Μέθοδο του Euler

Π.Α.Τ

$$\begin{cases} y'(t) = f(t, y(t)), & t > t_0 \\ y(t_0) = \eta_0 \end{cases}$$

Υποθέτουμε να έχουν μοναδική λύση $[t_0, t_f]$

$$f : [a,b] \times \mathbb{R} \to \mathbb{R}.$$

ή $$f : [a,b] \times \mathbb{R}^m \to \mathbb{R}^m \qquad m > 1$$

h: μικρό θετικό "βήμα", "μέγεθος της διαμέρισης"

$t_0, \; t_1 = t_0 + h, \; t_2 = t_1 + h = t_0 + 2h, \; \ldots, \; t_f = t_0 + N \cdot h$

$[t_0, t_f] = [t_0, t_1] \cup [t_1, t_2] \cup \ldots \cup [t_{N-1}, t_N], \quad \underline{\underline{t_N = t_f}}$

Προσεγγίσεις της $y(t_0), y(t_1), \ldots, y(t_N)$

αριθμοί $y_0, y_1, \ldots, y_N$

Δεν προσπαθούμε να προσεγγίσουμε την καμπύλη

$\{ y(t) : t_0 \le t \le t_f \}$

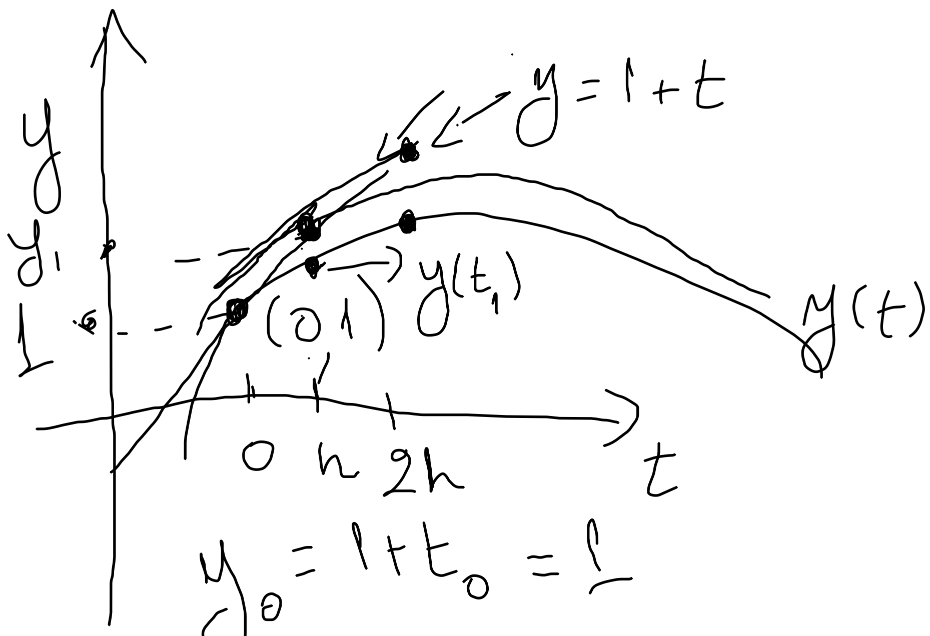
Π.Α.Τ. $\begin{cases} y'(t) = (1-2t)\, y(t), & t \geq 0, \\ y(0) = 1 \end{cases}$

Έχει μοναδική λύση στο $[0, 0.9]$

Η ακριβής $y(t) = e^{\left(\frac{1}{4} - \left(\frac{1}{2} - t\right)^2\right)}$

$t = 0$, $y(0) = 1$, $y'(0) = (1 - 2 \cdot 0)\, y(0) = y(0) = 1$

$y'(0) = 1 \Rightarrow$ κλίση της εφαπτομένης στο γράφημα $\{(t, y(t))\}$



$t_0 = 0$

$t_1 = t_0 + h = h$

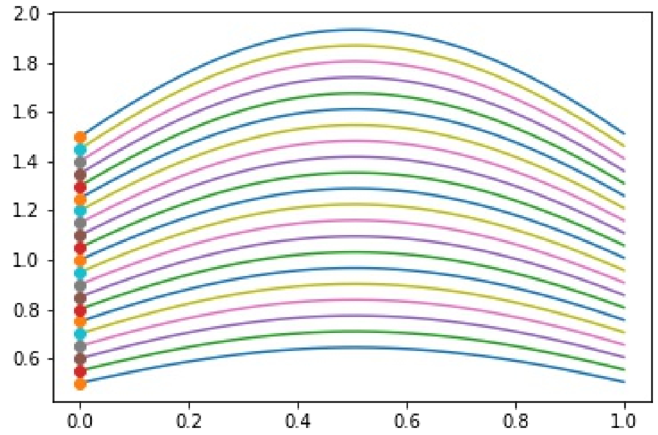
$y_0 = 1 + t_0 = 1$

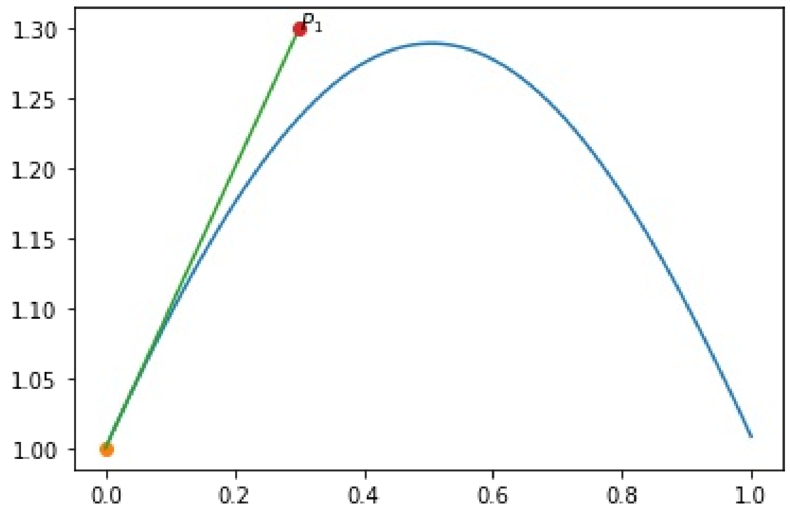
$y_1 = 1 + t_1 = 1 + h \approx y(t_1)$

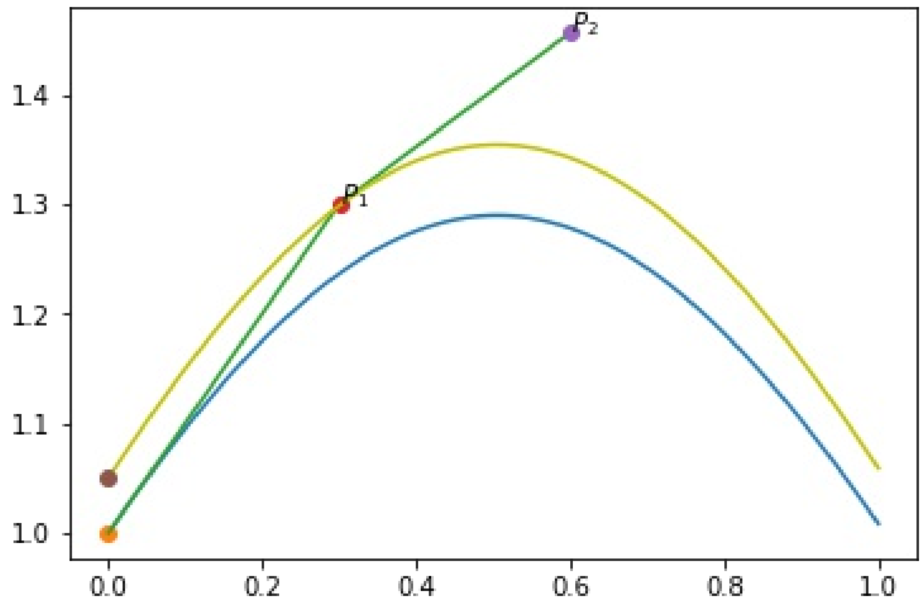
$h = 0.3$, $y_1 = 1 + 0.3 = 1.3 \approx y(0.3)$

$f(t_1, y_1) = (1 - 2t_1) \cdot y_1 = (1 - 2(0.3))\, 1.3 = \ldots$
$= 0.52$

$y'(t_1) = f(t_1, y(t_1)) \approx f(t_1, y_1)$


2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.0
0.2
0.4
0.6
0.8
1.0


P_1
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.0
0.2
0.4
0.6
0.8
1.0



P_2
P_1
1.4
1.3
1.2
1.1
1.0
0.0
0.2
0.4
0.6
0.8
1.0

$$y_2 \simeq y(t_2) \quad , \; t_2 = t_1 + h = t_0 + 2h$$

$$\frac{\tilde{y} - y_1}{t - t_1} = f(t_1, y_1)$$

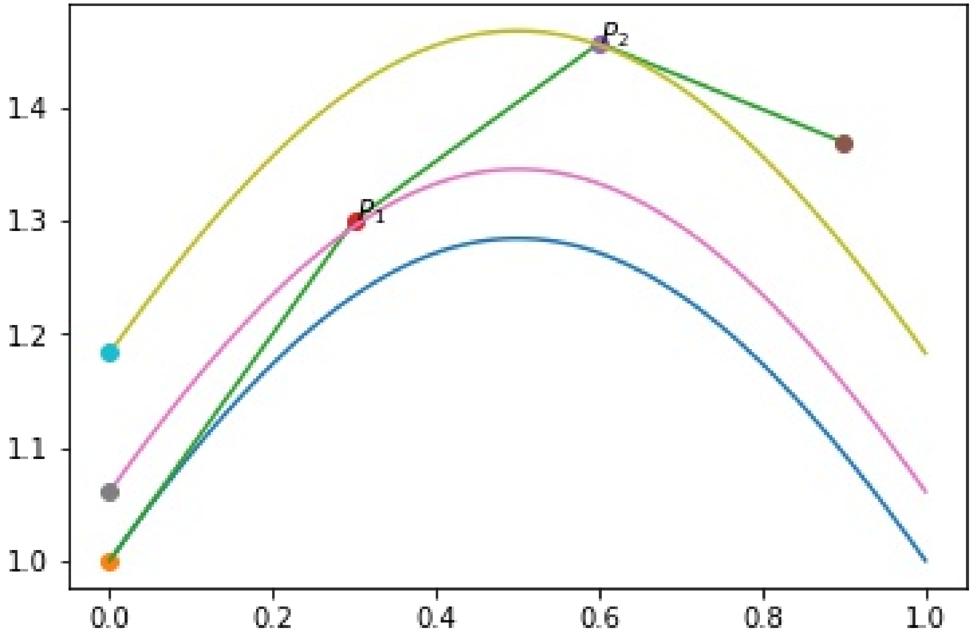
$$\frac{y_2 - y_1}{t_2 - t_1} = f(t_1, y_1)$$

P_2 σημείο

ή $\quad y_2 = y_1 + h f(t_1, y_1) \qquad P_2 = (t_2, y_2)$

$$y_2 = y_1 + h \cdot f(t_1, y_1) \qquad , \quad h = 0.3$$

$$= 1.3 + (0.3) \cdot (0.52) = \ldots = 1.456$$



$$y_3 \approx y(t_3) \quad , t_3 = t_2 + h = t_0 + 3 \cdot h$$

$$\frac{\tilde{y} - y_2}{t - t_2} = f(t_2, y_2)$$

$$f(t_2, y_2) \simeq -0.2912$$

$$y_3 = y_2 + h \cdot f(t_2, y_2) = \ldots \simeq 1.3686$$

$$P_1 = (t_1, y_1) \quad , \quad y_1 \approx \underline{y(t_1)} = y(0.3)$$

$$P_2 = (t_2, y_2), \quad y_2 \approx y(t_2) = y(0.6)$$

$$P_3 = (t_3, y_3), \quad y_3 \approx y(t_3) = y(0.9)$$

$$\underline{h = 0.3}$$

$$t_0 = 0, \quad t_1 = 0.3 \quad , \quad t_2 = 0.6 \quad , \quad t_3 = 0.9$$

$$\underline{h = 0.15}$$

$t_0 = 0$, $t_1 = 0.15$, $t_2 = 0.3$, $t_3, \ldots, t_6 = 0.9$

$y_0, y_1, y_2, \ldots, y_6 \approx y(0.9)$

$y_2 \approx y(0.3)$

$y_4 \approx y(0.6)$

$y_6 \approx y(0.9)$