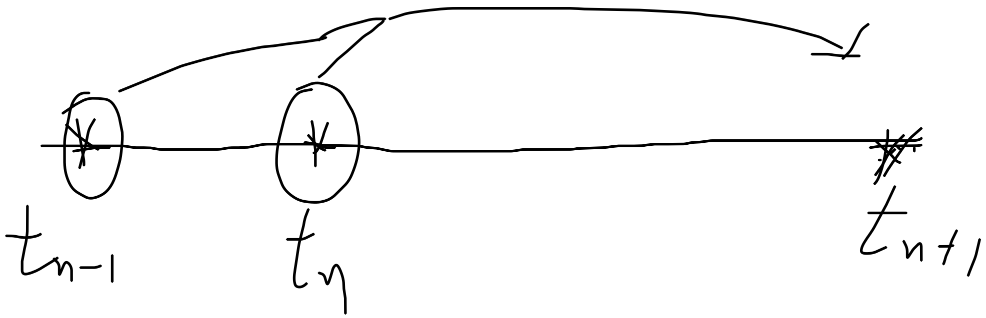
$$\Downarrow$$

$$y_{n+1} = y_n + h\left(\frac{3}{2}f(t_n, y_n) - \frac{1}{2}f(t_{n-1}, y_{n-1})\right)$$ Adams-Bashforth (2)



Άμεση Euler
Πεπλεγ. Euler
Τραπεζίου
TS(2)

Μονοβηματικές μέθοδοι



Μέσου
Simpson
AB(2)

2-βηματικές μέθοδοι.

⋮

Πολυβηματικές μέθοδοι.