

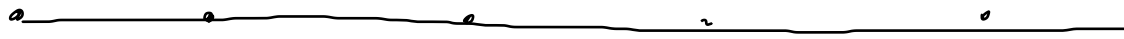
Παράδειγμα :

$$\begin{cases} y'(t) = (1-2t)y(t), & t > 0 \\ y(0) = 1. \end{cases}$$

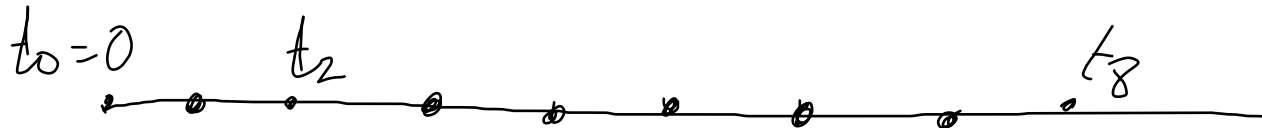
$$y(t) = e^{\left(\frac{1}{4} - (t - \frac{1}{2})^2\right)}, \quad h = 0.3, \quad h = 0.15, \quad t = 1.2$$

- Μέση Euler
- Πεντακυσμική Euler
- TS(2)

$$h = 0.3 \quad t_0 = 0 \quad t_1 = 0.3 \quad t_2 = 0.6 \quad t_3 = 0.9 \quad t_4 = 1.2$$



$$h = 0.15$$



$t_0 \rightarrow t_1$   
 $y_0 = 1 \Rightarrow$  - Approx Euler  
 $y_1 = y_0 + h \cdot f(t_0, y_0) = 1 + (0.3) \cdot (1 - 2 \cdot 0) \cdot 1 = 1 + 0.3 = 1.3$

Παράγωγοι Euler  
 $y_0 = 1$   
 $y_1 = y_0 + h f(t_1, y_1) = y_0 + h \cdot (1 - 2t_1)y_1$   
 $y_1 - h(1 - 2t_1)y_1 = y_0 \quad \Rightarrow \quad (1 - h(1 - 2t_1))y_1 = y_0$   
 $y_1 = \frac{1}{1 - h(1 - 2t_1)} y_0$   
 $y_{n+1} = \frac{1}{1 - h(1 - 2t_{n+1})} \cdot y_n$

Prüfung Euler

$$y_1 = \frac{1}{1 - (0.3)(1 - 2(0.3))} \cdot 1$$

$$1 - (0.3)(1 - 2(0.3)) = 1 - 0.3(0.4) = 0.88$$

$$y_1 = \frac{1}{0.88} = \frac{100}{88} \approx 1.1364$$

$$TS(2) \quad y_{n+1} = y_n + h f(t_n, y_n) + \frac{h^2}{2} g(t_n, y_n)$$

$$g(t, y(t)) = \frac{d}{dt} f(t, y(t)) = y''(t)$$

$$\begin{aligned} &= \frac{d}{dt} \left( (1-2t)y(t) \right) = -2y(t) + (1-2t)y'(t) \\ &= \left[ (1-2t)^2 - 2 \right] y'(t) \end{aligned}$$

$$g(t_n, y_n) = [(1 - 2t_n)^2 - 2] y_n$$

$$y_{n+1} = y_n + h(1 - 2t_n)y_n + \frac{h^2}{2} [(1 - 2t_n)^2 - 2] y_n, \quad n = 0, 1, 2, \dots$$

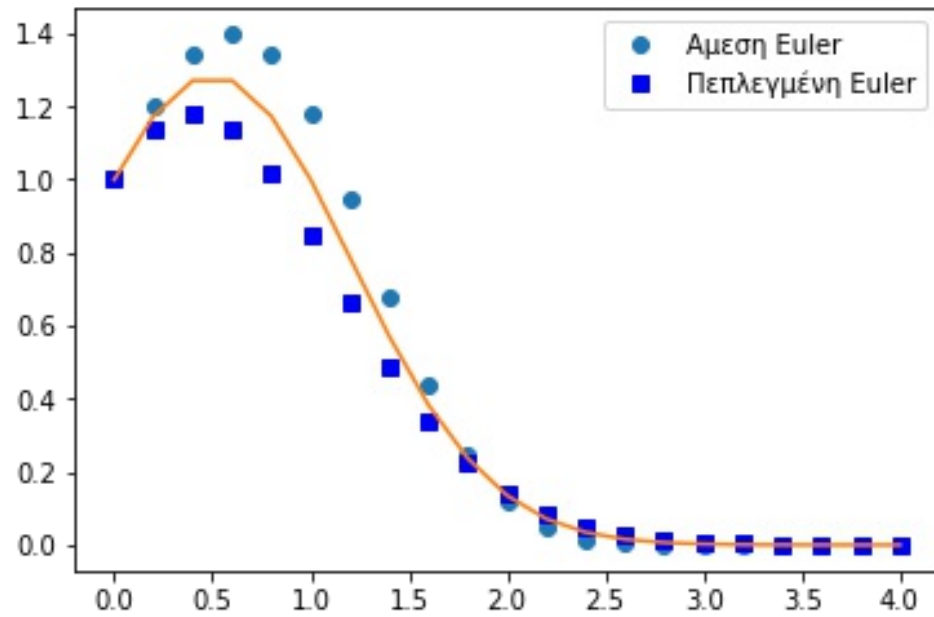
$$y_1 = 1 + (0.3)(1 - 2 \cdot (0.3)) \cdot 1 + \frac{(0.3)^2}{2} [(1 - 2 \cdot 0.3)^2 - 2] \cdot 1$$

$$\approx 1.2550$$

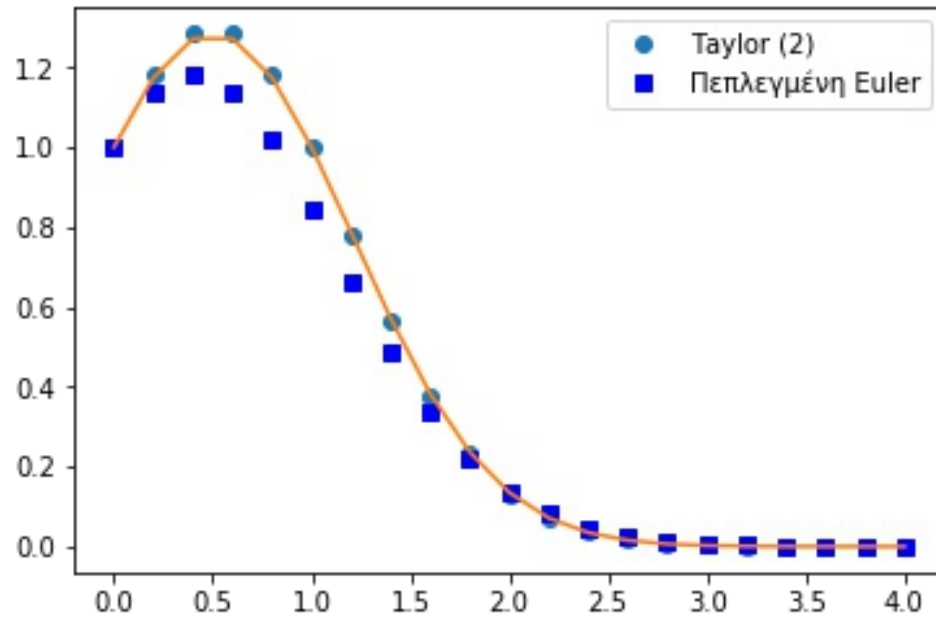
$h$	Euler (Alucon) $t = 1.2$	TS(2)	Euler (Alucon) Όγκια θραύμα.	TS(2)
0.3	1.0402	0.7748	0.2535	0.0118
0.15	0.9014	0.7836	0.1148	0.0031

TS(2) Τονικά θραύμα  $\delta_n \approx O(h^3) \Rightarrow$   
 Όγκια θραύμα  $O(h^2)$

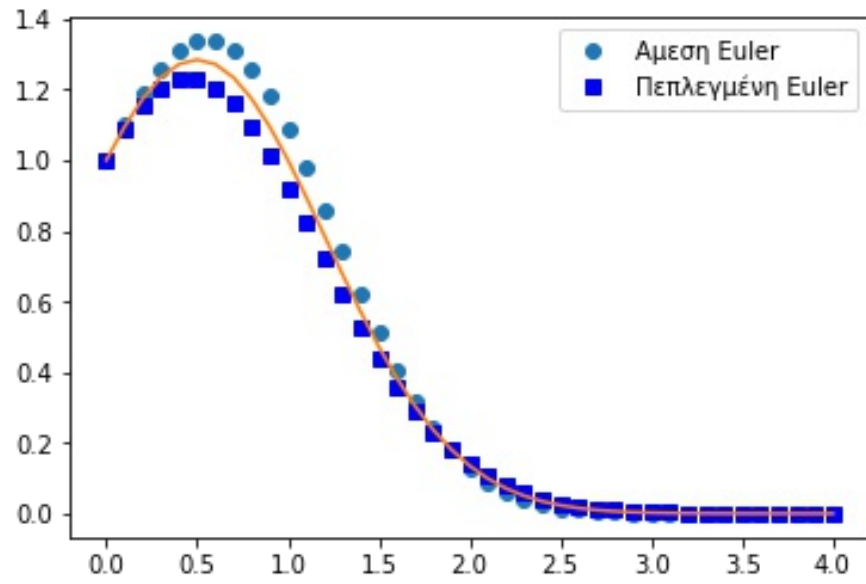
$h$	Όγκια θραύμα TS(2) / $h^2$
0.3	0.131
0.15	0.138



$$h = 0.2$$

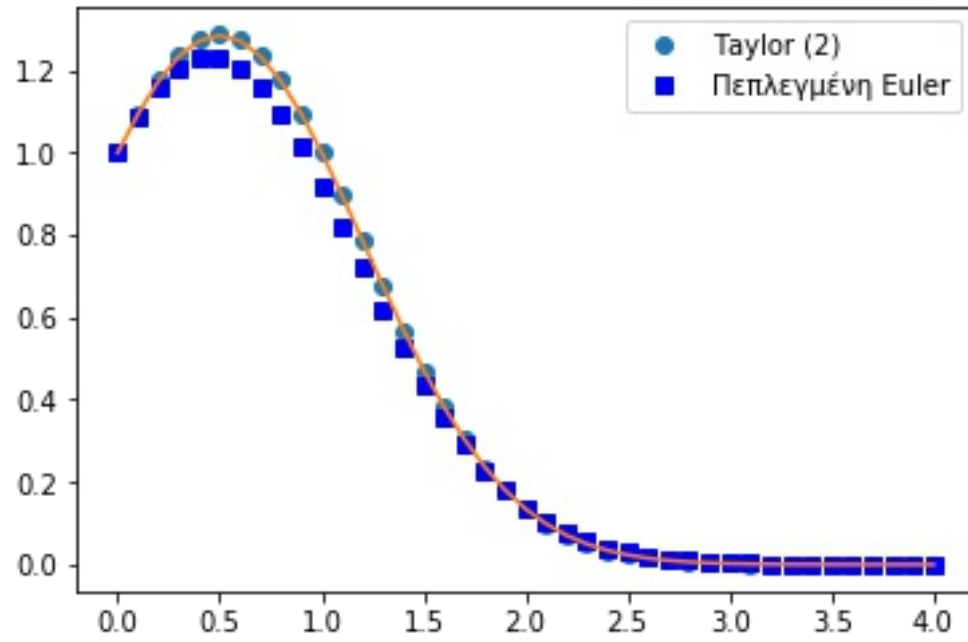


$$h = 0.2$$



$$h = 0.1$$





$$h = 0.1$$