9th Panhellenic Geometry Conference

29-31 of May 2009, Anogia Academic Village

Abstracts

Department of Mathematics, University of Crete

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Affine crystallographic and properly discontinuous groups

There is a long standing conjecture of Auslander (1964) which states that every affine crystallographic group has a solvable subgroup of finite index. Milnor (1977) asked if, more generally, every properly discontinuous affine group has a solvable subgroup of finite index. This more general conjecture was disproved by Margulis (1983). The Auslander conjecture has been proved in many cases, but is wide open in dimension at least 7.

I will explain the geometric and algebraic relevance of the notions and conjectures and will give an idea of the methods involved in the constructions and proofs in our joint work with Margulis and Soifer. We use dynamics of linear and affine maps. The connection with the Tits alternative and proximal maps will be pointed out.

Konstantin Athanassopoulos

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An algorithm for the equipartition of curves

In 1940 J.F. Pal posed the following problem: Let (X, ρ) be a metric space and let $c : [0, 1] \to X$ be a continuous curve such that $c(0) \neq c(1)$. Is there a partition $0 < t_1 < \cdots < t_n < 1$ such that $\rho(c(t_{i-1}), c(t_i)) = \rho(c(t_i), c(t_{i+1}))$, where $t_0 = 0$ and $t_{n+1} = 1$? K. Urbanik (1954) and M.A. Lopez and S. Reisner (2007) have given proofs of the existence of a solution using the Brouwer fixed point theorem. Our approach is based on the topological study of the zero level set of a certain continuous function defined on the n-dimensional simplex and leads to an approximate algorithm for finding all solutions with high accuracy. This is joint work with C. Panagiotakis and G. Tziritas (Computer Science Department).

Iakovos Androulidakis

Centro de Matematica da Universidade do Porto

Pseudodifferential calculus for singular foliations

We discuss the definition of pseudodifferential operators (bounded and unbounded) along the leaves of a singular foliation. The analytic index of such operators takes values in the K-theory of the C^*-algebra of a singular foliation, which is constructed by the method of bi-submersions. This is joint work with G. Skandalis.

Andreas Arvanitogiorgos

Department of Mathematics, University of Patras

Einstein metrics on compact Lie groups which are not naturally reductive

A Riemannian manifold (M, g) is called Einstein if the Ricci tensor $Ric(g)$ of the metric g satisfies $Ric(g) = eg$ for some constant e. For the case of compact Lie groups, left-invariant Einstein metrics have not been studied extensively. Even in low dimensional examples such as $SU(3)$ and $SU(2) \times SU(2)$, the number of left-invariant Einstein metrics is still unknown. The only complete work is the one by J.E. D'Atri and W. Ziller Naturally reductive metrics and Einstein metrics on compact Lie groups, (Mem. Amer. Math. Soc. 18 (215) (1979)), in which they obtained a large number of left-invariant Einstein metrics which are naturally reductive.

The problem of finding non-naturally reductive left-invariant Einstein metrics on compact Lie groups seems to be harder, and in fact this is stressed by the above authors. In 1996 K. Mori obtained non-naturally reductive Einstein metrics on the Lie group $SU(n)$ for $n > 5$, by using a method of Riemannian submersions (Left invariant Einstein metrics on $SU(N)$ that are not naturally reductive, Master Thesis (in Japanese) Osaka University 1994, English translation Osaka University RPM 96 – 10 (preprint series) 1996).

In the present work we prove existence of non-naturally reductive Einstein metrics on the compact simple Lie groups $SO(n)$ $(n > 10)$, $Sp(n)$ $(n > 2)$, $E6$, $E7$, and $E8$.

George Kaimakamis

Military Academy

Lorentz hypersurfaces in E_1^4 satisfying $\Delta H = aH$

A hypersurface M_1^3 in the four-dimensional pseudo-Euclidean space E_1^4 is called a Lorentz hypersurface if its normal vector is space-like. We show that if the mean curvature vector field of M_1^3 satisfies the equation $\Delta H = aH$ (a constant), then M_1^3 has constant mean curvature. This equation is a natural generalization of the biharmonic submanifold equation $\Delta H = 0$.

Themis Koufogiorgos

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3-dimensional contact metric manifolds with $Trl = c$ (const.)

In contact geometry, the Jacobi operator $l = R(., x)x$ plays a fundamental role. The class of contact metric manifolds with $Trl = c$ (const.) is particularly large, since, among others it includes the well known classes of Sasakian manifolds and (κ, μ)-contact manifolds. In the present work, a complete local description of 3-dimensional contact metric manifolds with $Trl = c$, is given. As a consequence of it, several new examples of such manifolds are obtained.

Michael Markellos and Vassilis Papantoniou

Department of Mathematics, University of Patras

Ricci solitons and contact manifolds

In [3], the authors introduced the notion of $(\kappa; \mu, \nu)$-contact metric manifolds in terms of a specifc curvature condition, where κ, μ, ν are smooth functions. Especially, they proved that a contact metric 3-manifold $[M; (\eta; \xi; \phi; g)]$ is a $(\kappa; \mu, \nu)$-contact metric manifold on an everywhere open and dense subset of M if and only if the characteristic vector field ξ is an harmonic vector field. Recently, Cho ([1]) proved that an odd-dimensional sphere does not admit a Ricci soliton whose potential vector field is the Reeb vector field. More generally, in [2] the authors proved that if the metric of a non-Sasakian $(\kappa; \mu)$-contact manifold $[M^{2n+1}; (\eta; \xi; \phi; g)]$ is a gradient Ricci soliton, then M is locally flat in dimension three, and locally isometric to $E^{n+1} \times S^n(4)$ in higher dimensions. Extending the above results, we prove that every 3-dimensional $(\kappa; \mu, \nu)$-contact metric manifold $[M; (\eta; \xi; \phi; g)]$ with κ non-constant smooth function on M, $\kappa < 1$ everywhere on M and the function ν or μ constant on M, does not admit a gradient Ricci soliton.

In the sequel, we concentrate on the unit tangent sphere bundle T_1M of a complete Riemannian manifold $(M^n; g)$ $(n \geq 2)$ equipped with the standard contact metric structure $(\eta; \xi; \phi; \bar{g})$, where the metric $\bar{g}$ is homothetic to the Sasaki metric g_S and ξ is the geodesic flow of M^n. More precisely, we prove that the metric $\bar{g}$ is not a Ricci soliton with potential vector field the geodesic flow ξ. Moreover, we construct examples of potential vector fields with respect to the metric $\bar{g}$ of $T_1\mathbb{R}^3$.

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Evangellia Moutafi

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Some Classes of 3-dimensional Pseudo-symmetric Contact Metric Manifolds

We classify
a) the class of 3-dimensional pseudo-symmetric contact metric manifolds of constant type satisfying $\nabla_\xi \tau = 2a\tau\phi$, where a is a smooth function,
b) the class of 3-dimensional pseudo-symmetric contact metric manifolds satisfying $Q\xi = \rho\xi$, where ρ is a smooth function constant in the characteristic direction ξ,
c) the class of pseudo-symmetric complete (κ, μ, ν)-contact metric 3-manifolds of type constant in the characteristic direction ξ and
d) the class of 3-dimensional pseudo-symmetric contact metric manifolds with harmonic curvature and Trl constant in the direction ξ.

Ben McReynolds

Department of Mathematics, University of Chicago.

Controlling manifold covers

I will discuss a generalization of Selberg's lemma and some geometric applications. Most notable is the resolution of Nimershiem's conjecture on the geometry of cusp cross sections of finite volume, complete, hyperbolic n-manifolds.

John Papadoperakis

General Department, Agricultural University

Hyperbolic spaces and Mostow's uniqueness theorem

We study complete hyperbolic 3-manifolds without boundary, which are constructed by gluing hyperbolic ideal tetrahedra. The gluing is done along the faces of the tetrahedra. The resulting manifolds have constant curvature -1, except perhaps on some axis, where the angle is not 2π. An axis is a geodesic which comes from an edge of a tetrahedron. These manifolds are in general Gromov hyperbolic. Fixing the way of gluing and varying the angles of the tetrahedra we study the moduli space under isotopy. We examine the possibility of parametrizing this Teichmüller space by the angles of the axis and we relate it to Mostow's uniqueness theorem. We give typical examples.

Despina Papadopoulou

Department of Mathematics, University of Thessaloniki

Existence of pencils of lines with common invariants

In E^3 we study existence and construction of pencils of lines with common invariants such as curvature and mean curvature.

Constantine Petoumenos

Department of Mathematics, University of Patras

Biharmonic hypersurfaces of type M_2^3 in E_2^4.

A submanifold M_r^n of a pseudo-Euclidean space E_s^m is said to have harmonic mean curvature vector field if $\triangle \overrightarrow{H} = \overrightarrow{0}$, where $\overrightarrow{H}$ denotes the mean curvature vector field and $\triangle$ the Laplacian of M_r^n with respect to the induced pseudo-Riemannian metric. Submanifolds with harmonic mean curvature vector field are also known as biharmonic submanifolds. In the present talk, by using all possible canonical forms of the shape operator of the pseudo-Euclidean hypersurface M_2^3 in E_2^4 we prove that every nondegenerate biharmonic hypersurface M_2^3 in E_2^4, is minimal.

John Platis

Department of Mathematics, University of Crete

The $PU(2,1)$ configuration space of four points in S^3 and the cross-ratio variety

In the classical case, the configuration space of four points in S^2 is biholomorphic to $\mathbb{C} \setminus \{0,1\}$. Things are much more complicated in the configuration space of four points in S^3. We present the latest results on the subject and pose some open questions.

Gregory Soifer

Department of Mathematics and Statistics, Bar-Ilan University

Affine crystallographic semigroups acting on a hyperbolic space

Let $G_n = \mathrm{Aff}(\mathbb{R}^n)$ be the group of affine transformations of $\mathbb{R}^n$. The group G_n is the semidirect product $GL_n(\mathbb{R}) \ltimes \mathbb{R}^n$ where $\mathbb{R}^n$ is identified with its group of translations. A semigroup S of G_n is said to act *properly discontinuous* on $\mathbb{R}^n$ if for every compact subset K of $\mathbb{R}^n$ the set $\{g \in S : gK \cap K \neq \emptyset\}$ is finite. If a discrete group consists of isometries, then it acts properly on $\mathbb{R}^n$. But this is not true for an arbitrary discrete subgroup of G_n, e.g. for an infinite discrete subgroup of $GL_n(\mathbb{R})$. A semigroup S of G_n is called *crystallographic* if S acts properly discontinuous on $\mathbb{R}^n$ and there exists a compact subset K_0 of $\mathbb{R}^n$ such that $\bigcup_{s\in S} sK_0 = \mathbb{R}^n$.

If the signature of a non degenerate quadratic form B on $\mathbb{R}^n$ is $(n-1,1)$, then the form B is called *hyperbolic*.

The motivating question here is the following:

Question. (H.Abels, G.Margulis, G.Soifer). *Is the Zariski closure of a crystallographic affine semigroup leaving a hyperbolic form invariant a virtually solvable group ?*

Our interest in these questions came from our work on the Auslander conjecture. The goal of my talk is to explain a main ideas of our proof of the following

Main Theorem. *Let S be a crystallographic semigroup in Aff$(\mathbb{R}^n)$, $n \leq 3$. Then the Zariski closure of S is a virtually solvable group.*

George Tsapogas

Department of Mathematics, University of the Aegean

On the Mapping class group of a handlebody

For a surface Σ, the complex of curves introduced by Harvey is a simplicial complex on which the mapping class group $MCG(\Sigma)$ acts simplicially. This action is widely used for the study of $MCG(\Sigma)$. For a handlebody H, we introduce a complex of incompressible surfaces (in H) which naturally contains the complex of curves for the boundary surface of H. A bijection between the $MCG(H)$ and the group of automorphisms of the complex of incompressible surfaces is established by induction, where the important initial case of a genus 2 handlebody is studied by analyzing the links of the vertices from the graph theoretical point of view.

Charalambos Tsichlias

Department of Mathematics, University of Ioannina

3-dimensional contact metric manifolds with $\nabla_\xi \tau = 0$

In contact geometry, the tensor $\tau = \mathcal{L}_\xi g$ plays a fundamental role. In the present work, a complete local description of 3-dimensional contact metric manifolds with $\nabla_\xi \tau = 0$ is given. Moreover, we obtain several new examples of this class of manifolds.

Anestis Fotiadis

Department of Mathematics and Statistics, University of Cyprus

Dirichlet's problem for harmonic maps of symmetric spaces

We generalize the notion of harmonic maps to include maps between manifolds and consider the corresponding Dirichlet's problem for harmonic maps of symmetric spaces.

Ioannis Chrysikos

Department of Mathematics, University of Patras

Homogeneous Einstein metrics on generalized flag manifolds

A Riemannian manifold (M, g) is called Einstein if the Ricci tensor Ric_g is a constant multiple of the metric g, i.e. $Ric_g = \Lambda \cdot g$, for some $\Lambda \in \mathbb{R}$. If M is compact, Einstein metrics of volume one can be characterized variationally as the critical points of the scalar curvature functional $T(g) = \int_M S_g dvol_g$ on the space $\mathcal{M}_1$ of Riemannian metrics of volume one. At the present time no general existence results of Einstein metrics are known. Important progress has been made when M is a homogenous space [3]. In this case we are mainly interested in G-invariant Einstein metrics on M. Such a metric is precisely a critical point of T restricted on $\mathcal{M}_1^G$, the set of G-invariant metrics of volume one [1].

An important class of compact homogeneous spaces consists of generalized flag manifolds. These are the adjoint orbits of a compact connected semisimple Lie group G and are complex homogeneous spaces of the form G/K, where K is the centralizer of a torus in G. In [2], the Einstein equation explicity solved for all flag manifolds with three isotropy summands. We solve explicity the Einstein equation for all flag manifolds with two and four isotropy summands. First we classify these spaces by using the notion of t-roots. Next we compute the Ricci tensor for a G-invariant metric on $M = G/K$ and we present new G-invariant Einstein metrics on M. For the case which corresponds to two isotropy summands we also exam the nature of the new Einstein metrics, as critical points of the restriction $T|_{\mathcal{M}_1^G}$.

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