

# A SMALL REMARK ON THE DERIVATION OF THE PLATEAU ANGLE CONDITIONS FOR THE VECTOR-VALUED ALLEN-CAHN EQUATION

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ABSTRACT. We clarify a point in [1], concerning the derivation of the Plateau angle conditions for the vector-valued Allen-Cahn equation.

The Plateau angle conditions for the vector-valued Allen-Cahn equation have recently been rigorously derived in the interesting preprint [1]. However, it is not clear to the author how the estimates (30) and (31) in [1] are used for showing that the terms involving integrals of  $T_{ij}(v)\frac{y_j}{y_2}$  with  $j \neq 2$  tend to zero as  $R \rightarrow \infty$  in (27) of the latter reference. In this note we will clarify this point and verify that the aforementioned terms indeed tend to zero. To this end, instead of bounding the absolute value of these integrals (as was done in [1]), we will exploit a cancelation property that seems to have been left unnoticed in [1].

Throughout this short note we will follow exactly the notation of [1].

In what follows, we will show that

$$\frac{1}{R} \int_{-R \sin \psi_1}^{R \sin \psi_1} \int_{-R \cos \psi_2}^{R \cos \psi_2} T_{33}(v) \frac{y_3}{y_2} dy_3 dy_1 \rightarrow 0 \quad \text{as } R \rightarrow \infty$$

(the remaining terms can be handled analogously).

We note that

$$T_{33}(v) = \frac{1}{2} (|v_{,3}|^2 - |v_{,1}|^2 - |v_{,2}|^2 - 2W(v)) \quad (v_{,i} = \partial v / \partial y_i),$$

and  $y_2 = \sqrt{R^2 - y_1^2 - y_3^2}$ . Moreover, we point out that  $R \sin \psi_1(R) \rightarrow \infty$  and  $\psi_2(R) \rightarrow 0$  as  $R \rightarrow \infty$ . By virtue of Lebesgue's dominated convergence theorem, whose assumptions have been verified in [1], it suffices to show that, given  $y_1 \in \mathbb{R}$ , we have

$$\frac{1}{R} \int_{-R \cos \psi_2}^{R \cos \psi_2} T_{33}(v) \frac{y_3}{y_2} dy_3 \rightarrow 0 \quad \text{as } R \rightarrow \infty.$$

Letting  $\tilde{y}_3 = R^{-1}y_3$ , with a slight abuse of notation, we can write the above integral as

$$\frac{1}{2} \int_{-\cos \psi_2}^{\cos \psi_2} \left( \frac{1}{R^2} |v_{,3}|^2 - |v_{,1}|^2 - |v_{,2}|^2 - 2W(v) \right) \frac{R\tilde{y}_3}{\sqrt{R^2 - y_1^2 - R^2\tilde{y}_3^2}} d\tilde{y}_3.$$

Now, by Hypothesis 2 in [1], and a trivial application of Lebesgue's dominated convergence theorem, we infer that, as  $R \rightarrow \infty$ , the above integral converges to

$$-\left( \frac{1}{2} \left| \dot{U}_{12}(y_1) \right|^2 + W(U_{12}(y_1)) \right) \int_{-1}^1 \frac{\tilde{y}_3}{\sqrt{1 - \tilde{y}_3^2}} d\tilde{y}_3 = 0,$$

as desired.

## REFERENCES

- [1] N. D. ALIKAKOS, P. ANTONOPOULOS, and A. DAMIALIS, Plateau angle conditions for the vector-valued Allen-Cahn equation, arXiv:1210.0231

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