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ABSTRACTS

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Case Studies in the Shortcomings of Maple in Teaching Undergraduate Mathematics.

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In this paper, we will investigate some of the algorithmic inadequacies and limitations of MAPLE as well as the common misuses of the software when used as a tool in teaching undergraduate mathematics. We will present examples for which MAPLE produces misleading or inaccurate results. We will also refer to situations where MAPLE gives accurate, but incomplete, results which are misused or misinterpreted by novice users of the software, specifically the undergraduate students. The authors have over ten years of experience in using MAPLE as a teaching tool and some examples presented here are based on those classroom experiences. Other cases have been reported by our students, by our colleagues and in various newsgroups devoted to discussions on Computer Algebra Systems (CAS). Many of the previously reported software bugs, observed in the earlier versions of MAPLE, are now corrected in the most recent releases of the software. Therefore, we have used only the latest version of MAPLE in the examples presented here. For the sake of brevity, we have limited our discussions to the topics which are ordinarily covered in the first two years of a typical undergraduate mathematics curriculum such as limits, single and multivariable integration, series, optimization, ordinary differential equations and floating point arithmetic. We have also tried to limit our case studies to the most common features of MAPLE, specifically those features that are widely used by the undergraduate students who are new to MAPLE.

Introducing Experiments into a First Course in Calculus

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It has become widely accepted that mathematical software can contribute significantly to the learning and understanding of Mathematics. In particular, the visualization capabilities of software packages and Computer Algebra Systems, students can explore function behavior and phenomena that would be impossible without the use of computers. A Mathematics instructor has a wide choice of software tools to consider for use in undergraduate courses. Yet, the problem remains how to construct interesting problems that would challenge the student and where the technology is an important tool assisting in the exploration, yet allowing one to reflect, analyze, modify one's thinking until the appropriate conclusion is reached. In this paper, I give examples of challenging problems within the conceptual reach and understanding of Calculus students. These problems were given in the fall 2001 to students taking a first course in Calculus. A characteristic of these examples is that without technology it may be difficult for students to do the analysis and to obtain the answer, yet the technology and its visualization capabilities provide the student with a mechanism for experimentation and testing, allowing them to modify their hypothesis and their thinking to lead them to a solution. Second, in these problem tasks, there is not one correct answer and the answer can be given to different degrees of generalization allowing students to go as far as they can in their analysis. In these assignments, a written component was added so that the students can reflect on their own thinking. They were required to do a write up showing the steps used in their analysis and an explanation of why the conclusion they arrived at is a valid one. Technologies used in these experiments are the TI-89/92 calculator and the dynamic software "Autograph".

Compounding ratios: a musical approach on mathematics education

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In this study I shall consider educational aspects of the development of ratio and proportion, focusing on the arithmetization undergone by these concepts in the light of the relations between mathematics and music. Since such relations, even if confined to the context of ratio and proportion, are fairly wide-reaching and also that the process of arithmetization is quite complex, we shall concentrate mainly on the instructional aspects of a structural peculiarity presented in such a fascinating dynamics. This peculiarity is the so-called compounding ratios, a curious feature present in the structure of ratio since the Classical Period whose irregular transformation into the operator multiplication is quite representative of the importance of theoretical music in the arithmetization of ratios. As a consequence we shall also point out features of the differences between identity and proportion, which are capable of being didactically explored with a mathematic-musical approach. The reason for choosing music for the present approach is not only historical, but more specifically didactic insofar as the subtle semantic differences between compounding and multiplication and also between identity and proportion are clearer if one thinks of ratios as musical intervals when looking at such constructs. Grattan-Guinness argues that the well-known difficulties in teaching fractions can be alleviated by converting the latter into ratios, and thus using a musical approach. These considerations corroborate the need to explore didactically specific contexts in which differences between given constructs manifest themselves more clearly. In order to fulfill the aforementioned aim we shall first of all introduce some historical aspects of ratio in mathematical-musical contexts as well as of the corresponding structure in which compounding makes sense, and then follow these with examples of the practice of compounding on the monochord and by the didactic-epistemological aspects that underlie such a practice.

Main Theme: Distance Learning

Secondary Theme: Mathematics And Other Disciplines

Virtually Face-to-Face Mathematics Instruction

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For the last five years, Arizona State University has been actively involved in the research and development of online learning environments. During that time, we have come to understand how technology can support and enhance the teaching and learning process. Accordingly, we have developed two on-line courses, MAT 117, College Algebra and MAT 210, Brief Calculus. We believe quality instruction is not compromised when classes are conducted virtually, rather than in a face-to-face classroom. Through the use of advanced and emerging technologies, the same high educational standards can be adapted to anytime/anywhere learning environments. In these courses, the learning is dynamic and interactive. The courses are delivered on Blackboard Course structure, design, proper and appropriate pedagogy, interactive tips and student results will be discussed.

Main Theme: Technology

Secondary Theme: Preparation of Teachers

Effectiveness of Graph Calculator in Solving "Equations and Functions" Skills for Omani Prospective Mathematics Teachers'

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A significant amount of research has been conducted into the effectiveness of Graphics Calculator as a tool for instruction and learning within pre-calculus and calculus course, specifically in the study of function and graphing. This study aimed to investigate the effectiveness of graphic calculator in solving Equations and Functions for Omani prospective teachers, as a new technology tool used with Omani students. The main objectives of the study were: to determine the limitations of the graphic calculators skills needed to teach that topic in Algebra course. Twenty-five students from Mathematics / Computer department in College of Education were conducted in the experimental study, graphic calculator used in this study was: Casio cfx 9850 +. Results of the study showed that: A graphic calculator requires new skills to be used effectively (as numerical, graphical, symbolic and translations). Graph Calculators is an effective tool to enhance Solving Equations and Functions Skills for the Experimental group of the study.

Modern Problems of Mathematics Teacher Training in Russia

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In Russia the course of the experimental psychological research of students' training at all five years at teachers' training colleges and independent professional activity of mathematics teacher with 1-6 years of teaching experience three crucial periods of their professional development are defined, connected respectively with: the end of the 1st year at college; the end of the third and beginning of the fourth year at college; the end of the first and beginning of the 2nd year of independent work at school. To our opinion, the pedagogical process of mathematics teachers' training should be treated at formation of an integral system of professional-pedagogical activity. The first, professional, stage should be devoted to formation of the subject knowledge and skills, aimed at formation of the nearest specific generalisation of basic educational elements of school mathematics; the second stage, when knowledge of mathematics becomes fundamental, it acquire profound theoretical generalisation, which on the third, methodological stage is integrated into the structure of professional activity as a means of realisation of the pedagogue's teaching and educational functions. Founding is a process of creating conditions (psychological, pedagogical and methodological-organisational) for actualisation of basic structural units, which reveal their essence, integrity, relations between the subjects in the direction of professionalism of knowledge and shaping teachers' personality. Conception of founding of school mathematical elements (knowledge, skills, abilities mathematical methods) presupposes development in the process of mathematical training of students the following components:

- determination of contents of basic schooling element;
- contents of levels and stages (professional, fundamental and technological)
- founding technologies;
- methodological adequacy of elements (founded elements).

In the proposed system of prospective mathematics teachers' mathematical education a fundamental role is played by a pedagogical technology of visual-model teaching of mathematics. It makes possible to achieve stochastically guaranteed results of teaching of various qualitative levels of learned material as well as integrity of representation of basic mathematical structures.

**TRAINING OF UNDERGRADUATE TEACHERS IN NIGERIAN
UNIVERSITIES: FOCUS ON PROBLEMS OF EFFECTIVE
INTEGRATION AND ATTITUDE OF STUDENTS TO COMPUTERS
IN MATHEMATICS INSTRUCTION.**

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It has been argued that the ways teachers were taught and the behaviour patterns they developed for coping with feelings exert a long pull on their teaching. It follows, therefore, that if teachers are to comprehend and appreciate the nature of mathematics and mathematical thinking, they must experience as learners, the kinds of mathematical knowledge and thinking that they are expected to teach. Similarly, if teachers are to appreciate the use of instructional materials and tools, especially the new technology (computer), in mathematics instruction, they must experience as learners, and be exposed to the use of such technology in mathematics teaching, as a model of what they themselves might do. The study was designed to investigate the attitude of undergraduate mathematics education students to computer usage and the problems facing the effective integration of computers into mathematics instruction in Nigerian Universities. Three hundred undergraduate mathematics education students and thirty mathematics educators were selected for the study through stratified random sampling technique. Two sets of questionnaires- one for the students and the other for the mathematics educators were used, for data collection. Percentage, means and t-test statistic were used for data analysis. The results revealed non-availability of manpower and computers in the universities for the training of mathematics education students due to inadequate funding of higher education in Nigeria. Therefore, the student teachers are not exposed to computer usage in mathematics instruction. Some of the recommendations made include: more money should be made available for the universities to enable them acquire both human and material resources for effective integration of computers into mathematics instruction at undergraduate level.

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Main Theme: Technology

Secondary Theme: Innovative Teaching

Let the students explore algebra with CAS, TI89.

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Let the students explore algebra with CAS, TI89. These symbolic calculating tools, CAS Computer Algebra System, will change the way to teach mathematics more than the start of using graphing calculators did. With these tools we will get more time over to discuss the concepts of mathematics, more time to let the students explore algebra themselves and more time to increase the understanding of mathematics. The question is not if, but when and how, we should use CAS in our math classes. In my presentation I will show some examples how to work with TI89 and simultaneously reinforce the concepts of mathematics. All students in my class use TI89 and the age of the students are 17-19 years old.

Main Theme: Distance Learning

Secondary Theme: Technology

Teaching an Interactive Mathematics Course for Liberal Studies over the Web

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As part of the Liberal Studies requirements at Indiana University of Pennsylvania (IUP), all undergraduate students must take at least one Mathematics course. Most students with no requirement determined by their major take MATH 101 Foundations of Mathematics. Faculty members teaching this course are expected to select materials and topics to accomplish the goals of the course (develop an appreciation for the nature, breadth, and power of mathematics and its role in a technological society; and introduce useful mathematics or mathematics related to student interest). These goals are difficult to accomplish since most of the students that enroll in the course have had a minimal amount of mathematics in high school and also exhibit a high level of anxiety towards the subject. To meet this goal the author began developing interactive class materials using Scientific Notebook in 1999. Scientific Notebook is an inexpensive program that combines an easy to use front end to TeX and Maple. Using this program solves both the problem of posting mathematical content easily on the Web and opening computationally complex applications of Mathematics to students with limited background. A pilot study was conducted with two classes to test the feasibility of using the materials and delivering them via the WWW using WebCT. After the study materials for a semester long course were developed. Several sections have been taught since using these materials and Scientific Notebook. Several sections of this course have been offered as part of the Distance Education offerings by IUP. Lessons learned in the development and teaching of the course will be shared.

A modern and effective approach to differential equations

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This paper starts from some considerations at the end of a classical lecture on differential equations for first year Engineering students. To introduce this subject, teachers usually give the analytic definition, illustrate common equations and give examples of the Cauchy problem. In each case a given function is shown to be a solution. Verifying by hand that $y(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t)$ (c_1, c_2 in \mathbb{R}) is a solution of $y''(t) + \omega^2 y(t) = 0$ (ω in $\mathbb{R} - \{0\}$) requires many boring calculations, and students turn their attention away from what the objective of the lesson is. Students appear confused as testified by their questions: is the content of this lesson needed for the exam? Where did the solution come from? So the authors changed the approach. The same equations are presented starting from real example (growth of a population, harmonic motion etc.). Students are stimulated to propose some functions they guess are solutions and, using a Computer Algebra System (CAS), they can prove by themselves if they are right or not. The CAS help them to concentrate on the problem of founding the solution and not on the calculations! As the equations come from physics, electronic, statistics, and model real events, CAS allow students to manipulate real data (often big numbers) stimulating them to think about the models and they are motivated to study the subject because it is required in their real life! Moreover CAS allow to visualise both the equation and the solution and students can "see" that a solution is indeed a family of functions and they can "prove" that a function is fixed as soon as they fixed a starting point, so they are introduced to the Cauchy problem. This paper shows the benefits the authors had using CAS in introducing differential equations: more effective lessons than the "classic" ones and overcoming of misunderstandings observed in the past.

Two components in learning to reason using definitions

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This paper discusses the transition to the use of formal definitions in mathematics, using the example of convergent sequences in Real Analysis. The central argument is that where in everyday contexts humans categorize objects in flexible ways, the introduction of mathematical definitions imposes a much more rigid structure upon the sets so defined, and hence upon the acceptability of different types of argument. The result is that, in order to have their reasoning accepted in proof-based mathematics courses, students must do two things: 1. align their notion of what mathematical objects belong to a given set with the extension of the defined set, and 2. (more fundamentally) learn to express their reasoning about such sets exclusively in terms of the definitions or other results traceable to these. The importance of these two components is illustrated using two examples. First, a student whose idea of what objects belong to the set of convergent sequences does not closely correspond with the definition, and whose reasoning is therefore insufficiently general. Second, a student whose set corresponds well to that given by the definition, and whose work is arguably more mathematically sophisticated, but who still does not "succeed" since he fails to reason using definitions in the required way. Finally, pedagogical implications are discussed, with particular reference to tasks that require exploring the extension of defined sets. We consider the role of collaborative student work in promoting awareness of a broader range of examples within such sets. Further, we suggest that there is often a gap in the structure of the tasks that students are asked to complete; that many would benefit from tasks which begin with a term and require students to generate examples, in addition to the more usual task of beginning with an example and establishing its membership of a set.

The Potential of Multimedia in teaching and learning of Mathematics

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One of the challenges currently facing the education sector is how to exploit the potential of integrating an Interactive MM environment into teaching and learning. The aim of this paper is to develop a new multimedia interactive learning environment for supporting both teachers and students in the teaching and learning of different mathematical skills. The characteristics inherent in Interactive Multimedia Learning Environment facilitate the progression from lecture driven towards self-accessed and student directed learning by removing the historical time and space constraints. It also provides learners with a wide diversity of learning options by removing the restrictions of a single style and mode of learning. These MM tools facilitate the possibility of shifting the focus of learning from ensuring students have precisely defined knowledge to developing students' competencies and talents. More targeted assistance can be given as the system 'frees' the teacher from responsibility of supervising students and allows a more concerted effort to be given to struggling students. The capabilities of interactive multimedia tools to support the learning process have been explored in many research works with many predictions being made about the impact that these technologies can make in the curriculum. With the integration of Interactive Multimedia into the traditional classroom, the role of teachers has shifted from the traditional perceived role of the 'teacher' to that of a mentor or facilitator. Educators and psychologists are studying the affects that these technologies can have on the way we teach and learn. Moreover, this has lead to the focus of the research being an investigation into the crucial role that these technologies can play in shaping the role of both teachers and students. Initial findings of this research work, has highlighted the need for more research in developing specialised multimedia environments that can assist in teaching and learning of mathematics. The foundation of this research work concentrates on the development of specialised MM environment that can support mathematics. There are some basic requirements that need to be considered when developing such applications and therefore it has been planned that during the requirements, design and implementation phases, opinions from both teachers and students will be incorporated utilising the prototyping method. Early experimentation is important to ensure that the system is properly configured to the user requirements. Only an incremental design and implementation process allows for such early feedback. Therefore, during the implementation of the initial version of the system, it will be necessary to undertake different pilot studies for investigating both the functionality and the usability of the system. This is especially true when it is necessary to experiment with a prototype to determine learning strategy and usability requirements. After developing the final version of the system, an evaluation study will be undertaken by designing and performing a variety of experiments in different local schools. The purpose of these experiments is to ensure that the system functioned satisfactorily, namely that teachers and students could use the system and to investigate the effects of using this type of technology on the learning process.

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Main Theme: Education Research

Secondary Theme: No secondary theme

Mathematics
**REFLECTION ABOUT PROBLEMS INVOLVING TEACHING AND
LEARNING OF GEOMETRY**

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We propose a didactical reflection about problems involving teaching and learning of Geometry at different levels of Brazilian-Ensino Fundamental, and also an analysis of the main results of a study work with teachers. The proposed work is divided in three phases. In the first one, we study the methodological fundamentals of our research. In the second part, we analyze the wording of those teachers who participated of the research project. At last, we analyze some factors that might produce the difficulties that teachers and students face concerned the teaching and learning Geometry at different levels of Brazilian-Ensino Fundamental.

On the effect of Using Automated Reasoning in Teaching Discrete Mathematics

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The field of automated reasoning is concerned with the ability of computer programs to reason about given knowledge and deduce a new one. A number of automated reasoning programs do in fact exist, and they had provided great assistance in solving a wide variety of problems that may include answering open questions, designing hardware circuits, and verifying correctness of theorems' proofs. Such tools are rich enough to teach students various mathematical concepts like problem representation (whether in propositional or first-order predicate calculus), quantification, simplification, substitution, splitting hard cases into smaller solvable ones, answer justification, and different ways of deduction like resolution and factoring. The existence of computer programs (tools) that do in fact reason is, to many individuals including those who are familiar with computers, clearly a surprise. Their effectiveness is amply demonstrated by examining their role in answering open questions from mathematics, designing and/or validating the design of logic circuits, verifying the correctness of proofs and programs, and constructing bases for mathematical domains which students need to understand before working vigorously in those domains. This paper can be viewed as an invitation to using those automated reasoning tools in teaching elementary discrete mathematics. An analysis to problem solving leads to the identification of three categories of problem solving techniques: numerical, data-processing, and reasoning. Some problems depend on some combination of the three for a solution to be found. Although, most computers programs currently in use focus on the first two categories, there do exist programs that reason. Examples of such programs are OTTER (McCune 1994), GANDALF (Tammet 1997), SETHEO (Moser 1997), and THEO (Newborn 1997). Any of these programs can be given some statement and asked to prove it is true. It can also be given some statement and asked to find a counterexample to establish the statement is false. The usefulness of a computer program that functions as a reasoning assistance or as a tool to teach concepts in mathematics depends on how the specific problem is presented to the program, the power of rules employed to draw conclusions, and on the effectiveness of the strategies used to control the reasoning process. These three areas - representation, inference rule, and strategy - are vital resources to students learning mathematics. With regard to representation, first-order predicate calculus give students skills needed to understand and work on theories such as terms which can be identified as variables, constants, or functions. The other two areas - inference rules and strategies - direct the students to concentrate on learning the desired theory. The basic procedures - drawing conclusions, rewriting those conclusions into canonical form, testing each for significance, comparing each with information already retained, and integrating the new ones into a database of gained information - on which an automated reasoning program relies are skills required in mathematics. Furthermore, some of the existing automated reasoning programs employ domain-dependent knowledge in their aim to model the reasoning process. While other programs hope to achieve the same objective by using a domain-independent system for modelling mathematical theories such as number theory, set theory, algebra, and analysis. In this paper, we will model number theory using THEO. This tool uses resolution-refutation and factoring inference rules and employs two strategies namely the set-of-support and unit-preference (Newborn 1997).

An Analysis of a Web Forum in Distance and Face to Face Teaching of a First Year Mathematics Subject

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Almost all of the subjects taught at Charles Sturt University (CSU) are supported with electronic communications. The electronic communication facilities provide students with communication tools such as direct e-mail to the subject lecturer and a subject web forum to enhance student-student-lecturer communication and, hence, learning. The web forums allow for open discussion, at the convenience of the students. The advantage of employing a subject web forum to enhance traditional distance education is that it alleviates some of the problems encountered by distance education students that internal students do not normally face. We discuss the benefits of a subject web forum for both external and internal students. We present the results from a questionnaire designed to discover the perceptions of students regarding their experience with a first year mathematics web forum. We will examine the opportunities presented by a web forum such as establishing a frequently asked questions (FAQ) database for first year mathematics subjects. The usage statistics of the subject web forum will also be presented. We will also discuss the infrastructure and human resources needed to develop such a subject web forum in Atılım University, Turkey and to make this forum available to other universities through Turkish Higher Education Council (YOK).

**DEGREES OF INFLUENCE OF THE BEHAVIOURS TAKING
PLACE IN THE PROCESS OF SOLVING VERBAL PROBLEMS
ON THE SUCCESS OF PROBLEM SOLVING**

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In this study, 65 second and fifth graders students in a primary school were observed while they were solving verbal problems in terms nine critical behaviours derived from Polya's stages of problem solving process in accordance with problem solving, and an attempt was made to determine to what extent each of these behaviours could point to the success of problem solving. Main aim of the study was to prove the power of these behaviours in problem solving, and to make use of these in teaching. A problem solving test was given to each of the classes where this study was carried out, and the students were divided into two groups as successful and unsuccessful according to the test-points they received. Later four problems from each test given were selected, and the students' behaviours during solving these problems were analysed, and their levels of showing the above-mentioned nine critical behaviours were determined. In turn, using these levels, the discriminant analysis was applied, and a new classification was made to find out to what extent these levels were efficient in determining the success or the failure of these behaviours. The results indicated that the second and fifth graders did not show the behaviours in a consistent way during the problem solving process, and also that the behaviours differed in influencing the success of problem solving. As a conclusion, it was found that the levels of the indication of the behaviours exhibited by the students during problem solving can be indicators of success in problem solving, and that this can be used in teaching problem solving.

The influence of the environment in the learning of the Linear Algebra

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This paper compares the results attained by a control group working with traditional methodology with those of an experimental group using an application software program called SCILAB. The focus is on linear algebra (matrices, determinants and linear equation systems) which forms part of Mathematics II, one of the core subjects in the B. Sc. econ. course at the Faculty of Statistics and Economic Sciences, National University of Rosario, Argentina. This survey comes under the Teaching Mathematics with Computational Tools. It is financed by Program 202 for the promotion of Scientific and Technological Investigation set up by the National University of Rosario, Argentina. The conclusion is that the experimental group proved more successful. The objectives of the experiment were as follows:

assessing how far a computer tool can help with learning linear algebra at university level.

determining the impact of computational tools on university linear algebra students' competence in solving problems.

Further effects of the new computational environment on learning linear algebra were also considered. These objectives were completed by comparing the levels attained by both groups, control and experimental alike. SCILAB is a software system developed by France's Institut National de Recherche en Informatique et en Automatique, INRIA. It has been conceived to provide experts in applied mathematics with a powerful calculus tool. It uses the syntax of the MATLAB system. This system is kept as the interpreter and offers the greatest possible similitude to ordinary mathematical writing. It allows the manipulation of mathematical objects such as vectors, matrices and polynomials. It is also an open system because it allows the user to create new functions in a simple way.

Teaching Mathematics Using the Idea of "Research Problems"

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The most mathematical tasks found in high school and early college textbooks begin with the words: "simplify the following algebraic expressions .", "calculate the following" or "solve the inequality.". Mathematicians, however, more often deal with more open problems, where the main aim may be to establish whether the object with the given properties exists at all, or whether the given assertion is valid in principle rather than to simplify or calculate something. Undergraduate mathematics courses for preparing high school teachers might benefit from including a number of such "higher - order" tasks. By the "research problem tasks" concept we shall mean those that are based on subjectively difficult theorems or mathematical constructions that are initially not known to a particular student (or he is unfamiliar with the proof modus operandi). There are such tasks that a student, when solving them, encounters the necessity to investigate mathematical models of configuration which are new to him, non-standard connections, existing between such models, properties of figures, and at the same time he has to find and establish a logical scheme of reasoning. Solution of a research problem task results in the established and well-founded algorithm of solution for the total class of similar problems or heuristic device, the scientific idea that, after being justified and generalized, can be used and recommended for the solution of other similar nonstandard problems. The proposed method is found to considerably intensify and advance the process of students.

ISSUES ON INTEGRATING CAS IN TEACHING MATHEMATICS: TEACHING CONCEPTS IN CALCULUS WITH DERIVE

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Even if, to day there are no clear answers to teach concepts in Calculus using CAS. To attract student teachers to use this system, they must be familiarized with the software and developing CAS teaching methods. On August, 2001 we hold on a seminar and workshop on this subject to present TI-92 and its applications. We applied a questionnaire on the student teachers to get their initial attitudes and their opinions about the Graphing Calculators and the software DERIVE. We found out that no one of the student teachers (n=67) saw this teaching tools and software in their educational terms but they are reluctant to learn and use this software. So, we decided to design a project named "Teaching Concepts in Calculus with DERIVE with the Research Group in Mathematics Education in Middle East Technical University (METU). So, we decided to teach the operation system of TI-92 in six hours for their needs to study for the activities designed for the project. . We continuously have got the impression that if a learner performs the requested tasks carefully he (she) will explore concepts him(her)self and may discover a new relationship in CAS environment. After this preparation, we gave five activities, which were designed in the form of worksheet to study by themselves as a group involving two student teachers consecutively. The activities were purely concerned with the concepts decay and growth, linearity, local linearity, limit by approximation, uniform continuity, right proportional, arithmetic and geometric sequences. At the end of these activities it was asked the student teachers (n=24) to construct a concept map showing the relations among these concepts. An experiment with this approach, using DERIVE, is discussed in this paper and presentation. The average rate of use of information and communication technology (ICT) and instructional materials in education system of Turkey is less than ten percent, but the percentage is increasing gradually. However, the trained mathematics teachers and some instructors in Turkey trying to use and implement TI-92 and the software DERIVE in teaching various mathematics topics. Because, both CAS calculators and computers are valuable cognitive tools which enable us trivialisation, visualisation, experimentation and concentration (Kuzler, 2000). it is important to prepare teachers for the future needs of their students (Zehavi, 1996). Technology-rich curricula can help meet the demands of the new standards for more inquiry based learning and new content, and can support more sweeping change that goes far beyond what is envisioned in the standards (Tinker, 2001)

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INTEGRATING TI-92/CAS IN TEACHING CONCEPTS FROM CALCULUS: HOW IT EFFECTS TEACHERS CONCEPTIONS AND PRACTICES

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Although there are many efforts, trials and experiments in many countries, there are no clear answers how to teach various concepts effectively in mathematics, in particular pre-calculus and calculus, by using CAS today. To inform and train a group of prospective mathematics teachers (PMTs) and practising teachers in Turkey we have attempted to organise a series of seminars and workshop for the last few years. One of them was hold for a group of PMTs who got their BSc degree from department of mathematics in various universities in Turkey on August 2001 as an integral part of ongoing projects at the Middle East Technical University (METU) in Ankara guided and directed by the researchers. Before the experimental study, we interviewed several PMTs first, and administred a questionnaire to get PMTs' attitudes and opinions about the graphing calculators (GC), namely TI-92 and the CAS-software DERIVE. We found out that none of the PMTs ($n_1 = 67$) had any idea about GC and CAS; and were reluctant to learn and use the cognitive tools in learning and teaching mathematics. Therefore, we decided to inform and train the PMTs, who participate in the teaching certicate courses (a special program) taken place at Selcuk University in Konya for a couple of weeks, and share our experience with other experts as well as improve the curricula by designing new instructional materials and implementing new strategies in teacher education and traing. Various activities, which concern with some basic concepts of calculus, namely decay and growth, linearity, local linearity, limit by approximtion, uniform continuity, arithmetic and geometric sequences, about the use of TI-92/DERIVE were designed and implemented in the course. At the end of the training, we asked some volunter PMTs ($n_2 = 24$) to construct a concept map showing the relationships mong the foresaid concepts, and reflect their views about the use of TI-92/DERIVE in teaching/ learning mathematics. In the presentation, we will report about our experiences made with the PMTs at the certificate courses, and show the sample of materials designed to teach various basic concepts in calculus. We oncentrate on how the PMTs can apply their knowldege from mathematics and pedagogical courses in teaching of mathematics and use TI-92/ DERIVE.

Role of a Peer Support Program in Learning

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A newly established institution Sabanci University offers highly challenging and interdisciplinary programs. The institutional structure and academic programs are based on the utilization of interdisciplinary approaches, and the traditional departmental structure does not exist providing the students the opportunity of choosing their academic programs, to realize their goals. The students during their first two years of university education are required to take the same courses independent of their future aims, to be engineers, natural scientists, political scientists, economists, historians, art historians or artists. The university has a guidance system that includes various units to promote student success and to support the realization of the academic programs. This paper aims at presenting an academic support program that is structured as a subdivision of the guidance system and run totally by undergraduate students. The task of motivating the students with different interests and diverse backgrounds as well as giving equal opportunity to each student in the assessment of their class work, calls for extra effort. Unsurprisingly one of the main aims of the program is to motivate and encourage students to understand mathematical concepts, mathematical modeling and to use mathematical tools in various contexts. To reach the aim program offers extra curricular activities in line with the university's academic programs and is subject to systematic evaluation. Program activities, office hours, tutorials and workshops are held by freshmen and sophomores in a friendly atmosphere encouraging peer discussions and sharing academic knowledge and experience. Evaluations and statistical results revealed the significance of peer support as well as the role of the program in building a learning environment and a healthy academic campus climate.

Polynomials in the Context of Linear Algebra: Expressions? Sequences? Functions? Vectors?

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Linear Algebra Textbooks offer different definitions for polynomials: Polynomials as expressions over a ring. Polynomials as infinite sequences over a ring. Polynomials as functions from the ring or field of their coefficients into itself (function-polynomials). The equality of function-polynomials is defined as that of functions, namely point-wise equality. The equality of expressions or sequences is defined via their coefficients. Two problems arise. First, it is not self-evident that two real function polynomials are equal only if all their corresponding coefficients are equal. Second, the coefficient-criterion does not hold for function-polynomials over finite fields Z_p . Fifteen college students took a one-semester first linear algebra course. The software Iset1 was used in the introductory parts of the course. The use of Iset1 promoted the introduction of finite fields and vector spaces over them. Hence a distinction between the different interpretations of polynomials arose. Vectors were first introduced as tuples, and later defined as elements of a vector space (systemic definition). Polynomials were defined as functions and dealt with mainly over R . Twelve students were interviewed after the course. I will report findings related to the concept polynomial and its relation to the concept vector.

Modelling and Interpreting of Experimental Data

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A lot of time is spent in traditional math courses for detecting the shape of graphs according to given expressions instead of training the more realistic situation to find functions fitting given data. Using CBL (Calculator Based Laboratory) and CBR (Calculator Based Ranger) from Texas Instruments it is quite easy to transfer real data obtained by experiments to graphic calculators for further mathematical manipulations. Various practical as well as mathematical skills of the students are trained by carrying out experiments, analysing the results and finally using functions for fitting data points obtained by the experiments. We report about experiments being carried out in the years 1999 until 2001. In eight different groups consisting of students at the age of 16 to 18 experimenting with CBL, CBR and TI-92 was integrated within regular classes. About 50% of the students were girls. A special course for high ability students at the age of 14 was installed during the school year 2000/01 also carrying out experiments with CBL. In 2000 a group of students were testing the water quality in regular classes using CBL and ion selective probes from Vernier. The students were really motivated. According to questionnaires and feedback forms they enjoyed practical work and felt free of the 'pressure of learning'. We forced the students to document their work and their results. Writing summaries of the experiments they understood the background of the respective experiments and some of the students wished to repeat the experiments to obtain better results. Interpreting results was difficult for the pupils especially in the course of testing water quality. It was also new for the students to work in groups. They had to dedicate work to different group members according to their abilities. Finally, it was quite difficult to find a fair grading for the students according to their individual achievements.

A Didactical Classification of Probability Problems Linked with their Formulation

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Our research aims at the study of the relationships between problems [1] involved in the teaching of mathematics at school and those encountered both during the historical development of mathematics and in educational research. We study the common elements and features of the formulation of these problems as well as the way they influence each other, aiming at the improvement of mathematical problems used in classroom contexts. This improvement is related to their content and the way they are presented, so that they will have epistemological account and be related as well as be improved by research results. The interrelations that will be presented focus on Probabilistic Problems and their teaching to 5-11 year-old children. Before coming to these interrelations, we initially collected problems, which were found: 1) in governmental school books [2] different for each level as well as in several published books [3], 2) in published articles [4] and conference proceedings (problems which have been used in researches), 3) in history, philosophy and epistemology books. Moreover, after registering the features of their formulation, we created categories and sub-categories, in which each problem was incorporated. Resulting data were statistically analyzed by Factor Analysis methods [5] in order to classify the problems and obtain the appropriate taxonomy. Research results and conclusions will constitute educational material for teacher training because, although Probability Theory is a very important and socially useful branch of mathematics [6], it has been observed that there are many difficulties [7] in their learning as well as teaching process.

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The role of mathematical patterns connected with some numerical Algorithms in mathematical education

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In mathematical education and teaching we need to show the relation between an abstract subject like mathematics and the world of reality in which we live. It is important also to discover that, owing to abstraction itself, mathematics found its way to various areas of sciences like physics, biology, economy etc. Examining some phenomena (physical. Chemical.), we try to find abstract mathematical pattern (equation, figure.) which describe these phenomena. It appears that in mathematical language the same mathematical equation may describe more than one different phenomenon simultaneously. For example harmonic motion and the motion of circulated current are expressed by one differential equation. Coming into new mathematics concepts was mostly connected with practical needs like the necessity of solving some problems which could not be solved by classical methods or explaining reiterated phenomena under some conditions in some environment through representing them by some mathematical equations. It is not easy to find a convenient pattern for given phenomena because the complicated structure of the real world forces us to a sequence of elimination for purpose of simplicity and this may prevent us from understanding the essence of the problem considered. On the other side if we want to study the problem with high precise it shall be very difficult to find a pattern which takes all elements of phenomenon into account. The question now is how to choose a mathematical pattern relatively simple and good enough to describe given phenomenon? We attack this problem by suggesting some new patterns using, also, some numerical algorithms. This may help in understanding the interpretation of given patterns or in describing given problem (or phenomenon) by a new means through the use of the advanced technology especially the technology of computers. Some example in physics and biology will be given.

ON THE INTEGRATION OF KNOWLEDGE: GEOMETRICAL INTERPRETATION OF THE PROPERTIES OF FUNCTIONS

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The effective use of mathematics requires the ability to discriminate how various concepts from a particular mathematics discipline are to be used and how to integrate them in the solution of a problem. A large body of research has been conducted on student understanding of specific Calculus concepts, but little has been done on how students integrate and apply this knowledge when they face complex problems. The purpose of this research is to contribute to knowledge in this direction. We designed a study to examine research questions about the difficulties that students face, and the strategies they use, when integrating mathematical knowledge. Twenty-seven of the most successful university students from two different universities in two different countries participated in the study. They were interviewed while solving specific tasks related to the analysis and interpretation of information in the context of Calculus about the properties of functions and their graphs. The interview questions examined several calculus graphing concepts in more than one way, in order to determine if the students were consistent in both their answers and difficulties. The theoretical framework used in the design of the project and the analysis of the interviews is based on the notion of the Schema Triad from Action-Process-Object-Schema (APOS) theory and the interaction of more than one schema (JRME 2000, Vol.31, No. 5, 557 - 578). Detailed analysis showed that these students, although very successful in terms of their class work, had difficulties in integrating particular properties of functions and derivatives and relating their application to sketch graphs of functions. In particular, students had difficulties coordinating information about different properties of a function across different intervals in the domain and determining the role of continuity in those situations. The results obtained highlight some cognitive and curricular issues to take into account when designing and teaching Calculus courses.

WRITTEN META-COGNITION AND PROCEDURAL KNOWLEDGE

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Sierpiska in [4] contrasts the positions that Vygotsky and Piaget hold on the ability of writing to influence thought and mental development. She states that, "Piaget would not claim that the activity of communication can change the course of development. On the contrary, he would claim that development is a precondition for a person to express him or herself clearly in writing." On the other hand of Vygotsky she writes, "Vygotsky was claiming that writing can have an actual impact upon development." One explanation for Piaget's position, advanced by the Haapasalo and Kadjevich [2] is based on the relationship between procedural knowledge and conceptual knowledge inherent in models of learning and development based on the work of Piaget. In such models, meta-cognitive reflection upon procedural knowledge and conceptual thought occur in latter stages of learning after the individual has achieved proficiency with the procedural knowledge, which characterizes the initial stage of learning. In this research we analyze procedural knowledge as measured by course average on the one hand and on the other hand conceptual thought and meta-cognition as measured by scores on writing exercises used as independent variables to study their relationship in promoting development as measured by the students' G.P.A. This teaching - research was done at a community college with students enrolled in the remedial courses of elementary algebra and basic skills mathematics. The writing exercises were designed in accord with the framework of Sfard [3] and graded using the scoring rubric set forth in Countryman [1]. Our results indicate that writing scores were independent of cognitive proficiency in promoting academic development. Thus, they provide the evidence against Piaget's view when interpreted as the position that written mathematics designed to promote meta-cognition and conceptual thought is dependent on the mastery of procedural knowledge in promoting intellectual development.

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A Case Study in the Evolution of Calculus Reform

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Calculus reform came to the University of Wisconsin-Eau Claire, a regional, comprehensive, primarily undergraduate Midwestern US university of approximately 11000 students, in 1995 when two instructors attended a workshop led by Ed Dubinsky. The reform program, which has affected the three semester calculus sequence for math and science majors, has evolved considerably, but preserves the basic feature of a computer lab experience (now with Maple 6) which provides an active learning environment where students, working in cooperative groups, can construct their own understanding of concepts. This paper extends and updates a paper by the same authors presented at the First ICTM. The results corroborate other research projects, which show that the reform students perform about the same as traditional students on skills-based tests, while showing an advantage in terms of attitude, communication, fluency with technology, and problem solving abilities. On the other hand, the program has at times proved divisive among students and faculty. In order to provide a common experience for the students, all calculus sections calculus, beginning in the fall of 1999, were required to include a lab experience. In order to coordinate the lab and lecture components, faculty found they needed to collaborate more closely. While some faculty valued the greater communication, others did not like the restrictions. Students would often fail to see the connection between lecture and lab because they took place in different environments. In 2001 the department received funding from the National Science Foundation to equip a classroom with networked laptop computers. The room, which has round tables to facilitate cooperative learning, makes it possible to effectively blend lecture and lab experiences. This fall some instructors taught in this dedicated room, resulting in less need to coordinate sections. Preliminary results indicate that both students and faculty are satisfied with this structure. Although we have compromised the goal of a common experience for all students, we have a structure in which all students have a lab experience, and which accommodates both reform and traditional teaching styles.

On some important aspects in preparing teachers to teach mathematics with technology

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The introduction of technology in the classrooms at all levels of education has brought forth a need to change some teaching practices. Together with the modernization projects of undergraduate instruction for Engineering and Sciences courses, it is especially important to focus the attention on Courses for prospective (and in-service) teachers in Mathematics. The use of technology in the teaching-learning activities can be regarded as a new communication language in developing the construction of knowledge. The recognition of this role of technology in education would contribute to a better preparation of the future teachers in selecting right teaching strategies, not only technology. This presentation aims first to discuss this aspect of technology in the undergraduate instruction, through a systematized classification of the use of technology in the classrooms based on the forms of activities, illustrated with examples. Furthermore, one of the advantages of the technology as teaching aid is the possibility of more realistic modeling in problem solving and interdisciplinary activities, so new and reformulated disciplines in the curriculum of teacher preparing courses come up. Regarding this aspect, we point out that the critical interpretation of the computer/calculator outputs demands an awareness of the kind of mathematics needed when using technology. That means that solving a problem with the use of technology requires from the user a deeper understanding of the importance of the concepts like units, scaling of units, significant figures, approximation/numerical methods, parametric representation and implicit representation, interpolation methods, structure of algorithms, etc., along with the proper theoretic concepts underlining the problem. The careful use of technology as a teaching strategy would enrich in this way the lectures and the preparation of activities by teachers. The second aim of this presentation is to call the attention to this aspect of teaching with technology, exhibiting some examples collected from experiences with prospective teachers.

The Graphing Calculator in Mathematics Education in Danish Upper Secondary School · Teacher attitudes and experiences.

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After being an Upper Secondary School Teacher for 11 years I started a ph.d.project in Mathematics Education in the summer of 2000. The presentation will report on some of the preliminary results. Denmark offers a rather unique situation, since it is now not only allowed but mandatory for students to use a graphing calculator at their final written exams in Mathematics in Upper Secondary School. This gives the possibility to actually base teaching on the assumption that all students have a graphing calculator. This has been the case since the summer of 1997. The central question in this research is whether this has generally had an impact on the teaching practices of the Mathematics Teachers in the Upper Secondary School in Denmark. A questionnaire among these teachers of Mathematics were conducted in order to clarify what they are doing, how they are using the Graphing Calculator and which are their main concerns at present. Following up on some of the results from the survey 4 teachers were selected for further collaboration. Together we developed 4 different ways of introducing the Concept of Derivative using the graphing calculator as a pedagogical tool. During this process the teachers were interviewed about their views on mathematics, teaching, learning and students. In Mathematics Education several interesting theories about concept development exist. In developing the teaching material, my ideas sprung mainly from some of these theories, while the ideas of the teachers came primarily from their experiences and their basic understanding of Mathematics. The cooperation between them as practitioners and me as a researcher has been an integrated part of my research method, investigating the possibilities for improving the students learning by using the graphing calculator, and it has been very fruitful.

Concept mapping as evaluation tool in mathematics

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My proposal is about the conceptual scheme of concept mapping as evaluation tool in mathematics. I present the basic rules of concept mapping (Hierarchical, Progressive differentiation, Integrative reconciliation) (Novak & Gowin, 1984) and consequently the theoretical background of concept maps which is consisted : On Ausebel's theory (1966,1978) that meaningful learning can be achieved when new data are incorporated in a web of relative components that make sense to the student. In Systemic theory which asserts that meanings and concepts are not sums but organised physical systems of behaviours (Paritsis,1986 ; Dekleris,1986) In Neurobiologist researches about function of human brain (Changeaux, 1988; Posner & Raichle, 1994) who suggest the important part of links and the correspondence with the conceptions, images and meanings. Eventually I refer in the use of concept mapping as evaluation tool and I present the outcomes of comparison the concept maps of two classes of mathematics in secondary education with a master concept map. Finally, I present my conclusion that concept mapping might be an important tool of searching the conceptual constructs under certain circumstances.

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Tartinville and Cabri II

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The first scholar who dealt with such subject in a very direct and clear way was Tartinville, once ·sadly· famous among the students of Liceo Scientifico. The name of the French mathematician, Tartinville, is exclusively linked to the problem of the qualitative analysis of the second degree equations, and he was widely studied at Liceo Scientifico, actually, that was the only method used to solve the problems assigned at the final exam at Liceo Scientifico up to 1969. Thanks to the protest carried out by B. de Finetti, this method in particular and the qualitative analysis of the problems, in general, disappeared both from the syllabi and from the final exam at Liceo Scientifico. Teachers who have taught this method usually believe that Tartinville's qualitative analysis is boring and cumbersome and it does not provoke any curiosity in the students but only a passive study of the subject; but we think that, not only the qualitative analysis of the problems, but also the method can be newly presented at school, using Cabri II software as useful tool for the teaching of geometry. This idea should be placed in a wider cultural environment recognizing the importance of ·external· events in the development of the mathematical thought. So we decided to use the computer to make the teaching of geometry lively, to animate the geometrical object, a characteristic diffused in the geometry treatises of the 16th and 17th centuries. Cabri II is an excellent support, in this sense.

SVG - A NEW DIMENSION IN PRODUCING INTERACTIVE NETBOOKS

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SVG - Scalable Vector Graphics is a 2D-graphics markup language based on XML. It is compatible with other web standards: HTML, XML Namespace, Xlink, Xpointer, CSS 2, DOM 1, Java, ECMA/Javascript, Unicode, SMIL 1.0, ... It allows us to include in HTML documents pictures described by their structure - composition of curves and shapes. Since the SVG viewer is not integrated yet into web browsers we need, to view SVG pictures, to install it as a plug-in. An excellent SVG plug-in was produced by Adobe. The SVG pictures are not static (as standard bitmaps GIF, JPEG, PNG). The SVG viewer provides options to zoom in (to see details) and out (to see global view), to move the picture, to search for text, ... Besides this, using built-in animation capabilities or Javascript program support, the pictures can be made alive and interactive. SVG pictures can be produced by drawing tools. But special programs for visualization of obtained data/results will produce most SVG pictures. In the paper we shall present the main features of SVG and discuss their educational usage. We can partition a SVG picture to several parts. Changing their attributes we can control their visibility. Using Javascript this can be done interactively allowing to the user to select the parts to be displayed. To support this we prepared SVG player - a collection of Javascript functions for controlling the visibility of parts of the SVG picture. Some applications of SVG player in interactive mathematical illustrations (for example, geometrical constructions) will be presented. We also prepared Logo2SVG - a library of redefined logo primitives that allows user to save a trace of the Logo turtle as a SVG picture. A ZIP with the last version of SVG player, Logo2SVG and related files is available at <http://vlado.fmf.uni-lj.si/pub/SVG/>

GEONExT - dynamic mathematics software in action

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The interactive geometry software GEONExT has been developed at the University of Bayreuth (from where it is available for free, see <http://geonext.de>) as a tool for experimenting with new approaches to mathematics at school, in particular geometry; in the latest versions also aspects of calculus are considered. The software's output is realized as Java applet, hence it can be used independent of system software and special browsers. It features all the constructions that are common in geometry and several elements from analysis (like drawing graphs). The main motivation for developing this tool was to have an appropriate means in order to realize new ideas for motivation and treatment of typical topics in school mathematics. So by now, there exists a collection of numerous work sheets as well as several complete learning environments, the themes of which include - golden section - pythagoras- reflection in a line - geometry of a triangle - regular polyhedrons- Durer's Melencolia I Many of them are rather close to standard curriculum, others include aspects going beyond that. Produced in a modular manner, the learning environments may be used either as complete units or just parts of it may be taken for demonstrations in lessons, for the independent use by pupils, for repetition or as a source of information. The advantages for teaching lie in an increased activity of the pupils, individual needs can be answered to a large extent. Experiments open new ways of approaching and understanding, which is supplemented by historical information as well as connections to other disciplines. The use of learning environments of course changes the role of the teacher: the emphasis now lies on accompanying and tutoring the pupils on their journey of discovery into the world of mathematics.

Main Theme: Technology

Secondary Theme: Mathematics And Other Disciplines

Teaching mathematics to engineering students with hand-held technology

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The École de technologie supérieure is a school of engineering that specializes in applied engineering and technology. Since the 1999 fall semester, the graphic calculators TI-92 Plus or TI-89 have been a compulsory purchase for every new student. The talk will give some examples of how we use these symbolic tools and will show how the calculator has changed the kind of questions we ask our students. We will focus on Calculus and Differential Equations. The talk will show that computer systems AND hand-held technology can be used by the teacher/student in order to teach/learn mathematics in a very original manner. One of our goal is to continue to teach "classical maths" with innovative approaches. This is possible if you can make use of technology in the classroom, when you need it, when you want it, without having to wait to go to the computer labs. And, when the teacher thinks that technology should not be used for some parts of an exam, students are not allowed to use the calculator!

Mathematics for Elementary Teachers: Making Sense by "Explaining Why"

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In order for prospective teachers to develop the reasoning and sense-making abilities of their future students, the teachers themselves must make sense of and reason about the mathematics they will teach. However, many prospective teachers have only experienced mathematics as the rote following of procedures, and are not aware that reasoning can be used to solve problems in non-standard ways, or that reasoning underlies the standard procedures in mathematics. Evidence will be presented that it takes work for prospective elementary teachers to develop this awareness: the standard procedures of mathematics are often taken as "given" and not open to question. One way to enhance prospective elementary teachers' abilities to make sense of and reason about mathematics is to engage teachers in "explaining why". For example, if a problem can be solved by multiplication, a prospective teacher should learn to use the meaning of multiplication to explain why multiplication is an appropriate operation to use. Because prospective elementary teachers may not initially recognize that it is possible to give such explanations, problems and activities must be chosen wisely in order to develop this ability. This paper will describe problems and activities that help prospective elementary teachers learn to explain why an operation (+, -, \times , or \div) is or is not appropriate to solve a problem, why the standard procedures and formulas of elementary mathematics are valid, why non-standard methods can also be valid, and why other seemingly plausible ways of reasoning are not correct. These problems and activities ask prospective teachers to produce arguments that are logical and make sense, but are not necessarily as general and formal as typical mathematical proofs. Criteria for writing good explanations will be given. Examples of prospective elementary teachers' work will be shown.

Geometry: Back to the Future?

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Geometry seems to be firmly back in the curricula of undergraduate mathematics in an increasing number of leading Mathematics departments, as witnessed by a growing offer of textbooks ([H] is a particularly beautiful example). A somewhat related phenomena is the mounting stream of documents stressing the need for a greater emphasis on geometry in the school mathematics curriculum (see for instance [NCTM] & [RS]). In this paper we discuss ways of providing, both in undergraduate and continuing education teacher training programs, effective means for adequately preparing the teachers who must undertake the task of promoting the change. The backdrop for our work is the recent history of teacher training in Brazil, where Dieudonne's war cry of "down with Euclid", back in the sixties, seems to have echoed for far too long and with particularly zealous fervour. To speed up the process of educating the teachers needed, we take advantage of the momentum provided by a stronger contemporary stimulus over the educational system: the need to incorporate ICT technologies into the school curriculum. That the maths teacher must have access to adequate preparation to cope with it successfully is also true in this case. We propose to deal with the two needs in a single program, dedicated to prepare teachers to integrate ICT into their classroom through the device of placing them on an environment where they use ICT to learn geometry. In this work we intend to present a strong case for two assumptions we made when starting this project:

the benefits of using geometry software as an integral tool in the undergraduate geometry courses;

the benefits, both cultural and mathematical, of revisiting, through the viewpoint of dynamic geometry, classical results in the geometry literature.

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Teacher in service training: analyse a priori in focus

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Teaching Mathematics in Brazil is strongly influenced by the traditional model, where the teacher explains the mathematical theory, presents some examples, resolve some exercises and, finally gives a list of exercises for students to solve in class and at home. In 80's, almost all the textbooks reproduce this model. In the last decade this scenario begins to change. Brazilian Standards propose giving a more active role to pupils and emphasise problem solving in mathematics learning. Following this tendency, some textbooks are organised according to these propositions: exercises are proposed not only to apply mathematics theory, they also helps students to build knowledge. Despite these efforts, even agreeing with the Standards orientations, some teachers present a barrier to modify their practices. One reason for this is that in a model which privileges problem solving, the role of building interesting problems for each content is central in teachers activities. This research deals with teacher in service training in this new context. We hypothesised that involving teaching on research similar activities could probe their competence to build learning situations. To investigate this hypotheses, we adapted a part of the Didactic Engineering (Artigue, 1988) method to build a Teacher Training Engineering. The chosen content was area of planar surface. Thirty mathematics teachers experienced this sequence, which took place during 8 meetings (around 50 hours). The Teacher Training Sequence comprised: elaboration, presentation and discussion of teaching plans to this content; an epistemological study about the genesis of this concept; presentation of some research results about area learning; textbooks and Brazilian Standards analysis; analyse a priori of mathematical problems; finally re-construction of their teaching plans. The productions of six teachers who did all the proposed activities in chronological sequence are analysed. We also investigated whether other teachers' competence and knowledge were improved.

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Technology: The metric unit of all subjects

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The world we live is becoming a global village. No society want to crawl any more like their grandfathers. The educational system is not an exception to this revolution. In the past teaching was more of a subjective thing, that depends on the teacher. This makes learning process more difficult. But the advent of computer has make all this a forgotten experience. Infact you do not need a human teacher that would be bugged up by the extreme noise of student asking questions. With computer in place, a teacher that was aforetime limited by space and rancour of student can teach the whole world right from his bedroom without any stress. Student can as well ask questions and submit assignment on line just as I have submitted this abstract on line. There are even packages that can teach and handle all student problems. All these the paper try to bring to forbear.

Main Theme: Curricula Innovation
Secondary Theme: No secondary theme

Ethnomathematics: a new look at cultural dynamics and the Political Power

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This work had as empirical experience a teaching training course and research in Ethnomathematics in which teachers of the first grades of teaching from the Educational district of Mizque (Cochabamba, Bolivia) took part. The above mentioned course was development within the referential mark from what the significance of the bolivian educative reform which establishes the perspective, rules and the norms of regulations for the development of the bilingual intercultural educational politics in this country. Thus and departing from discussions about education, interculturality, bilingualism and ethnomathematics I tried to understand the relations of tension and struggle for the maintenance, valorization, substitution, acceptance of the different forms of explaining and understanding in the process of production, organization, establishment and diffusion of knowledgwe inside these teacher's reality of work and performance. My conclusion is that this process which is associated to the ·cultural dynamics· becomes straight very clearly by the dominants groups· actions providing dominance situations, acceptance and cultural resistance.

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Implementation of students' research works in mathematics by using Microsoft Excel

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The report is devoted to the questions of implementation of students' research works in mathematics by using Microsoft Excel. The main aspects of the problem are: 1) Selection of a package is qualified by following requirements: minimal collection of tools, sufficient impressive means for presentation of the results obtained. The capabilities of a package environment should be equal to the level of students' knowledge. 2) The chosen research problem, which has unpredictable or unclear solution, should stimulate student's interest. It needs to be formed clearly and comprehensively, based on students' life experience and include interdisciplinary links. 3) The implementation of research work combines empirical and theoretical methods of research. The final results should be presented in proper way for further consideration and exploration. The report proposes the two following problems: the first research is based on the Didona's problem; the second is about how to find a trajectory with minimal period of time.

**Numerical Algorithms - enhancing presentation while maintaining
rigour in introductory courses:
A minimalist approach to course modernisation**

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Recent academic developments which include a changing student profile, the focus of contemporary research, and social trends have combined to pose significant challenges in first year undergraduate mathematics courses. Issues which arise include the content level, the need to maintain rigour, addressing the needs specialist and non-specialist students, the need to equip students with useful, applicable, techniques, and our desire to present a picture of the important problems and directions in modern mathematics and its beauty and excitement. Calculus reform has made a significant impact but more needs to be done. There are large areas of research in which the computer has the role of an experimental tool. The use of software packages is widespread. This has produced the need for something akin to an instinct which can identify the correct or incorrect functioning of a package or black box. To acquire this instinct some knowledge and experience of the behaviour of numerical algorithms is needed. As consequence the way in which calculus is taught needs to be changed. It also needs to change because the computer has caused major changes in the theoretical directions of mathematics. These influences can be used to enhance courses whose content contains the essential foundations of the subject. The foundations will not change, but investigations of numerical algorithms, for example, can pose the same fundamental questions that are to be found in texts dating back a century at least. Well founded approximation methods provide exact rigorous statements. Numerical experimentation can provide insight. There is no need to present a grab bag of computations whose output is of doubtful validity. This presentation will briefly review the knowledge levels of entering students; it will describe some important applications and it will attempt to show how some of the challenges can be met.

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Main Theme: Education Research

Secondary Theme: No secondary theme

Useful Mistakes

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We discuss several examples of wrong proofs used as a teaching tool to improve the understanding of the theory. Many times students consider a theorem as a formula and do not pay attention to the conditions that have to be met. Incorrect proofs and mistaken use of theory can serve as an effective way of emphasizing the relation between the conditions and the conclusion and that it is essential to make sure of a formula's applicability before using it. The examples are taken from Calculus, Linear Algebra, Differential Equations and are based on our experience of teaching Mathematics to undergraduate engineering students.

Critical factors and prognostic validity in mathematics assessment

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High school mathematics is traditionally more procedural than conceptual in character, as well as formally less rigorous, than is mathematics at the university level, and hence puts less demand on logical reasoning and conceptual understanding. To find an instrument to make a reasonably good prognosis for success in undergraduate mathematical studies, it is therefore necessary to look closely at the demands of the future mathematical activities rather than only more narrowly at what has actually been accomplished at the high school level in terms of content and methods. In this paper the development of a short test for prognosticating academic performance in mathematics is discussed, and the results from a group doing the test when entering university is related to the results on their first mathematics courses. Based on research literature and an analysis of the demand of the courses, the design of the test was built upon ten factors that were found to be critical for passing the mathematics courses in the educational programme being considered: conceptual depth, control, creativity, effort, flexibility, logic, method, organization, process, and speed. The critical factors cut across the content-process distinction and are expressions of a holistic view of mathematical performance. To prognosticate academic performance it is necessary to identify important nodes of integration in the web of mathematical ideas, concepts, skills, forms, affects, and so on. The critical factors constitute vertices where the different dimensions of mathematical thinking meet. In the paper the construction of the test is discussed, and the results show a strongly significant correlation to performance on the target undergraduate mathematics course. A notion of prognostic validity of the test is outlined and discussed. The paper shows how test construction, analysis and interpretation of the outcome, depends heavily on what the result is going to be used for, and how a mathematics assessment design by necessity leads into discussions about the nature of mathematics and the understanding/performance of mathematics. What seems to be typical in mathematical problem solving is that many of the critical factors are involved in one problem solving process and must be combined for success.

An engineering bridging course - success or failure ?

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The aim of the study was to evaluate the success of a one-year undergraduate bridging course in Engineering (PBS) offered to educationally disadvantaged students, with special emphasis on the role of mathematics in addressing and overcoming some of the problems encountered by Engineering students. These problems include the inability to relate classroom examples to the real world, and the impotence of students to make approximations and estimates without calculators. The study briefly describes the aims and structure of the course which has a two fold purpose: firstly, to teach students how to think, guess, estimate and approximate, to use common sense and to translate real-life problems into mathematics; secondly, to provide the framework and basics of the first year Engineering mathematics course. Examples of the type of mathematics problems that teach students how to think clearly and creatively are cited. The mathematical performance of this group was compared with the performance of a large control group of Engineering students in each of the three years of study. This was replicated for four groups of PBS students. A statistical comparison of the groups showed that there are significant differences in the average mark and the pass rate at first year level. In the second and third year, there are differences in the groups, but these are not statistically significant. This implies that the advantage of the bridging class has been maintained throughout the years, and this group is indistinguishable from the other students. Details of the subsequent career of some highly successful graduates are given as well as comments and reflections of some of the students who have graduated since the inception of this program in 1986. Universally, these students consider this was the best year of their lives where they learned many skills which were never taught again in their studies.

The Global Classroom: A Live E-Learning Environment

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The Global Classroom is an NSF-supported CCLI project. As a distance learning modality, it seeks to address the high drop-out rate among students that register for largely asynchronous, text-based distance learning courses while at the same time providing learning opportunities “anytime anywhere.” The Global Classroom is a computer-mediated learning environment that emulates important features of the “brick-and-mortar” classroom. It provides virtual “eyeball-to-eyeball” contact through live interaction via high quality audio and video. Applications on the instructor’s or a student’s computer can be shared either one-to-one or one-to-many. . It supports synchronous, collaborative use of common software packages including Mathematica, Maple and various commonly used software packages, such as wordprocessors and spreadsheets via the Web. Control of a software package opened on one machine can be passed to a person at another machine in another room or even another geographic location. The applications could also be opened in a “self-exercise mode.” For example, if the instructor opens a Maple worksheet in this mode, the worksheet appears on the student’s computer and opens the student’s copy of Maple. The worksheet can then be completed by the student, saved on his/her computer, and then sent back to the instructor. Students can be assigned to “meet” for collaborative activity in small groups, independent of their respective geographic locations. Live sessions can be recorded automatically and stored in a web location accessible to both students who attended the session (for review of class activity) and for viewing by students unable to attend the particular session. These recordings contain all activities, material presentations, audio presentations and class discussions. The author will demonstrate the Global Classroom from both the instructor and student viewpoint, describe project activities, both completed and planned.

How can students' understanding of graphical aspects of the integral concept be enhanced?

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This paper is based on results of a research project on students' understanding of fundamental calculus concepts and on ongoing research on teaching strategies that may assist students in their efforts to develop a conceptual understanding of calculus. In the initial research project 630 students from three South African universities were subjected to diagnostic calculus tests (a pre-test and a post-test). Fifteen of these students also participated in individual interviews that were structured around specific test items selected from the written tests. The analysis of students' written and verbal responses to test items revealed significant information regarding the nature and characteristics of students' knowledge and understanding of key concepts of the calculus. An understanding of students' mathematical understanding is an important first step in the development of mathematical tasks and specific teaching strategies in order to provide students with worthwhile opportunities to become aware of the limitations of their own conceptions and to develop a conceptual and relational understanding of calculus content (concepts, symbols, algorithms etc.). The development of a conceptual understanding of calculus content is usually stated as one of the main objectives of the calculus reform effort. Since mathematical tasks are central to students' learning, the nature of mathematical tasks and teaching strategies that encourage students' active involvement when dealing with those tasks are closely related to the aforementioned objective. Findings of the current study suggest that meaningful communication of mathematical ideas amongst students and lecturer can be a key factor in the development of a conceptual understanding of calculus concepts and of the network of relationships among them and should therefore be fostered. This paper deals with students' understanding of graphical aspects of the integral concept and proposals concerning mathematical tasks and teaching strategies that prove to be effective in assisting students to develop powerful conceptions of the integral.

Text Structures in Teaching Mathematics

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Any good learning material must try to anticipate the learner's problems. The author should take into account that the reader is not with him and understanding his good intentions. Any given text can be understood as an alphanumeric string that is a rather annoying structure. We can distinguish three dimensions of the text: line, column, and the block one. There are usually many internal relationships between parts of the string. The transformation between linear and structured text can be explained as two opposite processes: aggregation and decomposition. Natural destruction of the text linearity can be applied to implications, classifications and parallel formulations. The modern word editors offer a large amount of possibilities for structuring texts. For several years, the Department of Mathematics of the TU in Liberec tries to observe the influence of a mathematical text written in structures on an acceptance of lectures and textbooks. The research started in 1999 and continued in 2000 and 2001 with the goal to verify what type of a mathematical text is better for students - classical linear or structured. The results of the student's polls are presented and discussed which were passed through in exercises of Mathematics. Hundreds of students of five faculties at the TU in Liberec participated in them. The last polls of our research show a shift in the direction of the structured form. According to the student's answers the structured versions of the text are appreciated. We could also read many remarkable, wonderful answers in the students' questionnaires. It could be very interesting for psychologists and pedagogues.

Main Theme: Mathematics And Other Disciplines

Secondary Theme: No secondary theme

Mixed Mathematics

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When mathematics are relevant in a domain of science, this is more than a simple application. With the expression "mixed mathematics" we use of an expression of the classical age (XVIIth and XVIIIth centuries), expression which seems to me more relevant than the expressions as "applied mathematics" or "application of mathematics". The construction of this "mixed" has multiple forms. In some cases, the mathematics can be constructed in the inside of the domain of science, so the calculus in the development of the mechanics with Newton and Leibniz ; it is only later than we can consider mechanics as an application of calculus or calculus as a tool for mechanics ; in other cases, we use mathematics because we recognize structural analogies between the science we consider and a chapter of mathematics. As an example of this mixed mathematics, we can give elementary geometry (the geometry of Euclid) considered as the study of solid bodies ; an other example is given by the theory of magnitudes and the notion of proportionality. After this, we can ask the question of the place of mixed mathematics both in the teaching of mathematics and in the teaching of other domains ; I shall speak only about physical sciences (geometry, mechanics, optics).

Contents and nature of preparation of mathematics teachers

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We present a design of formation for mathematics teachers, and some examples of the work we offer to future teachers. The objects that the didactics of mathematics want to study are not exactly the same as a naive or only professional approach could identify as pertinent ; didactic tools are efficient to analyse teaching situations, and anticipate ways of learning, but are not always easy to communicate to future teachers, knowing that these students often get a very formal conception of mathematics during their university courses. What are the conceptions of novice teachers on the mathematics to be taught ? on teaching practice ? How can we bring these conceptions to light ? What are the means at our disposal to make these conceptions evolve ? Is theoretical didactical knowledge efficient to make future teachers broaden their conceptions of mathematics ? Their conceptions on how to teach mathematics ? What are the situations that can be submitted to the novice teachers and help them understand the articulation between advanced mathematical notions and the contents of what they are going to teach themselves ? Which complex learning situations can be introduced in the preparation of young teachers with a reasonable chance that they unable themselves to teach with these situations ? Are these situations the same that are useful to understand mathematics for themselves ? Which pedagogic knowledge is necessary to help teachers drive complex learning situations ? Does the preparation take care of this knowledge, and how to do it, or is it left to the teachers' own initiative? We will present the principles with which we build aids novice teachers, the mathematical and didactical questions we submit to them, and the teaching reflexive documents we use to help them.

Animations to Illustrate Ill-Conditioning and an Introduction to Matrices using Maple

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Computer packages, usually Maple, are used in most of our undergraduate courses but less so in the lower level courses. For the students of land information (surveying), they undertake the equivalent of first year engineering mathematics in a year and a half. In the fourth semester they undertake the course MA067 designed specifically for them consisting of two thirds numerical linear algebra and one third introduction to complex variables and conformal mapping. In the third year the surveying students take a course 'Geometry of Surfaces' which is a classical differential geometry course with one lecture and one lab session (using Maple) per week. All calculation and assessment is done using Maple in the computer lab. This course has run successful for a couple of years. However the students find that it is difficult at the start of the course with the necessity of learning differential geometry and Maple concurrently. In MA067, some Maple was introduced. Only one hour per week was available for the classwork practice sessions and the Maple lab sessions. This resulted in the availability of five sessions for Maple. These were devoted to a general introduction to Maple and then work with entering matrices (using the Matrix data type in Maple 6) and solving linear systems of equations (matrix equations). Students used Maple to solve exercises that they had solved the week before 'by hand' in the practice class. To add interest, following an expository lecture, a Maple presentation was given to reinforce the understanding of ill-conditioning. Maple animations illustrated ill-conditioning and this was contrasted with an animation of a well conditioned system. These animations were strikingly effective and appreciated by the students. Besides anecdotal feedback, we report on a feedback survey designed to investigate student attitude to the Maple component of the course.

**Learning to teach algebra:
an Italian experience with reference to technology**

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The Ministry of Education (MPI) and the Italian Mathematical Union (UMI) have carried out a project for inservice teacher training in algebra. It consists on three stages. Firstly a group of 20 selected teachers attended a series of lectures. The lectures were videotaped. On the ground of these lectures the teachers produced didactic materials (forms, references, etc) recorded in a CD. The final products (videotape and CD) are sent to the schools all over the country to be shown to mathematics teachers. Four lecturers developed the subject (the teaching and learning of algebra) according to the following streams: · general educational issues based on international literature in the teaching and learning of algebra· algebra and information technology· a new approach to algebra through number theory, history and algebra. The present work reports the technological part of that teaching equipment. The use of a computer algebra system can improve the teaching of algebra, helping the teacher in several ways. However several difficulties can show up in the classroom use of CAS: elementary "pencil and paper" algebra rules and procedures are not always the same as the tasks performed by a machine, some problems arise in the relationship between algebra and graphics. The aim of the lessons and of material produced is: 1. to give the teacher a good knowledge of a CAS (Derive); 2. to explain the problems that can arise in the classroom use of symbolic math; 3. to give a hint for the solution of these problems..

What Is Modern in "Modern Mathematics"? How Should Modern Teaching Reflect This?

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It is commonly held that what distinguishes modern mathematics is the availability of high-speed electronic computers and pocket calculators with graphical capabilities. Consequently, mathematics is usually taught in schools and undergraduate courses as if Euler and Gauss were our contemporaries, with electronic gadgets replacing tables and sliderules. We discuss cultural changes in mathematics over the past two hundred years overlooked by such an approach, specifically the rise of rigour and algebra. These have altered the face of mathematics, providing a deeper understanding of many important results, by providing a unified, coherent setting. At the same time, these developments have increased the power and scope of mathematics, by enabling it to deal with non-quantitative problems, by making many computations accessible to computers and by making it more applicable to other disciplines. We use, in particular, the Fundamental Theorem of Calculus as an illustration and offer a programme for teaching calculus in a manner which accommodates these developments and eases the student's path to further studies.

The Mathematics Bridging Course at the University of South Australia

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The Division of Information Technology, Engineering and the Environment at the University of South Australia runs a Bridging Program with courses in Mathematics, Physics, Chemistry and Communication. The term 'bridging' usually conjures up the concept of a group of students already enrolled in a university degree obtaining some aid in a discipline to fill in gaps in their background knowledge. At the University of South Australia the goal is to provide an alternative pathway for prospective students to gain access to a science or engineering degree program. It is designed for people who either have a gap in their science background or have a comprehensive background but at some time in the past. The students study full-time for one semester or part time for two semesters. The author has been the coordinator of the mathematics component in the Bridging Program for several years. The innovative methods that have been devised to try and fill some of the gaps in the students' background will be canvassed. Traditionally, computer software in mathematical education has been primarily used for problem solving utilising such packages as Matlab and Maple. However, spreadsheets are a remarkably capable tool for both "doing" and illustrating mathematics. Their wide-ranging capabilities, availability and affordability, make them an ideal tool. The specific characteristics which make them applicable for illustration of mathematical concepts are their almost real-time graph alteration and their recursive capabilities. Microsoft EXCEL has been extensively used in a series of laboratory sessions in the mathematics course. Specific examples of its usage, along with the students' attitude to this, will be presented. The second section will concentrate on how well the bridging course has prepared the successful students for the degree programs they have subsequently undertaken. This is a necessary part of the evaluation and revision of the mathematics course.

Function, Continuity, Limits, Infinity: Mathematics Cognition and Technology

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We will argue in our presentation that using technology in Calculus class may establish a new semantic activity for undergraduate students. We present an investigation for meaning production for limit, continuity, and infinity. This investigation took place in a calculus and in a teacher education courses in Brazil. It is part of a larger case study research; we will exhibit an example of the development process of student's actions. In Brazil as well in many other countries, Calculus Teaching is being scrutinized. Some authors defend that we should teach derivative before limits, some calculus text books are starting to offer exercises using calculators and computer technology, but research on the processes of teaching and learning of Calculus is still considered necessary. We argue that computer technology plays a much larger role in learning math than being a tool for learning. Moreover, using computer technology for doing things faster, it is not good or bad use, but that has nothing to do with mathematics cognition. We view computer usage as a prosthesis that enables the student to do things in a different way. This view will be extended in the presentation. In this presentation we will analyze calculus undergraduate students using computer software that are able to draw graphs, particularly Graphmatica and Maple. We will present data and findings of two tasks:

To comment on what happens when you draw a given discontinue function using Graphmatica. The students engaged in a lively discussion about continuity, function domain restrictions, number sets, semantics and syntax implications of using technology among others.

To comment on what happens when you draw other given discontinued function using Maple. Theoretical and methodological foundation and the results will be discussed.

Suspension of sense-making in mathematical word problem solving. A possible remedy

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In common teaching practice the habit of connecting mathematics classroom activities with reality is still substantially delegated to word problems. But besides representing the interplay between mathematics and reality, word problems often are the sole example of realistic mathematical modeling and problem solving. During the past decades, a growing body of empirical research (e.g. Freudenthal, Greer, Schoenfeld) has documented that the practice of word problem solving in school mathematics promote in the students an exclusion of realistic considerations and a "suspension" of sense-making and hardly matches the idea of mathematical modeling and mathematization. Several studies point to two causes for this situation: textual factors relating to the stereotyped nature of most frequently used textbook problems and presentational or contextual factors associated with practices and expectations related to classroom culture. Furthermore, it is noted that the use of stereotyped problems and the accompanying classroom climate relate to teacher beliefs about the goals of mathematics education. If we wish situations of realistic mathematical modeling, that is both real-world based and quantitatively constrained sense-making, we have to make changes: i) we have to replace the word problem solving with classroom activities that are more relatable to the experiential worlds of the pupils and consistent with a sense-making disposition; ii) we will ask for a change in the teacher conceptions, beliefs and attitude towards mathematics; iii) a directed effort to change the classroom socio-math norms will be needed. In this paper we discuss a teaching experiment in a primary school, based on the use of suitable cultural artifacts and on interactive teaching methods, that takes these three factors into account. The idea is not only to motivate students with everyday-life contexts but also to look for contexts that are experientially real for the students and can be used as starting points for progressive mathematization.

The "plus" provided by graphics calculators in teaching undergraduate statistics.

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It has long been accepted that the use of technology, in the form of computer packages, is beneficial in teaching undergraduate statistics. However, having recognised the potential of graphics calculators with inferential statistics capabilities, the relative roles of the different forms of technology were investigated. Initially, the focus was on calculators versus computers, evaluating the students' preferences. It soon became clear that it is technology as a whole that is important in a statistics course, rather than one particular form. Consequently, during 2001 the emphasis has been on providing access to learning with a whole range of technologies. Through surveys and interviews, the students have indicated that, whilst they recognise the need for computer packages in future work situations, their learning has been greatly enhanced by the use of graphics calculators. This seems to be due, in part, to their existing familiarity and confidence with the calculators as much as to the calculator's capabilities. Graphics calculators are required in the school leaving examinations in Western Australia and the majority of science students arrive at Murdoch University owning one that has statistical inference facilities. (Typically, about three-quarters of the students have a graphics calculator capable of statistical inference in their final examination.) The benefits to effective learning gained by incorporating, as an extra learning tool, facilities that the students already have at their fingertips have definitely outweighed any extra time required in developing appropriate learning activities.

Main Theme: Preparation of Teachers

Secondary Theme: Innovative Teaching

Significative Teaching in Engineering - Why not?

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In the paper there is studied the significative teaching of Mathematics in engineering, the significative teaching is studied in its theoretical and practical aspects, the problemical teaching of Mathematics is studied theoretical and practical, there are examples of using the problemical teaching in engienering, there are made many considerations about the Mathematics of the 21ist. century.there is studied the importance of the mathemmatical curriculum taking in mind the differents specialities of engineering where they are teaching. There are many practical and theoretical conclusions and recommendations.

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Main Theme: Preparation of Teachers

Secondary Theme: Innovative Teaching

Significative Teaching and Creativity in undergraduate Mathematics at the Universities

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In the paper are studied some important questions like Mathematical Teaching and its significative and creative teaching. There is a necessary introduction to the history of mathematical teaching and its 20th. century last tendencies. There is a critical study of that tendencies their good and bad sides. there is an study of the social importance of the mathematical curriculum and its teaching. There are studied the theoretical and practical aspects of significative and creative mathematical teaching. The problemal teaching in Mathematics is studied theoretical and practical. There are given many examples and indications for teaching undergraduted mathematics in a significative and creative forma. There are made conclusions and recomendations for teaching in that way.

SEMANTIC UNDER-LOADING: THE LESSON OF LOGS

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The extension of calculators to all second-level school grades terminates the use of logarithm tables. The majority of students have thus no exposure to logarithms prior to becoming first-year undergraduates. The task of teaching an understanding of logs which now falls to the college lecturer is better known historically for its failure than for its success. Under-loading of semantics in the area is to blame, and the mathematics community might mark the 400th anniversary of Napier's discovery with an appropriate pedagogy-driven remedy. Under-loading is the mirror-image of over-loading. The sharpest example of over-loading is the symbol \cdot used in fraction notation. This symbol carries at least four separate meanings (fraction, division-operation, ratio, rate) not all of which obviously intersect. The consequences for pedagogy in arithmetic and algebra are great. In semantic under-loading the domain of application of a symbol or term is restricted by the definition of a separate symbol, having precisely the same meaning, to apparently mean something different. This happens with the differentiation of the terms \cdot index \cdot and \cdot log \cdot (and to a lesser extent \cdot exponent \cdot). This leads in turn to the generation of spuriously distinct theories: "The Rules for Indices" and "The Laws of Logs". Mystification is accentuated by confused use of the term "power" due to an historical colloquialism. In a perverse way the advent of calculators provides space for the mathematical community, aware of its role in the confusion, to simplify the notation and integrate the semantics. A suggestion is made on how to do this without completely breaking with tradition. Not to undertake this task is to evade a central tenet of mathematics, that the definition of its symbols be sufficient and necessary. Besides, the use of over-loaded and under-loaded semantics accounts in part for both undergraduate alienation and a loss of understanding.

How to prepare students for a successful first year at university: an experience.

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During the last years, the basic mathematical knowledge with which Belgian students have enrolled has decreased a lot. This phenomenon results from various causes that we will try to outline. As a direct consequence, most of the students do not possess enough mathematical skills to follow the usual curriculum and face enormous difficulties from the start. In fact, only the better of them will go through these obstacles. In an attempt to give a chance to everyone, we have set up since 1999 a special system of support. Here are its main characteristics: a course has been added to the first year curriculum with the goal to deepen the understanding of high school mathematics. During this course, the students benefit from individual help from the teacher but also from a set of more advanced students who intend to become high school teachers. We will explain the organization of this course (unfolding, material covered,...) and will assess the students progresses. Our students also have access to another kind of support, more targeted to specific courses of the curriculum. A special session takes place once a week which focuses on the problems encountered by the students. Although this session is optional, the participation rate is high. We will show what makes this system work well and will analyze the positive effects on students successes.

Are students able to transfer mathematical knowledge?

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The ability to use mathematics in other disciplines is generally expected of all science and engineering students. Anecdotal evidence suggests that many students lack this ability. While there is a substantial body of research dealing with the transfer of training, and the transfer of mathematical skills to problem solving in everyday life, there is very little relating to the transfer of mathematics to other scientific disciplines. This paper reports on the development and trialling of an instrument which can be used to research the ability of students to transfer mathematical skills and knowledge to other disciplines. The instrument consists of mathematical problems set in various contexts. All the problems involve exponential and logarithmic functions, and are based on scenarios from physics, microbiology and computer science. In each case, any discipline-specific knowledge required to solve the problem is given, so that all the problems can be solved with mathematical knowledge only. The problems were initially written by a physicist, a microbiologist and a computer scientist. The instrument has been trialled with 50 first year science students at the University of Sydney. Performance on the instrument has been correlated against final high school marks, first year university results, and subjects studied. These results are presented. The paper also discusses some of the interesting issues which arose from the collaboration of a mathematician with academics from three other scientific disciplines. For example, differences in the ways the physicist, the microbiologist and the computer scientist used mathematics were apparent. Also, their use of mathematics was often quite imprecise. Such issues have important implications for the teaching and learning of mathematics, both as a subject in its own right and within other disciplines.

Multiple choice questions; can they help in learning proof?

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I teach a largish (250 students) first year 'computing mathematics' class based on Eppcite. One of the intended outcomes is that by the end of the course students can "understand and create proofs of simple numeric and algebraic propositions." Financial constraints mean that there is very little time for tutorial discussion, or critical grading of homework, relating to students' attempts at proof analysis and creation. In this paper I discuss an approach to address this shortcoming through the creation and publication of multiple-choice quizzes using the web-based system Socrates. Web-based learning cannot hope to replace interaction with an expert human teacher, but it does have some advantages; it is available at any time and at remote places, it can be worked on at any student's individual pace, and it has infinite patience. Here are several ways to try to improve the effectiveness of multiple-choice quizzes as an aid to learning proof: Question design: Ask about both correct and incorrect proofs and proof fragments, how to start a proof, how to continue from a given start, and how to provide counterexamples. Feedback: Use distractors that students are likely to 'fall for' and for which instructive instant feedback can be written (to be provided to the student by the system). Also provide reinforcing feedback for the correct response. Assessment: Employ an assessment scheme which rewards active learning via the system. Examples of the application of these principles are presented, along with a demonstration of the capabilities of the Socrates system from both the student's and instructor's viewpoint. The Socrates system has been specifically designed for mathematics; the instructor's input is in LaTeX and the on-screen mathematics is perfectly rendered in the browser via the widely available (and free) Adobe Acrobat Reader plug-in.

Epp S: Discrete Mathematics with Applications 2nd Ed., Brooks-Cole 1995.

Brooks M, Hewett M & Norton A: Socrates Web Quiz

<http://socrates.ise.canberra.edu.au/viewer/login.php>, University of Canberra 1998.

Main Theme: Preparation of Teachers

Secondary Theme: Curricula Innovation

Preparing Teachers-A Diagnostic Mathematics Course

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Preparing Teachers-A Diagnostic Mathematics Course At the University of Houston-Clear Lake (UHCL), undergraduate mathematics students preparing to teach complete a three-hour Diagnostic Mathematics course in which they work on a one-to-one basis with a child between the ages of eight and thirteen. The university students administer and evaluate diagnostic tests; conduct parent, student, and teacher interviews; and analyze measurement and screening data provided by the school. Based on these data, the student and the university faculty member develop an achievement plan for each student. Background Information Undergraduate students working toward elementary or secondary mathematics certification staff the UHCL Diagnostic Mathematics Clinic. Teachers or their parents typically refer children to the clinic. Information about the clinic appears in local newspapers and flyers describing the clinic are sent to area mathematics supervisors, elementary schools, and middle schools. There is a registration fee for the clinic and the children meet at the university one and one half hours each week for ten weeks. The parents and the child's mathematics teacher complete questionnaires about the child and mathematics. This presentation will describe the course and the impact of the clinical experience on undergraduate students. Questionnaires and survey instruments will be shared with participants. According to one university student, "When my student, Kristen, felt good about correctly answering the problems assigned to her, a little grin would appear on her face, and she would have this little smile that I interpreted as 'I'm Good.' This had to be one of the most warming experiences I have ever felt. I knew I had done well. Not only had Kristen accomplished the task of learning, but I had accomplished the task of teaching. Diagnostic teaching is an attitude that cares very much about each student's learning. I will carry this attitude with me into the classroom."

Mathematical Problems On the Internet

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At the Laboratory for Mathematical Modelling of Technology Processes at the Liberec Technical University we have prepared a program system suitable for automatic generation of typical mathematical problems for undergraduate students. The system creates (with the help of parametrisation) both problems and solution so that it can be used by teacher as well as students. The system is written in JAVA and uses database program MYSQL. Final output is a postscript file generated from the \LaTeX source prepared automatically by the system without any interaction of the users. The following fields are covered: differential and integral calculus of one and several variables, linear algebra, basic numerical methods, differential equations, probability and statistics, optimization, elements of high school mathematics. The system is available on the address `http://daemon.kap.vslib.cz/matematika` and its basic version is free for use. It is enough to login as `·host·` using the password `·host·`. For registered users the use of the system is free and offers more advanced features of the system. During the lecture we will present on-line use of the advanced features of our system.

Teaching and doing mathematics with 'under-prepared' students.

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This paper draws from a study that focused on a group of students enrolled for courses in undergraduate mathematics. These particular students had matriculated at schools where teachers had to do their best with large classes and minimal resources. Results from studies such as TIMSS suggest that these students had limited exposure to problem-solving techniques. The study began with the recognition that past practices might hinder the students' attempts to create their own solutions. Therefore, workshops were structured so they worked in small groups, using concrete materials, at investigative tasks. Initially, students were hesitant to experiment and lacked the confidence to engage in mathematical discussion. This reluctance may have been exacerbated by the fact that their first language is not English. However, findings showed that, firstly, the use of concrete models and manipulatives gave the students a practical and visual approach that they could later apply to developing their own representations in further investigations. Secondly, the vital role of cooperative discussion was noted as an important part of leading students to engage in their own individual mathematical interpretation, reflections, explanations and suggested solutions. Further, teachers were seen to have a decisive role in encouraging and supporting effective mathematical conversations. This paper drew inspiration from the writings of Arthur Powell and Alan Schoenfeld, as well as the research on cooperative work by Johnson and Johnson. It supports Biggs' belief that the key to teaching for quality learning lies in the words: 'what the learner does is actually more important than what the teacher does'. (Shuell quoted in Biggs (2000:xii) *Teaching for Quality Learning at University*) In conclusion, it is noted that the issues around teaching to develop ways of thinking that encourage problem-solving and abstract formulation are extremely complex. There can be no simple answers and the teaching practices and outcomes described in this paper are meant to invite further discussion.

The History of Mathematics in the Education of Mathematics Teachers: an innovative approach

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It is a widespread view that mathematics teachers should have a working knowledge of the History of Mathematics, for three reasons. First, so they can make lessons more lively and interesting by adding to the lessons stories about mathematicians of the past; second, to help students to develop a sense of Mathematics as a human production, always evolving; and, third, to develop a better understanding of the foundations of Mathematics. We understand there is a fourth reason: through that study the teacher can develop an understanding of the process of meaning production for Mathematics that would allow her/him a much finer reading of the learning processes in the classroom, as well as an understanding of the possibility of different meanings being produced for the 'same' mathematical object, for instance, 'linear equation', 'function' or 'dimension'. We have developed and conducted a course on the History of Mathematics for undergraduate students in which we read and discussed, over 30 two-hour sessions, four texts: (1) C. Wessel's paper on the analytical representation of directions; (2) G. G. Granger's text on the philosophy of style (a section related to Euclid's 'Elements'; (3) a section of A. Aaboe's 'Episodes from the Early History of Mathematics' (part of chapter 2); and, (4) a section from R. Hersh and P. Davis' 'The Mathematical Experience' (on the Chinese Remainder Theorem). We went from a primary source (difficult reading for them) to texts which discussed 'style', 'interpretation' and 'different presentations', aiming at helping them to develop an awareness of the processes involved in meaning production for Mathematics; as much as possible we related the current experience with the experiences they had as undergraduate students taking Mathematics courses. Data from the course will be presented and discussed.

Teaching and Learning Mathematics with Virtual Worlds

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A lot of mathematics and statistics is taught using textbook examples and exercises. It is difficult through these to give students a feel for the broad issues involved. Ideally students could carry out experiments in the real world to then model and analyse mathematically. However, this is not practical for large classes and the logistics may even detract from the learning. We propose using virtual worlds instead, allowing students to manipulate the parameters of a simulated experiment and record the results. These simulations should be messy, requiring the students to think about measurement issues and noise and how these impact on the mathematics. The results can then be used as a starting point for teaching, in place of the traditional exercise settings. We give experiences and feedback from several virtual worlds for statistics and discuss current work on virtual worlds for calculus.

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Main Theme: Technology

Secondary Theme: No secondary theme

Developing students' research skills in the process of studying numerical methods of solving equations with computer support

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The organization of students' research activities in learning mathematics especially algebra and mathematical analysis has some peculiarities because of abstract ideas, concepts, objects, specific solving methods and proofs. The process of studying numerical methods of algebra and analysis is complicated for students: they have to choose and implement the optimal numerical method or methods during problem solving. Students should have a sufficient level of developing research skills for successful result. The system of computer and methodological support (SCMS) of students' research activities in the process of studying numerical methods of solving equations is concerned. It's includes system of tests and help, students' worksheets, training tasks, teachers register. Using of SCMS step by step develops students' research skills.

Mathematics Education Laboratory and the construction of a (new) mathematics culture

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In a highly technological, communication society of knowledge which evolved at an alarming rate, is incontestable the importance of a solid mathematical and technological education that contributes towards, in a final analysis, a better quality of life for all citizens, on whom strongly depend a timely and efficient solution to a multiplicity of problems and problematic situations with which they are confronted daily. The active, effective involvement of the pupil in the process of construction of knowledge through the most diverse and significant laboratory activities possible, could be a precious contribution to the education of citizens capable of being at ease with uncertainty and change and, at the limit, to the construction of a technological culture which is not to be only recreational or technical, and of a mathematics culture which is not to be only instrumental, academic and decontextualised. It's in this context, and in the confluence of the triptic · research, training and innovation · that the project ICT and the construction of a (new) mathematics culture arises. The project seeks, as a final goal, to devise, develop and evaluate innovative strategies and methods for pre-service teachers, namely through laboratory approaches (recently) recognised as a way of obtaining significant mathematical learning, facilitators of the construction of a new mathematics and technological culture. It has as its main functional structure the LEM@tic · Mathematics Education Laboratory -- a facility that, by strongly supporting itself on ICT, will be structured on thematic and transversal ·stations·, covering the most diverse mathematics curricular areas, at various educational levels, assuming four main domains: mathematics history; task (types, connections, materials), curiosities and bibliography. These as well as the inherent aspects of Rentability of the Lab will be considered in the present paper.

REALISTIC CONTEXTS IN MATHEMATICS: THE HUMAN BODY.

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The necessity to use realistic contexts in the classroom to link mathematics to the students' reality is generally assumed in mathematical education. This is not only due to its methodological aspects but also to the emphasis on the learning of how to use mathematics as a very useful tool to analyse reality and understand the world around us. An example of this importance is the amount of research that can be found on the teaching and learning mathematics in contexts as in the Realistic Mathematical Education (RME) investigations. In order to fulfill this important task effectively it is essential that these realistic contexts verify some conditions to be more useful and successful in several and different aspects such as age, level, interests of the students, etc. It will be shown that one of the greatest and best source of specially interesting contexts is the human body itself that allows teachers and learners to experience a new approach to mathematics in the classroom. Nowadays due to the amount of information we have about the human body there is a link with mathematics which allows us to handle these data. Some examples will be given to take advantage of these realistic contexts in the classroom by bringing Biology and Mathematics together.

THE CORRELATION BETWEEN ATTITUDES OF PUPILS TOWARDS MATHEMATICS AND THEIR MATHEMATICS PERFORMANCE

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This study investigates the correlation between the attitudes of pupils towards mathematics and their mathematics performance in primary and secondary education in northern Cyprus. Initially, a mathematics attitude questionnaire has been given to approximately 2000 pupils in primary and secondary schools. As a second step, a performance scale which was prepared by the researchers to measure and evaluate the math performance of pupils was given to the pupils. On the basis of the outcomes of the questionnaire and performance scales, it is claimed that the attitudes and performance about mathematics are correlated. During the conference the results of the study will be given.

MATHEMATICS TEACHERS INITIAL TRAINING AND COLLABORATIVE WORK IN THE IUFM

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For several years, Technologies of Communications have been intensively used by the trainers in mathematics of the training college I.U.F.M. of the North of France. Use of the e-mail for the memoirs of maths, use of WEB resources for the history of mathematics, an on-line bibliographic data base, a data-base of memoirs and web sites created by trainers:

the C.R.E.A.M, a center of pedagogical resources for the trainees;

Mathadoc (A6-3), the electronic schoolbag of the secondary school teacher that the teacher can modify it the way he wants.;

LILIMATH: discovery workshops for a use in the classes received in 1998 the first prize of a national competition (cervod) of software tools for training;

FUNCTIONS allows an individual following-up of the works of the pupils (fourth price in that same competition) ;

GEOWEB, a site presenting some creations made by the pupils themselves on the solving of some open problems of geometry, has received in 2001 a national price for innovative methods.

All these groups of teachers use e-learning software for their network. In 2001-2002, a new program for the training in technologies has begun, in link with the memoirs. The trainers are themselves engaged in creation of ressources and animation of networks, and a workshop with the trainers engaged in the memoirs will analyze this new training. This program is part of a program called FORMASCIENCES, with laboratories of the North of France; the essential points of this program are the following : use of simulations in sciences, real teaching program, with references to distributed cognition and collaborative work.

This communication will give the essential point of this program and a paper will analyze the first year of training.

Main Theme: Mathematics And Other Disciplines

Secondary Theme: Innovative Teaching

Service with a Smile

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In this paper I will discuss some aspects of specialised service teaching, by which I mean the teaching of mathematics to an identifiable group of students with a shared primary interest which is not mathematics. I will first argue for the vital importance of service teaching in general, not because of its budgetary implications for mathematics departments, but because of its role in ensuring the overall health of mathematics as a discipline. I will then examine two key issues concerning the teaching of specialised service courses, namely whether mathematicians should teach such courses, and if they do, how they should approach this task.

**Learning to teach algebra:
an Italian experience with reference to history**

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The Ministry of Education (MPI) and the Italian Mathematical Union (UMI) have carried out a project for inservice teacher training in algebra. It consists on three stages. Firstly a group of 20 selected teachers attended a series of lectures. The lectures were videotaped. On the ground of these lectures the teachers produced didactic materials (forms, references, etc) recorded in a CD. The final products (videotape and CD) are sent to the schools all over the country to be shown to mathematics teachers. Four lecturers developed the subject (the teaching and learning of algebra) according to the following streams: · general educational issues based on international literature in the teaching and learning of algebra· algebra and information technology· a new approach to algebra through number theory· history and algebra. The present paper reports on how the history of algebra in relation to classroom practice was developed. It was taken as a common thread going across the algebraic development the method of analysis. This allowed to see in a different perspective the passage from arithmetic to algebra as well as to see the links between algebra and other parts of mathematics (in particular, geometry). Moreover many historical forms have been arranged to illustrate the main points of algebra development. These forms should help the secondary school student to get over the great difficulty in learning how to construct and solve equations and also the cognitive gap in the transition from arithmetic to algebra. All this work is in accordance with the recent research on the advantages and possibilities of using and implementing history of mathematics in the classroom that has led to a growing interest in the role of history of mathematics in the learning and teaching of mathematics. On the ground of the described activity, in which history was integrated in a kit of topic concerning the teaching of algebra, we are encouraged to make recommendations as to how history can and should impact the algebra curriculum in the future.

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Main Theme: Education Research

Secondary Theme: No secondary theme

FACTORS AFFECTING THE MATHEMATICS STUDY HABITS OF PUPILS

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This study reports on an ongoing survey of primary, secondary and high school mathematics pupils in northern Cyprus. The questionnaire was prepared by the researchers and was given to approximately 4000 students which is 10% of the population. The aim of this study is to find the relationship between the mathematics study habits of pupils and the performance in mathematics lessons indicated by the marks obtained in mathematics tests. Preliminary results, conclusion and recommendations will be presented in the conference.

A COMPARATIVE STUDY ON MATH EDUCATION RENDERED IN THE TWO SOCIETIES ON THE ISLAND OF CYPRUS

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There is no doubt every country has its own unique education system that complies with the demands of its society, culture, values and vision for the future. Math education is a long process starting with the first years of the elementary school, and continuing throughout one's education. The Island of Cyprus embodies two distinct societies having their own systems of education. But on the road to the EU certain parallelisms need to be set as it is the goal of this study on math education rendered in the two societies.

This paper will concentrate on K-1 to K-12 math education. The topics to be dealt with will cover math curriculum at all levels; weekly distribution of math classes, as well as their proportion to the other courses offered. The present century demands experimental teaching and learning, therefore, the study will aim to bring out the teaching/learning strategies in math education, as well. Students are expected to be more creative in class rather than resorting to rote-learning even in math education. The study will also try to shed light on class size. Finally the math teachers and their educational background will be looked into so as to check into their prior math knowledge.

These points will be treated from the standpoint of the math educational systems carried out in the two societies that form the Island of Cyprus, and the main goal is to bring about the similarities and differences, and to come up with a compromise so as to comply with the standards of EU as well. Therefore, certain statistical data obtained from EU countries will be used for the purpose of comparison. The study will end with the recommendations to establish a better and a more harmonious education that will break the barriers between the educational systems of the two societies.

Activity Mathematics

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Mathematics is something that people do. In the ages before the recent rapid developments in technology, this activity called 'doing mathematics' has been restricted to those who happened to be able to master a variety of artificial, mechanical, formal processes. The necessary conditions for one to be a candidate to practise mathematics have included for instance mastery of the mindless symbolic process of manipulation of formulae, and possession of the magician's box of techniques such as symbolic integration. Now though, technology allows freedom for many more people to benefit from being able to 'do mathematics', and for others to benefit from the results of that. 'Doing mathematics' has always been much more than just being able to carry out manipulations on paper. It is now easier to perceive it and to present it to people at large as a broader activity which enables one to gain insight into the world, encompassing a rich combination of communication between reality both internal and external, words, pictures, and numbers, and a formalised language. Thus the idea of thinking logically and analytically in order to make human sense of the world can receive more emphasis than the repetitive practice of mechanical skills. In propagating this wider view, mathematics becomes more obviously a 'people' activity. We present in this paper some of our own recent experience of positively developing courses, for students of mathematics and others, to incorporate developing technology. Packages involved include for example Mathsoft Studyworks, TI-Interactive and Cabri Geometry, but the important issue is not precisely which packages we currently use, but how we have changed what we now perceive as 'doing mathematics' now that rapidly changing technology is here to stay.

**Points collinearity and corresponding determinants evaluation.
Error detection and analysis in Analytic Geometry at the University level**

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We present the process of detection and analysis of mathematics students' systematic misconception in learning undergraduate Analytic Geometry. That misconception, not yet reported, in the bibliography, is difficult to observe, because in most of cases it leads to a correct answer by an incorrect reasoning. The starting point of this study was an examination question in Analytic Geometry for first year mathematics students at the University of Ioannina, concerning the examination of the collinearity of three points, given by their coordinates in a 3D Cartesian system. To answer the question the majority of the students calculated the determinant of the coordinates of the three points, which was different from zero, and concluded that the points were not collinear, without any other justification. Although the answer was correct, we suspected that it was based on an incorrect reasoning. We made the hypothesis that in the case of a vanishing determinant, the students will erroneously conclude that the points are collinear. Therefore, we added a complementary question concerning the collinearity of three non-collinear points, with vanishing determinant. Afterwards, we asked the students to solve the same problems with a different method, in order to provoke cognitive conflict and lead them to see the inadequacy of their method. These questions were presented to two groups (30 students each) of first year students in the tutorials of Analytic Geometry, two groups (20 students each) of fourth year students in the tutorials of Didactics of Mathematics and two groups (35 and 100 students) of fourth year students in the written examinations of Didactics of Mathematics. All groups belong to different generations of Mathematics students at the University of Ioannina. In all cases about half of the students used the method of the determinant and their answers confirm our hypothesis. Furthermore, for the majority of students, the only stimulus to solve the problems with a different method had not the expected results.

What is important for students to know prior to taking calculus?

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The evidence suggests that calculus is overloaded with rich concepts that are ancillary to calculus proper. We have found that some of these ancillary concepts can be studied earlier in a laboratory component for precalculus with the use Mathematica, a software product that includes a computer algebra system. Our work provides fresh answers to a question that has persisted since the advent of technology: what role does technology have in instructional use? We have been using Mathematica at Hunter College in a multi-section 1 credit, 2 hour course entitled Precalculus Technology Laboratory. This laboratory course has been given at Hunter College since spring 2000, and is a corequisite to the traditional 4 credit, 4 hour Precalculus course. Included in this laboratory approach are several nonstandard precalculus-level topics that are either impractical or impossible to treat using either paper-and-pencil methods or graphing calculators:

- Extensive mathematically-enhanced graphing in 1D and 2D.

- Creating animations.

- Finding the turning points for polynomials of any degree (without calculus).

- Applying concepts of local and global maxima and minima.

- Finding the point of symmetry for any cubic.

- Max/min problems for precalculus that involve only polynomials

Several of these topics were previously inaccessible at the precalculus level. In addition, we treat several other nonstandard topics:

- Distinguishing between plotting and graphing.

- Computing polynomial asymptotic behavior of rational functions as $x \rightarrow \infty$.

- Equality of polynomials theorem.

- Zeros of even multiplicity theorem

- Utilizing implicitly defined functions

Materials for this course have been put together in a text, *Preparing for Calculus Using Mathematica: A Laboratory Approach* (in press). It is hypothesized that by taking ancillary calculus concepts and making them accessible to precalculus students, the chances for success in calculus are improved.

USING MATLAB TO SOLVE A CLASSIFICATION PROBLEM IN FINITE RINGS

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Although most linear algebra problems can be solved using a number of software packages, in our judgment MATLAB (MATrix LABoratory) is the most suitable package. MATLAB is a versatile and powerful, yet user - friendly software package designed to handle wide - ranging problems involving matrix computations and linear algebra concepts. MATLAB incorporates professionally developed quality computer routines for linear algebra computations. In this paper, we make use of elements from MATLAB to devise a program that helps in determining the structure and classification, up to isomorphism, of a naturally arising class of finite associative local rings. We demonstrate this in the case where the finite local ring has a finite residue field K of characteristic p , although our results apply in fact over any field K .

The inverse numerical function concept and computer software learning.

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There is a dialectic between the effect of the instrument (·artefact· and utilization schemes) on the construction of mathematical knowledge and the effect of mathematical knowledge on the ·instrumentation dimension· of ·instrumental genesis· process. The availability of tools such as spreadsheets means that decisions have to be made concerning didactical approaches. This proposal reports on data from a study of trainee teachers who are preparing to teach statistics and computing in French agricultural education schools. I investigating the effect of the "artefact" on the comprehension of the inverse numerical function concept while determining the significant value in a hypothesis test. These teachers usually encounter this difficulty. For example, one of the obstacles in the simultaneous learning of mathematics and computer software seems to be that the strategy of trial and error for solving problems is not encouraged in the teaching of mathematics in France. Indeed, some trainee teachers do not solve the problem alone. The following theoretical frameworks are combined : Pierre RABARDEL (1999) : Cognitive approach to instruments Michèle ARTIGUE (1998) : Didactical engineering. My observations protocols are built on experimental situations which enable the study of spreadsheet utilization schemes, where the spreadsheet is considered as an ·artefact·. I decided on a clinical rather than a statistical approach to the experiments and the observations. The study used video recordings with transcripts. The data collection protocol enabled us to focus on two aspects of a praxeology, and the filmed session allowed the analysis of the tasks and techniques used by the subjects. After the a priori study of different the kinds of obstacles (mathematical, statistical, didactical, epistemological and computing) that this task involved, I will give some results : techniques used by the trainee teachers and effective observations of the obstacles encountered.

PROBLEMS OF TEACHING ANALYSIS IN COLLEGES

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"Functional Technique of Teaching" consists primarily of the following characteristics:

- (i) Explaining the scope, the structure (nature) and contents of the core knowledge, in their true spirit.
- (ii) Familiarizing the language of mathematics discipline.
- (iii) Bringing to the limelight the axiomatic of the systems and foresee the implications of the deductions thereof.
- (iv) Switching over from computational skills to conceptual abilities - about mathematical modes of thought.
- (v) Reflecting the philosophical, historical and educational aspects of mathematics.

Such a functional and instructional technique of teaching, will primarily aim at developing concepts, relations and structures. Students frequently complain that much of their first year consists of pernickety and tedious proving of the obvious, in the great cause of rigorous proof. This suggests to me that there is a serious lack of motivation. In this paper, we shall discuss in detail the problems of teaching Analysis with reference to Functional Technique of Teaching.

The difficulty Level of Function Representations Among University Students

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The concept of function is of fundamental importance in the learning of mathematics and has been a major focus of attention for the mathematics education research community over the past decade. The understanding of functions does not appear to be easy, given the diversity of representations associated with this concept, and the difficulties presented in the processes of articulating the appropriate systems of representation involved in problem solving. Aspinwall, Shaw and Presmeg (1997) asserted that in many cases the iconic (visual) representations can cause cognitive difficulties, because perceptual analysis and synthesis of mathematical information presented implicitly in a diagram often make greater demands on a student than any other aspect of a problem. Although there are a lot of studies dealing with students' conceptions of function and their difficulties in coming up with the function concept, there is more to be uncovered with function's representations and the connections between these representations. Aims of the Study: This study purports to contribute to the ongoing research on translations between representations in functions by identifying the pattern and levels of difficulty of fundamental modes of function representations. Furthermore, the study aims at identifying variables, such as translations from one mode of representation to another that might explain the order of difficulty of representations and suggest a coherent instructional procedure for teaching them. Method: A written test was administered to all first year students studying mathematics at the University of Cyprus (N=38). The test consisted of 24 tasks with functions expressed in algebraic, graphical and diagrammatic forms. Each correct answer was marked as 1 and each wrong answer as 0. Results: The main contribution of the present study was the identification of hierarchical levels among the graphical, diagrammatic, and symbolic representations of mathematical functions. An association was verified between the students' ability to handle various functions, and the translations among representations of functions.

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BASIC-ALGORITHMIC TECHNOLOGY in TEACHING of UNDERGRADUATE MATHEMATICS

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We introduce the computer-oriented Basic-Algorithmic Technology (BAT) for teaching and learning basics of mathematical courses at the undergraduate level. It is based on creation of computer files: the basic didactic materials (BDM) and algorithmic didactic materials (ADM). A special symbol-graphic language has been worked out to create BDM and ADM. These didactical materials are compact, illustrative and free from language barriers.

Main Stages of the Basic-Algorithmic Technology:

Stage 1 Basic structurization of instruction material To extract the basic information from the portion of chosen instruction material. By basis we will mean a minimum subset of knowledge and skills that are necessary for learning all the instruction material. The basis has a two-functional orientation: the theoretical basis and the practical one.

Stage 2 Basic didactical materials To develop computer oriented basic didactic materials (BDM). These didactic materials contain symbol-graphic formulations of basic definitions (def.), theorems (th.), formulas (f.). Forming an interactive, creative environment to organize a dialogue in the process of training is the main objective of developing these didactical materials. The BDMs on the Functions of a Complex Variable are offered as a demo model.

Stage 3 · Algorithmic didactical materials To work out computer-oriented algorithmic didactical materials (ADM), that contain algorithms of proofs of basic theorems and algorithms of solutions of key problems. ADMs are oriented on developing skills of algorithmical thinking. As an example of ADM we suggest an algorithms of proving some of the theorems.

Using of Basic-Algorithmic Technology will allow to fulfil the following tasks: will provide each student with an opportunity to learn the bases of undergraduate mathematical courses using basic and algorithmical didactic materials (BDM and ADM); to organize cooperation between teachers of mathematics for creation of BDM, ADM database and tests f

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Main Theme: Preparation of Teachers

Secondary Theme: No secondary theme

Math for Elementary School Teachers: Geometry and Problem Solving.

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The writing intensive program is a cross-disciplinary initiative designed to help students learn how to communicate within a discipline. Each academic subject has its own criteria for what constitutes a valid argument. The writing intensive program helps students to learn these "ways of knowing" and to communicate effectively with their writing. For example, as a WIP TA in the mathematics department, I helped students write convincing (informal) mathematical arguments concerning basic geometric concepts. In general, learning to write a convincingly in an academic subject is taught informally. Students often mimic the style of academic writing they read in their textbooks, and professors communicate standards by making corrections on student papers. The writing intensive program is an effort to make this process explicit. Writing intensive program teaching assistants from all involved academic areas met once a week to discuss strategies, review current research on writing and communication and to resolve problems. This collaboration allowed us to share strategies and ideas that worked in classrooms across the disciplines.

PRE-SERVICE TEACHERS' PERFORMANCE IN SOLVING MATHEMATICAL PROBLEMS WITH CALCULATORS

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Problem solving is the focus of mathematics learning and teaching. One of the main issues is ·How can the problem solving performance of students be improved?·In this process, teachers have key roles, and affect the students' performance if they have enough experience and use cognitive tools, eg calculators, computers etc effectively. But how can we be sure whether the teachers by themselves use the technology in solving problems? This is a general issue and question, and it includes various sub-problems in training teachers in both pre-service and in-service education. To investigate the aforementioned issue and particular problems, an experimental research was conducted in Balikesir University, Turkey on 147 prospective teachers in 2001. The prospective teachers were divided into four groups, two of which were experimental groups (EGs) and the others were control groups (CGs). The treatment lasted in two weeks. The EGs did the prepared activities with calculators while the CGs did them with pencil-and-papers. The reports of all the pairs were collected, analysed and evaluated. It is found that there exists a significant difference between the CGs and EGs in the Polya's steps of understanding the problem and looking back. The overall effects of calculators on prospective teachers' problem solving performance were positive in various dimensions, i.e. solving problems with more than one way, generalizing the results. It is also found that there was a statistically significant difference between the mean scores of two groups of prospective teachers' performance. These findings highlight the issues on teacher training and the ways of introducing, integrating and implementing the calculators in the teaching and learning of mathematics.

Using Technology and Cooperative Groups to Develop a 'Deep Understanding' of Secondary School Geometry

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In Fall 2001, the Conference Board of the Mathematical Sciences (U.S.A.) released a list of recommendations on the mathematical preparation of prospective teachers. These included recommendations that prospective teachers take courses that "develop a deep understanding of the mathematics that they will teach" and that "prospective teachers should develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching." In 1997, Ohio University significantly revised the 'Foundations of Geometry' sequence taken by prospective secondary teachers. The revised course uses a significant amount of group work and technology to plant the seeds for a deep understanding of the geometry taught at the secondary level. By using software programs and manipulatives, students begin building an understanding of non-Euclidean and Euclidean geometry from the outset of the course. The use of cooperative group work, and written reports on group projects, develops student's writing and oral communication skills. A major goal of the course is to give the students the experience of 'doing mathematics'. During the course, the students use the experience gained using software programs and manipulatives to develop their own axiom systems and use these systems to prove theorems. This paper describes the overall structure of the course, how and where various learning aids are used, and discusses the effectiveness of the course in promoting a 'deep understanding' of the secondary school geometry curriculum. The assessment is based on student work and journals collected during the first four years the course was offered in its current form. The evidence suggests that the students improve their ability to prove theorems and develop a good understanding of models and axiomatic systems. The paper also discusses some shortcomings of the course. Of note is that, in some cases, the use of technology inhibits effective group work.

Main Theme: Preparation of Teachers

Secondary Theme: Technology

Effective Integration of Technology in Mathematics Teacher Preparation Courses

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Common themes in requirements for prospective mathematics teachers include mathematical modeling, problem solving, technology, and communicating mathematics. In this presentation we will discuss student presentation of projects and demonstrate how prospective teachers can use technology to analyze the predator prey model. In our courses we assign student projects to provide the opportunity for students to communicate mathematics. We require that students work in teams of two or three to prepare a typed paper complete with references. Moreover, we require each team to make a formal presentation of their project to the entire class. We permit the students to use the technology of their choice in the preparation of the paper and for their presentation. For example, the students can use the technology of the TI-89 computer algebra system to explore differential equations and interpret solutions from three different points of view: graphical, numerical, and analytical. Slope fields and graphs of solutions or direction fields and solution curves in the phase plane contribute to better understanding of long term behavior of the model. Tables of approximate solutions using Euler or Runge-Kutta methods also provide information. Graphs and tables of exact or approximate solutions can be compared on a split screen. The deSolve command of the TI-89 can be used to compute exact symbolic solutions to many 1st- and 2nd-order ordinary differential equations. Matrices, eigenvalues and eigenvectors are also easily handled on the TI-89 to determine the exact solutions to systems of ordinary differential equations.

Main Theme: Mathematics And Other Disciplines

Secondary Theme: No secondary theme

"How can architecture be helped by mathematics?"

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As everybody knows, mathematics has been the basis of all sciences. Specially, engineering, medicine, applied sciences such as physics, chemistry, biology and so on. It is also known that mathematics has always accompanied or, at least, tried to follow arts in their diversities, codes, geometrical signs, harmony and proporcionality researches. The aim of this work is to develop the relevance of mathematics in contemporaneous architecture. Namely, its more representative forms of cultural or sport buildings. As in our's time (posmodernism) the architectural object has a great exuberance, such as in Gothic's with its ogival forms, or in Barroco'swith its vaults and spheric callotes, we require notions of topology. The classical linear algebra and analytical geometry are insufficient for the purpose. For an architecture's graduation, with a nowadays vision of the utility of the technology, the mathematics must understand that students would wish and be able to apply forever mathematics in their professional work. Indeed, it is important that mathematics do not fall down in forgetfulness, and students might profit from mathematics and topological geometry as previous requisites for their imagination and poetics. Nevertheless, let us remember that harmony, expression or quality of the actual worth of messages may not be explained rationally by mathematics, but in character of sentiment or sensibility.

Electronic simulations of collaborative study groups in calculus courses

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Studies have shown that students' understanding of mathematics is deepened through working with collaborative groups, both within and outside of class. Yet undergraduate students, at least in the United States, rarely organize study groups on their own, even if teachers encourage the practice. Why is this the case? Several of the more plausible reasons are:

- a shortage of time, for increasing numbers caused in part by living away from their peers.
- not really believing such interaction will help (sometimes because they haven't been successful in earlier study groups, and sometimes because they think of knowledge as something that must be passed along to them from the professor).
- fear of having to carry more than their share of the burden in a group discussion (again perhaps based on past experiences with groups).
- shyness.

To address these difficulties, we are developing computer-simulated study groups for undergraduate calculus students. Just as a flight simulator allows pilots to gain the knowledge and skills needed to fly an airplane without risk of life, the GroupLearn simulations will allow individual students to gain an understanding of calculus from seeing several ways to look at a concept. They will also learn the skills needed to participate in an effective group, without risk to their self-esteem. The simulations are being based on scripts written by professors experienced with collaborative groups in calculus and are being tested on diverse students in a variety of academic situations. The project is under the aegis of the Institute for Studies in Educational Mathematics (ISEM), a non-profit organization that coordinates grant-sponsored work involving different educational institutions.

Effective Integration of Computing Technology in Mathematics at USTL

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During the last four years, I have been engaged in the production of multimedia units for the French National program called "On line University", software for the first two years of universities of sciences, in physics, mathematics, chemistry and biology. A presentation on the national site : <http://www.uel-pcsm.education.fr/>, and about 900 hours of training courses are already available. In mathematics, the units are shared between the six universities. The production carried out in Lille : a unit including "Elementary set theory and logics", "arithmetics", "analytical geometry", a unit "Complex functions" and a unit "experimental mathematics". In 2000-2001, I have used this software with my students : one fourth of the teaching was given in a multimedia center. I have observed the advantages of multimedia teaching for students. I have also given a 26 hours teaching in History of Mathematics, with use of WWW resources, and an e-learning platform helped me to organize a collaborative work of the students. During 2001-2002, a generalization of this experience have been decided by the staff of the Mathematics department of the U.S.T.L. The experience of creation of multimedia was both extremely enriching and really difficult : it implies a new way of writing mathematical teaching material. With the project "On line University", the presentation will include the analyse of these difficulties. The biggest problems are encountered to convince the teachers that such multimedia material really improves the teaching and is usefull for them as well as for the students. In U.S.T.L., each year, a new progress has been done in that direction.

**A Matter of Modes:
Towards Flexibility in Mathematics Education for a New Millennium**

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The pressures on undergraduate Mathematics students and educators in Australia have increased dramatically over the last decade. A softening of entry prerequisites, substantial increases in the number of mature age, part-time and alternative entry students, and high school curriculum reforms that have de-emphasised manipulative skills have all contributed. It is becoming increasingly difficult to meet student learning needs within the contact hours available. What is required is a "postmodernist" approach to mathematics learning support which improves learning outcomes through close personal interaction between students and teachers rather than through access to materials online. We are fortunate at QUT that the needs of Mathematics cohorts have been given explicit recognition in the form of a Large Teaching and Learning Development Grant. The main product of the grant is the Mathematics Access Centre (MAC) which offers voluntary learning support programs to approximately 700 first-year students per semester. These programs include weekly enabling tutorials that address weaknesses in foundation skills (assumed knowledge) and workshops to improve mathematical problem solving skills. A web-based diagnostic testing facility provides students and staff with valuable early feedback on preparedness levels and "black spots". The MAC also operates a Drop-in Centre and web site (<http://www.maths.qut.edu.au/mac/>). A strong commitment to program evaluation and educational research are cornerstones of the MAC's mission. We have begun to build a comprehensive data archive which includes student demographic information, equity status, approaches to learning (using Biggs' SPQ [1]) and performance measures. We will present data that suggest disparities in mathematical preparedness within cohorts are effectively removed by participation in enabling tutorials. Workshop evaluations indicate that students perceive an improvement in their problem solving skills. This qualitative finding is supported by examination performance data. Preliminary SPQ data indicate significant positive correlations between examination performance and the deep and achieving approaches to mathematics learning encouraged in workshops.

[1] Biggs, J. B. (1987) Student approaches to learning and studying. Melbourne: Australian Council for Educational Research.

Teaching Statistics and Academic Language in Culturally Diverse Classrooms

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The last decade has seen a substantial increase in the cultural and academic diversity of commencing tertiary education cohorts. The challenge for mathematics and statistics educators is the development of curriculum measures which address the language related difficulties of language minority students [1] and improve learning outcomes for all students. Our focus in this paper is on enhancing language and communication skills in culturally diverse undergraduate statistics cohorts. Most students have difficulty adjusting to the formal language requirements of academia. Non-English speaking background (NESB) students can have particular difficulty with the reading and assessment demands of Western universities if they are not adequately supported [e.g. 2]. This is especially problematic when discrepancies between verbal and written expression and true intellectual ability result in assessment penalties. What is required are curriculum models which focus on what students do as opposed to deficit models which focus on who students are [3]. We describe curriculum development in two subjects designed to teach language skills in statistics. Both subjects require students to engage with academic language and to develop statistical discourse skills relevant to modern professionals in the quantitative sciences. Methods used to encourage this include explicitly teaching academic reading techniques, and group research projects that are peer assessed. The projects are designed to develop statistical concepts within the context of professional practice and to address key competency requirements of relevant professional associations. We will present data that suggest that NESB students have more difficulty than ESB students on "traditional" statistics assessment tasks and describe curricula interventions that assist those students to achieve their academic potential. The reaction of students to these developments has been very positive. The quality of the work is impressive and students improve both their statistical knowledge and their reading and writing skills.

[1] Cocking, R. R. & Mestre, J. P. (1988) *Linguistic and Cultural Influences on Learning Mathematics*, Hillsdale, NJ: Lawrence Erlbaum Associates.

[2] Ballard, B. & Clanchy, J. (1997) *In the classroom. Teaching international students: A brief guide for lecturers and supervisors* (Chapter 3, pp. 27 - 43). Deakin, ACT: IDP Education Australia.

[3] Biggs, J. (1999) *Teaching international students. Teaching for quality learning at university: What the student does* (Chapter 7, pp. 121 - 140). Buckingham, UK: Society for Research into Higher Education and Open University Press.

Factors affecting students' perceptions of difficulty in calculus word problems

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This study set out to determine what factors affect a student's perception of the difficulty level of a word problem, to construct a hierarchy of difficulty and to investigate whether experts and novices perceive difficulty levels differently. To this end, word problems were defined and characteristics were isolated that differentiate between such problems. A typology of problems was compiled in which the problems were characterised by the following features: the type of response requested, which could be calculation or interpretation; the context, which could be concrete (realistic) or abstract; and the number of representations, either written language alone, or written language and a diagram. The geometric problems were presented with two-dimensional and three-dimensional options, and the readability of the problems was measured. Five problems were chosen from this typology and a questionnaire was distributed. Responses were received from 660 first-year university students and 20 expert mathematicians. The questionnaire required the students to rank the five problems in order of difficulty, without solving the problems first. The data was analysed to obtain a hierarchy of difficulty, in which several word problem characteristics were compared relative to one another. The results confirmed the findings of other theorists in measuring relative effects on difficulty perception of word problem characteristics. The results indicated that familiarity had the largest effect on perception of difficulty, followed by the context (realistic or not) and lastly the presence or lack of a visual representation. The expert responses did not form as clear a hierarchy, suggesting that, among experts, perception of difficulty is subjective.

Mathematics or Computers? Confidence or Motivation?

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The use of computers in the teaching and learning of undergraduate level mathematics raises many still unanswered questions about the relationships between students' perceived abilities and attitudes towards mathematics and computers, both separately, and interactively, and their performance or levels of achievement, on assessment tasks. This paper reports on an investigation of the correlations between first-year mathematics students' performances on a range of assessment items, and the following affective factors: their levels of confidence in their ability to do and learn mathematics, and their motivation to do so, their levels of confidence in the use of computers, and their motivation to do so, and their attitudes to technology in the learning of mathematics. The study targeted a class of 176 students in a typical Australian first-year Calculus and Linear Algebra course. Support for the use of MATLAB was integrated into their learning, and students did both hand exercises, and tasks requiring the use of technology, in tutor-supported weekly computer laboratory sessions. Established scales, well tested for internal consistency and reliability, were used to assess students' individual levels of confidence with mathematics, confidence with computers, mathematics motivation, computer motivation, and attitudes to technology in the learning of mathematics. Where appropriate, scatter plots and correlation coefficients are offered to illustrate the relationships between the students' mean scores on each of these scales, and their achievement levels on each of 5 assessment items (three assignments and two examinations). The trends revealed thereby are discussed in relation to the nature of the tasks in each of the 5 items of assessment: the balance of technology tasks and hand exercises required in each, and their relative level of complexity. Data gathered on preferred learning styles are used to further illuminate the achievement levels of students with a range of confidences and motivations.

Student Centred Learning in a Matrix Algebra Course

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A new approach was tried in presenting a matrix algebra course to students with differing abilities and diverse needs. The course had previously been presented using fairly traditional methods where the emphasis was on the transmission of knowledge from the lecturer to the students (using partially completed notes which students filled out during lectures). While exam results were reasonable, based on well practised examples, the course was fairly narrow and prescribed. A change to a more student centred approach was effected using the following mix. (1) The lectures paralleled a text book (Linear Algebra with Applications by David Lay), which closely followed the Linear Algebra Curriculum Study Group recommendations for an appropriate core syllabus responsive to client needs, and using a matrix-oriented problem solving approach. The resulting lectures were reasonably informal, encouraged students to read the text for themselves, attempted interaction and incorporated some technology. (2) There was a weekly computer laboratory session using Matlab, where students were encouraged to work cooperatively in pairs on problems from the databank of Matlab exercises available with the text, and on other projects that emphasised understanding and demanded written interpretation. (3) There was a weekly individual exercise, which provided a variety of question types from routine computations and standard algorithms to short proofs that required an understanding of key concepts. The paper considers such questions as how successful the course was from the viewpoint of lecturer and student, how Matlab was used not only for calculating more realistic examples but to aid understanding, and how well the group approach worked.

The Definite Integral: A Coordination of Two Schemas

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The report presents the results of the teaching experiment concerning students' understanding of calculus concepts done in a large mid-western university with the group of 32 students of engineering, science and mathematics students. The instructional method in the course included guided discovery learning activities suited for a cooperative learning mode and a computer environment.

The research question addressed here is: What is the mental development of the schema of definite integral amongst students? The analysis of students' clinical interviews suggest that the construction of the concept of the definite integral proceeds through the coordination of two distinct constructions, the visual schema of the geometric approximations to the area and the numerical schema of the infinite converging sequences. We identify the sources of the main difficulties with the understanding of the concept, viz., the absence of a clear understanding of sequences and their limits, and the absence of the coordination of this scheme with the visual scheme of Riemann sums. We suggest to correct this situation by introduction of the precise ϵ - N definition of the sequence prior to the discussion of the definite integral, and in developing on this basis a clear coordination between the two involved schemes. We provide several instructional strategies grounded in the ISETL programming language, which could achieve that objective. In our emphasis on the coordination of the visual and numerical schema as one of the foundations of understanding of the concept, we confirm and extend the Case theory [1], who sees central conceptual structure-organizing feature of children's domain specific processing- as originating by integration of two primary schema, the digital or sequential, and spatial or analogic. In identifying the absence of coordination between the two as a source of students' difficulties, we confirm Duval's work [2], who pointed out the importance of coordination between different registers.

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Duval, R.(2000), Basic issues for research in mathematics education. In Nakahara,T. and Koyama,M. (Eds) *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, Hiroshima, Japan, July 23-27, 2000.

The Teaching Experiment and The Teacher Researcher. Emancipation of the Concept

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The Standards 2000 place high demands on the mathematics teacher of the reform [1]: "Teachers must balance purposeful planned classroom teaching with the ongoing decision making that can lead the teacher and the class into unanticipated territory from the point of view of effective mathematical and pedagogical knowledge." The proper attitude in such a situation is that of a Teacher-Researcher, that is a professional who can investigate students' learning processes in the context of classroom teaching to gain needed knowledge. However, till now, teaching research (action research, classroom research) has been hampered by the inability to make the contact between the particularity of classroom situation and the general educational knowledge base of the profession.[2], and thus its usefulness as a research tool has been limited. This presentation offers a new profile of Teacher - Researcher based on a careful composition of ideas centered around Action Research with the ideas centered on the concept of the Teaching Experiment of Vygotsky school in Russia [3], which removes that limitation. Where Action Research relies on the individual self-improvement of the teacher in the classroom, the Teaching Experiment of Vygotsky views the same classroom as one of the sites of a large scale investigation of learning processes designed using a general theory of learning. A composition of both approaches will be described, which will bring forth the new professional profile of the Teacher - Researcher, a professional equally at home in the classroom with the daily work of the teacher as well as with that of a researcher for whom the same classroom is understood as her or his experimental field, the significance of which may be of the global educational character. Examples of the professional development program and chosen examples of teaching research in undergraduate mathematics education will be included in the presentation.

[1] NCTM (1998) Standards 2000 Principles and Standards for School Mathematics: Discussion Draft, October 1998, NCTM

[2] Noffke (1994) Action Researcher: Towards a New Generation. *Educational Action Research* 2(1) 9-21. Cited in Doorrr and Tinto, *Paradigms for Teacher centered Classroom Based - Research in Lesh, R. and Kelly, A. (eds) Research Design in Mathematics and Science Education*. Lawrence Erlbaum, Mahway, New Jersey (2000).

[3] Hunting, R.P. (1983) Emerging, Methodologies for Understanding Internal Processes Governing Children's Mathematical Behavior. *The Australian Journal of Mathematics*, v.27 (1).

An analysis of the production of meaning for the notion of basis in Linear Algebra

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In this study we have investigated the production of meaning for the notion of basis in Linear Algebra, supported by the Theoretical Model of Semantic Fields, proposed by R. Lins (2001). It was conducted in three parts: (1) a historical-critical study, based on secondary sources, in which the key question was 'in which semantic fields were operating the mathematicians who constituted the notion of basis in the historical process of emergence of the elementary notions of Linear Algebra?'; (ii) an analysis of Linear Algebra textbooks, to investigate meanings which could be produced for the notion of basis from their reading; and, (iii) interviews with students of a first course on Linear Algebra (undergraduate mathematics degree), aiming at eliciting the meanings actually produced by them while engaged in solving proposed problems. The study allowed us to identify several and distinct meanings for the notion of basis being produced, coming from our many 'informants'; it had as a general objective to gather information which could help us and other professors a better reading of the classroom dynamics in a Linear Algebra course.

LINS, Romulo (2001) 'The production of meaning for algebra: a perspective based on a theoretical model of semantic fields'. In: R. Sutherland et al. (eds) 'Perspectives on school algebra', Kluwer Academic Publishers, The Netherlands

Derivation and integration with Cabri and graphic calculators

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I will first present a script for an introduction of the derivative function using the dynamic geometry software Cabri and how students use these tools in order to realize a written report of this activity. I will continue with cabri experiments letting us to draw the tangent line on any point of any usual function curve and to conjecture their derivative formulas (this work includes trigonometric functions, log and exp functions). At least, I will present under Cabri one utilisation of Euler's method to visualize antiderivatives of usual functions. As a conclusion, I will give a few examples of testing exercises that can be asked to our students after such a training. Level: 16 or 17th students of french Highschools

**"Cooling-off":
the phenomenon of a problematic transition from School to University.**

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This paper will investigate the transition from School to University focussed specifically on mathematics. It will explain how students negotiate their response to the changes in the dynamics of the teaching and learning milieu. In particular, it will consider an important new viewpoint on the well-documented cognitive difficulties that first-year Mathematics Undergraduates encounter: their developing loss of interest in mathematics which we call the "cooling-off" phenomenon. The paper is focussed on a study based on a close qualitative observation of 12 students who were followed from the last year of school through their first year at a prestigious mathematics department at a UK university. The data illustrate the development of the attitudinal profile of the students and the persistence of their beliefs about the nature of mathematics. We will consider how these persistent beliefs influence their 'didactical contact' (their view of their role and the teacher's role in the teaching/learning process). Comparing extracts of interviews from both school and university will highlight some of the subsequent difficulties in students' abilities to engage with learning and doing advanced mathematics. We will develop a theory which links the characteristics of the "cooling-off" phenomenon which, we hope, will simplify our understanding of some of the affective aspects of the transition to advanced mathematical thinking. The paper will finally propose ways in which the mathematical community can diagnose the symptoms of "cooling-off" phenomenon and embark upon an adjustment of the mathematical courses in order to deal with it.

COMPUTER ALGEBRA SYSTEMS IN A MULTIVARIABLE CALCULUS COURSE AND CENTER OF GRAVITY PROBLEMS

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In teaching a multivariable calculus course, two main difficulties facing the student are to draw surfaces in three-dimensions, and to setup and calculate tedious triple integrals. The added third dimension causes great difficulty even to the well prepared student who has successfully finished a two semester single variable calculus course. The student must now suddenly think in three-dimensions. Visualizing and drawing the corresponding three-dimensional surfaces pose a significant challenge to the novice. To alleviate the problem, the student and the instructor must resort to the modern technologies. Computer algebra systems (CAS) such as Mathematica and Maple are well equipped to handle such tasks. The paper focuses on a certain important topic of a multivariable calculus course, namely the center of gravity of solid objects. We will show how to use the CAS Mathematica to evaluate tedious triple integrals arising in calculating the center of gravity. Mathematica can also be used as a visualization tool to draw the graphs of three-dimensional solids under consideration. Usually a standard multivariable course only considers the center of gravity of fixed solids. However, with the aid of Mathematica, the students are in an ideal position to consider variable solids as well. Thus, the paper introduces the novel concept of the locus of the center of gravity of certain types of variable solids. The paper also illustrates several facets of A CAS in undergraduate education – the usage of a CAS as a computational tool, visualization tool, experimentation tool, and a conjecture-forming tool.

Students' Assumptions during Problem Solving

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This study analyses the written work of a group of undergraduate students (doing mathematics, physics or computer science degrees) that took part in a ten-week, problem-solving course. The course was structured with the objective of introducing students to vocabulary and concepts that could help them reflect on their own solving processes and share their experiences. During the course, students were required to solve problems and encouraged to develop a rubric for recording their ideas and experiences. Students' strategies for solving the "Fault-Free rectangles" problem were analysed*. The analysis suggested that students' "hidden" assumptions about the nature of the solving process that they needed to adopt could be classified into two different groups. On one hand, the majority of the students seemed to have worked as if the problem was one of developing a method for dealing with the situation. On the other hand, a smaller number of students seemed to have thought of the problem as one of discovering a single key idea which, once found, would naturally lead the process towards a solution. This classification also suggested that students who worked under the apparent assumption that a method should be developed for solving a problem tended to be more successful than students whose worked consisted of searching for a key idea that would form the basis of the solution. Since this study looks at the solving process of a single problem (solved individually) by a group of students, it opens questions about whether different assumptions would work better for other problems and about the flexibility of a single student to adopt different positions in different situations. * The problem is the following: "These rectangles are made from 'dominoes' (2 by 1 rectangles). Each of the large rectangles has a "fault-line" (a straight line joining opposite sides). What fault-free rectangles can be made?"

On the production of meaning for the notion of Linear Transformation in Linear Algebra: Kika and Vivian speak

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In our study, based on the Theoretical Model of Semantic Fields (Lins, 2001), we analysed the production of meaning for 'linear transformation' (LT), aiming at producing elements to support a further reflection on the teaching and the learning of Linear Algebra. As part of the study we have conducted interviews with two students of a first Linear Algebra course (undergraduate mathematics degree), seeking to elicit the meanings they were producing for that notion while engaged in trying to 'talk about' particular (and non-usual for them) LT's presented to them. Two of the aspects considered in the analysis were the meanings being produced (and the kernels thus involved, see Lins 2001) and the texts being produced (notations, diagrams, writing, speech, gestures). For instance, we have found out that the students always tried to find a way to visualise the LT's in question (as one may visualise the usual R^2 as a geometric plane). This study is part of a broader project ('A framework for the mathematics-content courses in the university preparation of mathematics teachers') and aimed at producing elements that allow an adequate reading of the process of meaning production in the classroom, leading to new approaches to deal with students' difficulties and to new approaches to the classroom practices of mathematics professors engaged in mathematics teacher education. LINS, Romulo (2001) 'The production of meaning for algebra: a perspective based on a theoretical model of semantic fields'. In: R. Sutherland et al. (eds) 'Perspectives on school algebra', Kluwer Academic Publishers, The Netherlands

**PRINCIPIA PROGRAM: EXPERIENCES OF A COURSE WITH
INTEGRATED CURRICULUM, TEAMWORK ENVIRONMENT AND
TECHNOLOGY USED AS TOOL FOR LEARNING.**

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The ITESM's teaching model has evolved in the last years. Nowadays, several abilities, attitudes and values (AAV's) are taken into account without forgetting the development of knowledge in students. These AAV's include teamwork, the use of technology as a tool in learning, self-learning, problem solving, among others. Within this evolution process, several problems were identified in the former model used in ITESM to teach mathematics and engineering. These problems involved both teachers and students. For instance, there were poor knowledge retention in students, courses were too centered in algebra instead of developing mathematical reasoning and rules and algorithms were preferred rather than practical applications in the areas students are usually interested. Principia is an engineering academic program which comes out from the idea of overcoming those difficulties. The main purpose of Principia is to develop a mathematical, physical and technological culture in students that will make them able to analyze and solve complex problems. This is achieved with the integration of different subjects in one unique program where the classroom and learning environment are considered. Principia has been planned and implemented for the four first semesters of engineering school. Some of the basic tools used by this program are problem based learning (PBL) and heavy use of computer technology. The are five fundamental principles in Principia:

- a) The integration of the curriculum for mathematics, physics, and computer sciences.
- b) Collaborative learning.
- c) Teamwork.
- d) The emphasis in mathematical modeling.
- e) The use of technology in the learning process.

With all these elements, Principia has evolved as an integrated program that considers objectives, knowledge, methodology and an evaluation system. In this work, we present our experiences in Principia over three generations of students and some statistical and comparative results.

Assessment of Solving Types of Problems in Algebra

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The evaluation as measuring and investigating activity represents a relevant element of the teaching-learning process and their interactions, contributing to the educational changes. As a requirement of investigation and in order to improve the learnings in mathematics, it is necessary to contribute to the development of certain abilities considered as fundamental in the student formation, as the solving of problems is. This ability is of priority in the proposal of the Educational Reform, which at present is being developed in Chile at secondary level. The new curriculum structuring which articulates around fundamental objectives and minimum contents, considers an area called "algebra axis", for the common and differentiated formation in mathematics, which assists in the development of the abstraction capacity by using the algebraic language and modeling daily life problem and of the science field through different functions. The deficiencies detected in this area of the school mathematics at secondary level, has permitted to consider the problem solving as a relevant and pertinent element in the educational process. This recognition of the problem solving in algebra, has permitted a wide analysis by considering various approaches. Some of them are the concern for clarifying in a general way its meaning as an activity, as a study of solving strategies and others in relations to their classification. Our classification is inserted in this Reform frame, in terms of differentiating types of problems in algebra and of evaluating its solving at secondary level. The types of problems have been considered according to their nature as routine and non - routine, and according to their context as real, realistics, fantasy and purely mathematics. Non - routine problem, in the sense that the student does not know a procedure or routine of solving previously established. An algebra problem belongs to the real type if this problem is effectively produced in the reality, and involves the student's action. It is of the realistics type if it is susceptible to be really produced or if it is a simulation of reality or of a part of it. It belongs to the fantasy type if it is imaginary without a basis on reality. Finally, a problem is of the purely mathematics type if it is referred exclusively to mathematics objects. Under this frame of Educational Reform this classifications will permit the evaluation of learning achievement in algebra, considering different level at secondary school in Chile.

EFFECTS OF USING ADVANCED CALCULATOR (TI-92/CABRI) ON LEARNING TRANSFORMATIONAL GEOMETRY

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The present study aims at to find out the effects of using TI-92/Cabri in teaching transformational geometry on student teachers' attitudes, geometric thinking levels and achievement. The subject of study, i.e. the participants in the introduction to geometry course, were 78 students from freshmen elementary school mathematics education students at Hacettepe University, Ankara, Turkey. Three instruments were used in the present study to find out the relevant factors and effects of TI-92/Cabri supported geometry teaching/learning. They are van Hiele geometric thinking level test (VHL), geometry attitude scale (GAS), and a set of calculator supported instructional materials (CSIM). The researchers have designed CSIM by taking into account the points of views of constructivism, and the use of TI-92/Cabri. Furthermore, the instruments were administered before and after the implementation. The data were gathered by means of the designed instruments and analyzed by using PC-SPSS. In the analysis, several tests were used in order to understand the effects of various factors on the attitudes and achievement. This statistical analysis compares the mean scores of each group and reveal whether this difference significant or not. More specifically, in order to determine student's geometric thinking levels 25-item VHL was used. The items in the test represents the five geometric thinking levels proposed by van Hiele, and developed by Usiskin in Chicago. The GAS, which consists of 37, 5 point Likert type items, was used in order to determine students teachers' attitudes toward mathematics. The items represent 4 dimensions of attitude: interest, anxiety, importance and enjoyment. Factor analyses revealed that these 4 dimensions are valid and reliability coefficient of this scale is 0,89 for the first administration and 0,90 for the post administration. To test the hypotheses, a pre-experimental research design was implemented. The freshmen of Hacettepe University students, who will be teachers after the graduation, were taught 3 transformational units: translation, rotation and reflection in 3 hours a week, totally 9 hours. Instruction was contributed with TI-92/Cabri. The students were encouraged to engage in dialogue both with their classmates and the implementers and ask questions to each other. The instructor encouraged them to think by asking thoughtful, open-ended questions. During implementation they were received some worksheets to help them clear their ideas related with what they did by the calculator. These worksheets were prepared to help prospective teachers on discovering some important aspects and rules of transformational geometry. At the end of the instruction and experiment, we tried to find out the effects of using TI-92 in teaching transformational geometry on students teachers' attitudes, and geometric thinking levels. In the presentation of the paper, the details of the analysis and the sample of CSIM will be explained.

Practical purposes in teaching of Mathematics of future high school teachers

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Given the fact that the student purposes in teaching of Mathematics of future high school teachers is an agent of his own learning process, a teaching/learning methodology is proposed in which:

The concepts are built upon each one's experience and concrete situations; ·

The concepts are dealt with through different points of view and with increasing level of strictness and formalization;

A stronger connection between mathematics and real life, technology and matters of other subjects is established;

The selection of activities plays a particularly relevant role, for these should bring the student to conjecturing, experimenting, testing, evaluating and reinforcing his skills of autonomy and cooperation. Experimental problems and situations are the starting point so that the student gradually grasps the formalization of the concepts through intuition. The use of the graphic calculator, which is not only a tool but also a source of activity, investigation and learning, should be compulsory in order to prepare the students for a society in which Informatics play a highly relevant role in the resolution of scientific issues. The analysis of real life situations and the identification of mathematic models, which enable their interpretation and resolution, constitute an opportunity to come to grips with the scientific method. Investigation activities, which are excellent means of developing research skills, should approach not only mathematical situations but also others related to Physics, Economy and Geography. Role of the teacher The role of the teacher in the becoming concrete of this methodology is to be both a regulator and an invigorator of the teaching/learning process, providing motivating situations and adopting a strategy which implies the student in his learning and develops his initiative, stimulating investigation through mathematic modulation. In this presentation, we will tackle the logistic function as a means to interpret real life situations, namely the increase of a population. We will also deal with an investigation in the field of Physics analysing the light intensity of a lamplight.

Impact of Formative Field Research with Children on Applications of Modulo Structures upon Preparation of Teachers

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Researching directly with children in school settings on the accessibility of mathematical ideas is analogous to the laboratory of a scientist where theory is discovered and validated. Discovering accessible ideas for children leads to researching potential applications for teachers and preparation of teachers. This has been central to the evolution of research on applications of modulo structures to arithmetic over the past several years. The author has extended the idea of applications of modulo structures to checking arithmetic of rational numbers expressed in various numeral bases. The following problem, in base seven, has been chosen as an example because it represents a difficult problem to solve and check in rational number arithmetic: -6.0432 divided by 0.34 is -14.65 WR 0.0034 , and checks with the cast out of six being 3. The author has determined that the idea of cast outs is accessible to children as soon as they are conserving a one-to-one correspondence and can engage in developmental numeral structures. Also, the author has determined several enhancing techniques for implementation of the idea as children become progressively more mathematically sophisticated, numerically. Not only can children access and apply the ideas, but also, the ideas/techniques impact conceptual understanding/applications of numeral structures. NCTM and others consider checking of arithmetic by application of calculators an abuse of technology as an educational tool. Formative field research with teachers and pre-service teachers, based upon these applications of modulo structures, has led to significant changes in teacher preparation courses. Further, the implications suggest a renewed interest in modulo structures for pre-calculus. The author proposes to share the fundamental accessibility of applications of modulo structures to arithmetic and how such has impacted the preparation of teachers, with implications for pre-calculus courses.

Modelling around the triangle

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Mathematical modelling is an important aspect of mathematics teaching. Often only one way is gone or one method is used to solve tasks with the help of components of modelling in lessons of mathematics. But in the case of real modelling practical problems out off the school the helpful strategy or method solving the problem is not clear or determined. Often several and different methods has to be used to find a solution for a practical based problem. Consequently pupils should learn in mathematics that modelling means using different methods to overcome one and the same problem. But often mathematics teaching is directed by teachers to declare one focussed method and to show later how to use this method for different classes of mathematical tasks. In the lecture the idea of mathematical modelling will be described at first. Around the example of a central and important geometrical content – the formula for the area of triangles – shall be shown the understanding of using different kinds of modelling in mathematics teaching. For it concrete examples of tasks for pupils to find the formula for the area of triangles shall be shown and described and the idea of a complete teaching sequence will be shown for the participants of the lecture. Consequently the aim of the lecture is to speak about possibilities to teach modelling in schools as early as possible with the help of an example for middle-aged pupils.

Co-operative learning as a tool for enhancing a Web-based Calculus course

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One aspect of concern when presenting a web-based course is the lack of personal contact. It is often difficult to stay committed and motivated when you are completely on your own, especially in a subject such as Calculus where discussion of the subject enhances your understanding considerably. Group work or co-operative learning is a means of addressing this problem. We work with large groups of students. We have developed a model that is used in a number of these courses. Our courses offer a number of group-based activities such as assignments and projects. Students are divided into small groups of three or four and it is expected of them to get together to discuss the subject matter and work on the assignments and projects as a group. In this paper we discuss our findings on the successes and pitfalls of our model. We firstly discuss the process of forming groups. We then investigate how students experience the co-operation with fellow students, their work ethics, their concerns and their suggestions. We also look at the case of students who prefer to work alone and perform better in doing so. We critically evaluate our model, we discuss how trustworthy the co-operation is between students and how to prevent students from riding on someone else's back.

**Mathematics-Specific Technologies: Effects on Preservice Teachers'
Problem Solving Strategies and Attitudes Toward Technology
Use in the Classroom**

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We investigate the effects of a laboratory-based capstone mathematics course on preservice teachers' problem solving strategies and their beliefs toward the use of computer algebra systems, graphing calculators, and geometry software in mathematics instruction at the middle school level. The course incorporates a discovery learning approach in a computer laboratory. It draws upon mathematics content from prerequisite courses in college algebra, elementary geometry, elementary analysis, and finite mathematics and focuses on the use of mathematics-specific technologies for mathematical problem solving. Readings ranging from research on teaching with mathematics-specific technologies to excerpts from documents that discuss the importance of incorporating technology in mathematics instruction are also required. Data analyzed includes: pre- and post-surveys on student beliefs about mathematics and the use of technology in mathematics instruction; pre- and post-tests on aspects of visualization in mathematical problem solving; daily in-class observations; and student work. Research indicates that preservice teachers' beliefs about teaching and learning mathematics are resistant to change. However, qualitative and quantitative analysis of the data suggest that over the course of the semester there was a positive effect on student beliefs and attitudes about the use of technology in mathematics instruction and an enhancement in the use of visualization in problem solving.

Are Mathematics for other disciplines different Mathematics?

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Going to the mathematical section of a bookshop one can find titles like: Mathematics for economists, Mathematics for the life sciences, Mathematics for the Social Sciences, Applied Calculus for engineers, Linear Algebra with applications to Economics, and so on. If one looks at the Statistics area the menu is even broader. Is there a different approach to what universities worldwide teach in basic mathematics or statistics for different disciplines and what students of Mathematics learn in their courses? My main point is that like a child who wants to learn playing football there are steps like how to kick a ball, how to stop it, i.e. solid foundations that must be learned step by step. Of course you can apply the method of learning by doing, but if you want to use your knowledge professional, with very few exceptions, sooner or later you will have troubles. A solid foundation of mathematical thinking and techniques is needed to undertake what can be called an application to other disciplines. The same problem can be found from an inverse angle: if a mathematics student want to apply mathematics to, say economics, he or she must unavoidable assimilate the foundation of that science to be able to understand what he is doing or wants to do. Mathematics is by definition a rigorous discipline and students of any other academic area have to learn not the mechanics of mathematics or a mechanistic mathematics where for this problem a receipt is said and for that other another receipt must be applied. It is more important to understand concepts that mechanics; nevertheless one dose of carpentry must also be trained. The implication of the above ideas is that curriculums, their design and its development must be carefully undertaken. My personal experience in teaching mathematics, mostly for economists, is that at a certain point I had to learn and understand economics to give advanced courses in mathematics for economists. At the first year undergraduate level, more important was to build the solid bases needed afterwards.

THE USE OF THE JIGSAW IN HYPOTHESIS TESTING

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The Jigsaw is a cooperative learning technique in which the class is first divided into expert groups that are assigned different but related tasks. New groups consisting of one member from each expert group are then formed. Each expert instructs the other members of this new group about what they have learned. In this paper, we provide two examples of how the Jigsaw could be used in an introductory statistics class. The first example is for presenting different sampling techniques and a more advanced application is for introducing hypothesis testing. In the later, we have found it effective to have expert groups use experiments and computer simulation to investigate a specific claim. It is hoped that these preliminary concrete activities will provide a smoother transition to understanding the formal and symbolic hypothesis testing framework.

Teaching Mathematics in Indonesian Primary Schools Using Realistic Mathematics Education Approach

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This paper presents a case study about employing Realistic Mathematics Education (RME)-approach to teach mathematics in Indonesian primary schools. There is a number of problems in mathematics instruction at primary school in Indonesia. For example, the approach that is used to teach mathematics is very theoretical, and many abstract concepts and formulas are introduced without paying much attention on aspects such as logic, reasoning, and understanding (Karnasih & Soeparno, 1999; Soedjadi, 2000). Besides, the teaching learning-process is always organized in a traditional (teacher centered) way (Somerset, 1997). In my research project (started in 1998) it is argued that Realistic Mathematics Education (RME) (see Freudenthal, 1973; Treffers, 1987; Gravemeijer, 1994; De Lange, 1987, 1998) is a very promising approach to improve mathematics teaching and make it more relevant for pupils in Indonesia. The research is conducted by employing type 1 of development research approach (see Akker & Plomp, 1993; Richey & Nelson, 1996). Many obstacles, such as the very dependent attitude of the pupils, the pupils were not used to working in groups, lack of reasoning capability, and lack of understanding of basic concepts were found when the pupils, who were used to the traditional way of teaching, dealt with RME. Nevertheless, this first pilot with RME had many positive impacts on the teaching-learning process in the classrooms. The difference in the learning behavior of the pupils found from day to day showed that RME is a potential approach. Based on the interviews with a number of pupils it was known that they like the new approach. They realized that there were some positive changes on themselves especially in reasoning, activity and creativity. The teacher themselves admitted the positive changes on the pupils' behavior after they dealt with RME-based lessons. In conclusion, RME is an approach to mathematics education developed in the Netherlands, but the research reported here shows that this approach is very promising to utilize in Indonesia.

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Main Theme: Curricula Innovation

Secondary Theme: Mathematics And Other Disciplines

Fourier Series and Elliptic Functions

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For some time this author has been trying to promote early introduction of Fourier series into the undergraduate curriculum, and beginning differential equations is just the place to do so. In such courses there is a broad range of students not just from mathematics who would benefit from exposure to arguably one of the most important applied mathematics topics. Elliptic functions are usually not encountered in the undergraduate curriculum, however, by viewing the elliptic sine, cosine, and denominator functions as solutions to certain nonlinear differential equations, it is easy to generate highly accurate Fourier series approximate solutions with bounded global error which represent these non-elementary functions. By doing so, one can then solve the large angle pendulum problem and other problems previously thought too advanced for undergraduates. The Fourier series technique is straight forward and can be used for any equation expected to have a periodic solution. The procedure is a fine example of how to use mathematical technology to investigate new and deeper problems, avoiding computational difficulties and concentrating on qualitative aspects.

Main Theme: Preparation of Teachers

Secondary Theme: Innovative Teaching

University of Southern Mississippi - Technikon Pretoria Masters Degree Program in Mathematics: a case study in teacher's preparation

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Since 1995, the University of Southern Mississippi and the Technikon Pretoria, with support from the National Research Foundation of South Africa, have been collaborating in offering a Masters Degree Program in Mathematics with the emphasis in doing mathematics on the computer. The main thrust of the program is to upgrade the qualifications of Technikon Staff members.]The program consists of three "winter schools" held for seven weeks in June and July. After successfully completing the three schools and a thesis, the students receives a masters degree from USM. By December 2001 there will be 15 graduates. The program is non-traditional in that is is "total emersion". At the heart of the program are two courses entitled "Computer Assisted Mathematics" taken during the first two winter schools. These are project oriented laboratory courses. Courses in odes, pdes, numerical linear algebra and analysis and given together with specialized courses on finacial mathe,matics (Black-Scholes equation) and wavelets. A cap stone course in real analysis finishes off the course work. The emphasis on mathematica technology has led to over 30 published articles on teaching from this perspective and the establishment of research programs for all students.

Parable Family: Integrating Technology in Math Lessons

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It is common to study quadratic functions in middle and high schools in Brazil. Regarding graphical interpretation, it is usual to analyze parable concavity studying the a parameter, and the number of real roots by discussing the delta (D). We propose a different task for undergraduate students. To observe the variation on the vertex of parable families when we fix two of the three parameters a , b and c of the formulae $ax^2 + bx + c$, and let the third vary. The students are using the winplot software. We will present findings regarding the graphical interpretation as well as the algebraic properties.

Information technology and coordination of representation systems in undergraduate mathematics teaching

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The goal of this paper is to present some ideas to design an introductory mathematics course for science students. The audience usually includes a good share of students with poor mathematical background. This is a major obstacle to an extensive use of mathematical notations (as customarily used in algebra and calculus courses). So multiple representations (verbal language, graphs, tables, formulas) must be used with emphasis on their coordination rather than on the learning of the specific methods of each representation system. To achieve these goals the help of information technology is needed. Computer Algebra Systems or statistical software are introduced to play a variety of functions: they provide various representation systems with related algorithms and usually allow some conversions, i.e. the translation of a representation from a system to another; they provide easier access to representations of mathematical ideas (like the graph of a function) and take charge of a wide amount of calculations (e.g. when generating a numerical table for a function). Coordination of registers and the use of information technology alone cannot promote students' learning if we do not design situations where students themselves take charge of their learning and reflect on the ideas they are dealing with. This may be achieved through verbal language, both in spoken and in written form. So questionnaires and tasks are often proposed to the students that ask them to explain in written form some mathematical idea. Moreover, discussions within groups of students (guided by a graduate tutor) are promoted, together with translation tasks from spoken to written verbal language. The presentation will focus on derivatives and on the development of students' related conceptions and use of language in the construction of meanings.

Mathematics Teachers Without Math: Responding to the Crisis

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In the once great state of California, as many as 40% of all current secondary (grades 9-12) teachers of Mathematics have never taken a Mathematics course beyond the first semester of Calculus, many have significantly less background. California State University Bakersfield is responding by developing an sequence of innovative courses leading to a Master of Arts in Mathematics. New courses, using modern technologies, have been -- and are being -- developed to introduce working teachers to contemporary ideas in both mathematics and pedagogy. This has proven a daunting challenge for us, since designing courses that are at once mathematically accessible and intellectually challenging, is not a problem that Mathematics Departments are accustomed to undertaking. The first course taught, Discrete Mathematical Modeling, exemplifies the strategies we are employing. By choosing difference equations, a topic traditionally absent in US Schools and Universities, and making a total commitment to Computer Algebra Systems in the form of the TI-92 Plus as our computational platform, we are able give an intellectually sophisticated treatment of material largely accessible to secondary students to an audience whose mathematical and computational backgrounds are widely divergent. Nonetheless, the course's subjects and techniques supply a review of fundamental algebraic concepts. We will examine the course structure, assignments and laboratories in detail and show where the material fits into the overall degree program.

Using environmental science to bridge mathematics and the sciences

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Environmental issues can provide an excellent way to connect mathematics with the sciences. At our small liberal arts college, faculty are working together to build an interdisciplinary curriculum for lower-division mathematics, biology, chemistry and environmental science. In our traditional calculus and statistics courses, we have introduced environmental materials, some adapted from outside sources and some developed through collaboration between mathematicians and scientists in the college. Many of these materials are small projects, designed for students to explore collaboratively, with the assistance of a graphing calculator, computer algebra system, or statistical software. Complementing the mathematics program, lower-division science courses bring science and mathematics out of the classroom and into the community, using local ponds, lakes, forests and greenways as science laboratories. Student and faculty teams collect data on the water quality in dozens of area ponds, the diversity of wildlife in more than 250 acres of nearby preserves, and the impact of a growing population on the environment. They then bring their studies back to the classroom and use mathematics and statistics to analyze and model their data. A series of three new "link" courses -- one-credit courses that are team-taught by a scientist and a mathematician -- focus on the analysis of student-collected data using increasingly sophisticated tools. The project is supported by National Science Foundation grant DUE-0088221. Our goals are for students to understand the interdependence of mathematics and the natural sciences, and be able to apply what they learn in the classroom to hands-on scientific studies. For both faculty and students, the project aims to integrate teaching, learning and research in a holistic form of scholarship. Preliminary data were collected in the fall of 2001, and a first assessment of the project's goals will be completed in the late spring of 2002.

Teaching Mathematics to Primary Teachers in Australia

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Primary teacher education is a relatively low status option for school leavers in Australia, when judged by competitive rankings of university entry scores. Although undergraduate primary education students have completed 12 years of study in mathematics, their knowledge is not always secure and their understandings are largely instrumental. Although most students are continuing from school, there are also mature-aged students (mostly women) among this cohort who have been out of education for many years. Not surprisingly, mathematics anxiety is manifest among many students, young and old. This paper will give details of an innovative approach to strengthening the foundation of mathematical knowledge as well as broadening the students' perspectives on the nature of mathematics itself, with a view to influencing the pedagogical approaches that the students will eventually adopt. The course content is based upon Bishop's (1988) 'six universals': counting, locating, measuring, designing, explaining, and playing. As with all educational endeavours, the paper represents a work-in-progress. It will analyse the theoretical foundations of the course structure, describe student responses, and evaluate the progress of this course which has run since 2000.

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**Do you understand me or I draw you a picture?
(visualization in teaching math)**

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In México, when it seems that somebody does not understand what we are trying to explain, we use to say: "¿Me entiendes o te hago un dibujito?" (Do you understand me or I draw you a little picture?). We use draws and pictures to make us understand, we use them to make things or concepts clearer to ourselves; in teaching math, we use diagrams, graphs, pictures, and drawings to explain concepts. Nevertheless, there is a resistance to use visualization in mathematics (Vinner 1989, Eisenberg & Dreyfus 1991). Eisenberg and Dreyfus assert that the student prefers the use of an algorithmic thinking instead of a visual thinking, claiming that the visual thinking involves higher cognitive skills than the algorithmic thinking. Duval (1995), says that the apprehension of a concept by a student is achieved only when that student is able to convert from one representation of the concept to another and vice versa in at least two representations. In my own experience, the use of graphs and diagrams in teaching mathematics, together with an adequate activities design, can foster a better understanding of the concepts involved. In this paper, I present several examples illustrating how visualization can improve the mathematics skills, performance, and understanding of students in math courses. The examples were taken from a 10th grade course. The purpose of the paper is to reflect on how visualization can be encouraged in a classroom, and what role can the technology (specially cas calculators and dynamic geometry software) play in foster visualization.

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Preservice teachers and mathematical proof

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Some preservice mathematics teachers reveal difficulties and negative attitudes when they study geometrical concepts and relations. Some years from now I´m having a problem: what can I do in the classroom to foster understanding and performance of students when they face proof in geometry? I need more knowledge about proof and teaching of proof. I have access to papers from Gila Hanna, Michael de Villiers, Tommy Dreyfus, P. Goldenberg, · After that, I start designing environments where students play active roles in trying to develop geometrical habits of mind. The goal is to develop in preservice teachers reasoning processes, meaning that the variety of actions that students take in order to explain to themselves, to others, what they see, what they do, what they conjecture and why they do it. I decided to use The Geometer´s Sketchpad (GSP) with the students to contribute to experimentation, to conjecture, to convince of the truth of the conjecture and to help them come to see proof as a form of explanation and understanding why, rather than convincing. One aim of the study is to analyse the performance of preservice teachers in proving, in order to answer to the question: what level of performance reveal preservice teachers in doing proofs? The participants in this qualitative case study are a whole class of preservice teachers in the second year of the maths and science course in a School of Higher Education, during a whole year. Data has been collected through geometric problem solving tasks and observations. To solve the tasks the participants need to use GSP, to make conjectures and to answer why they are true. We are in the beginning of data analysis, that is holistic, descriptive and interpretative. This presentation will try to answer the question and to draw some conclusions.

TACIT KNOWLEDGE IN CURRICULAR GOALS IN MATHEMATICS

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We describe an analysis of the current curricular goals in mathematics at K-9 to K-12 school levels in different countries. Based on Paul Ernest's view of mathematical knowledge, we consider school mathematical knowledge as multidimensional in the sense that it involves components from different domains, such as the cognitive, the social, the beliefs and the values domains. Further, most of those components are of a mainly tacit nature. From this perspective, we examine such curricular goals. We present evidence to support that those goals, in different countries, foster the learning, by students, of a mathematical knowledge that is mainly tacit. On the other hand, we argue that the curricular guidelines for the teaching of mathematics lack the supports to handle the processes involved in the learning of knowledge of that nature. Part of the current literature emphasize that that knowledge can be learnt but can not be taught as in the traditional meaning of the word teach, that is, transmitting or enouncing publicly some kind of knowledge teachers hold. The same literature, although not dealing specifically with mathematics teaching, suggests, for instance, that the act of teaching a knowledge that is mainly tacit is closely linked to the teacher's public actions while facing authentic questions. That is, when he is engaged in a situation which demands the use of his own tacit knowledge. We conclude by discussing some curricular implications for the teaching of mathematics at basic school resulting from these issues.

A Partnership Pipeline: Progress Report

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A recent initiative to create a new Education curriculum for Community College of Philadelphia began in September 2001. This three-year project, "Secondary Math and Science Teacher Preparation: A Partnership Pipeline", is funded by an ATE(Advanced Technology Education) grant from the U.S. National Science Foundation. The aim of the project is to create a new Education curriculum for students who intend to become secondary science or mathematics teachers. The two major goals of the project are to create articulation agreements with twelve colleges and universities in the Philadelphia region, and to develop two new courses, Foundations of Mathematics(Math 167) and Teaching with Technology (EDUC 226). The Foundations of Mathematics course will introduce fundamental ideas that form the basis of Advanced mathematics courses. Topics may include logic, sets, functions, methods of reasoning and proof, number theory, and algebraic systems. Main objectives will be improve students' ability to produce rigorous mathematical arguments, and to improve their mathematical communication skills. The Teaching with Technology course is required for future teachers who must be prepared and able to use technology in the classroom in an effective and appropriate manner. This paper will document the progress of a NSF-funded project, "Secondary Math and Science Teacher Preparation: A Partnership Pipeline", which aims to create a new secondary mathematics education curriculum at the Community College of Philadelphia.

The Integration of Interactive Excel tutorials into a First-year (Pre-calculus) course

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Inadequate mathematical skills and understanding act as a barrier to students wishing to study a variety of courses at university. At the University of Cape Town a first year course called 'Effective Numeracy' is offered to such students, with the objective of supporting their study of other subjects and preparing them for mathematics courses in later years. Addressing the problem of the lack of mathematical and quantitative reasoning skills in these students is very challenging, and calls for the use of various techniques and approaches. Excel workbooks coded with VBA have been found to be a very effective environment for creating interactive tutorials that students can use for self-paced study. The Excel tutorials constitute one third of the course (in terms of time and credit), and are very firmly integrated into the overall curriculum of the course. Although there are slight variations in timing of delivery (because the class is divided into three groups), the content of any tutorial session consolidates and enriches material covered in the classroom within the same week. A large part of the curriculum is devoted to pre-calculus, focussing particularly on the understanding of the function concept and the idea of slope. The design of the tutorials includes a custom-built graphing device which can be incorporated into any Excel workbook at every point that it is required for the execution of the exercises. This means that a student can easily produce the graph of a function without leaving the context of the tutorial and interrupting the interactive 'conversation' of the exercise. This paper reports on our experiences in implementing this multimedia intervention. Representative extracts of the tutorials will be presented, students' response to the course will be analysed and the results of a study of its educational effectiveness will be reported.

**Practice Makes Perfect on the Blackboard:
A Cultural Analysis of Mathematics Instructional Patterns in Taiwan**

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According to the TIMSS 1999 Math Report (Mullis et al., 2000), a sharp difference in math achievement lies between students in the U.S. and in several East Asian countries including Taiwan, where student math achievement ranks the 3rd among the 38 participating countries. This difference has been illustrated in Stevenson & Stigler's (1992) classical work on the "learning gap" in student performance, teacher practice and parental rearing between the West and East (Stigler & Stevenson, 1991). Stigler & Hiebert (1999) further contend in their book "The Teaching Gap" that teachers in Japan, German, and U.S. display different teaching patterns, reflecting that teaching is a cultural activity. This paper intends to know how the Taiwanese teachers teach math so as to produce such a remarkable achievement of their students in the international math competition. Math teachers in the middle school in the Taipei area are videotaped for a complete unit of instruction, lasting for 3-4 periods (hours). The videotapes were reviewed and analyzed by using both the quantitative (Classroom Instruction Scale) and qualitative methods (observation notes). The results show that when compared with findings by Stigler & Hiebert's (1999), math teachers in Taiwan, like their American counterparts, tend to review previous materials, present the problems, and have students practice problem sets at their seats as part of their everyday practices. It is the process of presenting the problem that reveals a great difference between teachers from the two countries. While American teachers tend to demonstrate the procedure of solving math problems and then have students practice the procedure at their seats, Taiwanese teachers are inclined to call upon some students to practice on the blackboard when the other students remain practicing at their seats. The cultural beliefs underlying the Taiwanese instructional patterns are further discussed.

WeBWorK -- Generating, Delivering, and Checking Math Homework via the Internet

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The WeBWorK system delivers homework problems to students through standard web browsers, giving them instant feedback as to whether or not their answers are correct. It has been developed and used extensively for calculus instruction and physics courses at the University of Rochester over the last six years and is currently in use at more than a dozen other institutions. Each WeBWorK problem set is individualized; so that each student has a slightly different version of each problem. When students complete their homework assignment, log onto the internet and enter their answers into a web browser, WeBWorK responds immediately, telling them whether their answer is correct and recording this fact in a database. WeBWorK does not give the correct answer. Students then correct a careless mistake, review the relevant material before attacking the problem again, or know that they need to seek further help with this problem (frequently via e-mail) from friends, the TA or the instructor. The students are free to try a problem as many times as they wish before the due date. They can also download and print typeset PDF versions of the assignment to work on away from the computer. A key educational benefit of this system is that students with incorrect answers, get immediate feedback while the problem is still fresh in their mind. Nearly all of our students complete almost all of their homework until it is all correct. Our surveys indicate that they are very happy with the instant feedback and the resulting control they feel over their education. WeBWorK's large collection of existing problems and its extensible macro framework (modeled on TeX) for posing questions AND checking the answers, allow each instructor to ask the mathematical questions they "should" as opposed to the questions they "must" because of machine limitations. By focusing on checking homework answers alone rather than also supplying guidance and instruction, WeBWorK plays to the strengths of computers, and avoids some of the difficulties inherent in trying to build "intelligence" into a computer program. WeBWorK collaborates well with existing educational practices such as traditional lectures, reform calculus, workshops, and expository writing.

See <http://webwork.math.rochester.edu> for more information.

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Main Theme: Education Research

Secondary Theme: Innovative Teaching

Web-Aided Spreadsheet Education: Myth and Realities

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The CAD-CAM-CAT-WAT comparisons are presented as seen from student`s and teacher`s viewpoints. Psychological issues related to CLASSROOM versus website teaching are discussed. The student and teacher timeframes in a web-aided spreadsheet education are investigated in detail. Structural and software provisions are proposed to facilitate development of website education. The paper is based on the author`s new book "Chess, Computers and Education".

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Main Theme: Curricula Innovation

Secondary Theme: Mathematics And Other Disciplines

Convex Sets and Hexagons

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Euclid presented his fundamental results about 300 B.C., but Euclidean Geometry is still alive today. We studied the new properties of convex sets and its inscribed polygons in a two dimension Euclidean spaces. As an application, these results solved a question in Geometry of Banach space. It may encourage other to know that the tools we give our students remain useful in modern research.

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Main Theme: Innovative Teaching

Secondary Theme: No secondary theme

DIFFERENTIAL CALCULUS OF SEVERAL VARIABLES WITH MATHEMATICA OR MAPLE

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The use of Computer Algebra Systems (CAS) in Mathematical Laboratories at Engineering Schools is increasing. Therefore, it has become necessary to undertake a new educational approach in the teaching of many mathematical topics. Obviously it is not possible to give the same lectures as in the 1950's. The CAS allows us to experiment and teach in "a different way".

HISTORY OF MATHEMATICS AS MOTIVATION: AN INNOVATIVE METHOD FOR TEACHING IN ENGINEERING

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Mathematics teachers in Engineering usually thought that the task of transmitting information was the only mission of the profession. Also we used to think that our subject was the most important in the formation of students. This is due mainly because we have been formed in a classical teaching method and we just repeat the same topics and use traditional methodologies. Fortunately, this situation has changed in the last years mainly due to the continuous changes in the topics of the subjects in Engineering. This fact has lead us to review the subject contents. Even more, the use of the computer as an important tool in teaching is nowadays so usual that, developing practices with computers using some mathematical software is not seen to be an extraordinary method or a new experiment. Nevertheless, one important thing has not changed yet: we keep developing the concepts in each chapter beginning with the definitions, later the results and finally the techniques for solving problems. In this paper we will detail our experiment in changing this routine in our subjects of applied mathematics in the faculty of Telecommunication in Malaga University. We believe that we can emphasize the greatness of Mathematics by looking back to the history of how mathematicians developed all the results we know nowadays. This is the main reason that leads us to include history of Mathematics as an introduction to the concepts that will be developed in each chapter. Also we consider some biographical notes of the more influential mathematicians in the fields we deal with. As an example, we will develop in this paper the history about derivation and integration of complex functions and some biographical notes. We will finish with the conclusions and the corresponding bibliography.

MATHEMATICS IN ENGINEERING. AN INNOVATIVE EVALUATION METHOD

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In this paper we will present an experiment with students from the faculty of Telecommunication who take Math courses. This experiment is based on the evaluation method and we have been developing it these last eight years. Evaluation is the most difficult element of the educational process because the teacher must check if the pupils have grasped the expected objectives by this evaluation. On the other hand, the purpose of evaluation is not only to classify, but also must be used to obtain information about the real knowledge of the pupils and the problems the didactic methods present. Even more, the results of the evaluation could lead the teacher to re-examine the objectives. Besides, it would be interesting to do this revision during the time of teaching the subject. In Mathematics the abstraction of the concepts, the precision of the definitions and the reasonings, are as important as the contents of the different subjects. A constructive way of teaching is necessary. The concepts must be developed one after another. Because of that it is very important the pupil can assimilate the contents of the subjects gradually, in order to get an acceptable level of knowledge. In Spain, the traditional evaluation method at the undergraduate level is only one final written exam. One of the main problems of this method is that it increases the luck factor in the result. It is not a formative evaluation for pupils. Teachers use the argument of the elevated number of students in the class as a justification for one final exam. In our opinion, the number of students is an important problem but not one that can not be overcome. Basically, our method is a process of continued evaluation. We will present the objectives, the method and the conclusions of our experiment.

USE OF THE COMPUTER IN MATHEMATIC TEACHING FOR ENGINEERS. A POWERFUL CALCULATOR?

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The use of computer as a tool in teaching has been quite extensive in University over the last years, especially in applied subjects. Nevertheless, we should wonder if the computer is under-utilized in mathematical formation. Nowadays, computers are being used in university teaching as a powerful calculator but they are not being used as tools to help in developing a substantial change in the teaching of mathematics. The use of them as a tool to foment the mathematical creativity of pupils has not been achieved. The use normally given to computers is just to deal with calculus with numbers, algebraic manipulations or graphics representations which is very useful in complementing or simplifying the traditional way of teaching but this does not lead to an improvement in mathematic teaching. This is due in part to the fact that teachers have not been taught to use the computer as a tool for improving teaching techniques and thus, we have not yet assumed the computer culture needed to develop some activities which lead the pupil to use the applied mathematics for their professional future. So, the future challenge is to overcome this situation and use the computer as a mathematical creativity tool. As this is not a work of a few days, it would be interesting to include practices with computers in this sense in any mathematical subject, even more in Engineering degrees. In this paper we will present our experiment in the use of computer in mathematic teaching, developed in the last few years in the degree of Telecommunication Engineering in Malaga University, trying in some way to foment the mathematical creativity of pupils. We will finish with the conclusions and the corresponding bibliography.

LEARNING WITH TECHNOLOGY: SIMILARITIES IN MATHEMATICS & WRITING

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Learning mathematics with numeric, graphic, and symbolic calculators has implication for the learner's development of symbolic representations. This presentation will examine the evidence on learning symbolic representations at different developmental levels with special attention to how controversial technologies--such as numeric, graphic, and symbolic calculators in math or word processors, spell checkers, grammar checkers, and graphic organizers in writing--help or hinder learning. Examples with representations of ·functions· and ·fractals· will illustrate the many facets of this symbolic concept learning. An extensive body of research clarifies how people learn symbolic information with and without technology. Three sources provide the basis for an original comparative analysis of the development of symbolic representations in writing and in mathematics. The first source draws on the work of a federally funded, national team of jurors, who completed a two-year review of research on learning writing and math with technology headed by O'Donnell. The second source builds on the work of a think tank of faculty members from two universities that met every two weeks on the Access Grid [Internet2] during January-September 2001 to address models for representing complex and multiple variables of human learning. Third, Gavosto conducted a visualization and computer modeling workshop for researchers, and it suggested possibilities beyond the calculator technology. Finally, broad implications and a theoretical model will integrate this information about symbolic representations. The conclusions will be consistent with the recent approach of the National Academies of Sciences on mathematics and learning that asserted ·mathematics can be said to be about levels of representation, which build on one another as the mathematical ideas become more abstract. For example, the increasing focus during the school years build facility with more abstract levels of representation” (National Research Council, 2001, p.19). The results of this research will benefit the learning of math.

The impact of mathematics teaching methods courses on teachers' teaching practices: A case study

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This paper addresses issues derived from an ongoing research, which attempts to explore the kind of support that inexperienced primary teachers need in their struggle to embed the development of their mathematics teaching within the demanding school programme. For this purpose I cooperate with a group of six novice teachers who work in a primary school in Piraeus, Greece. The research is conducted in group and individual level. In this study I investigate how and in what extent teachers' experiences from the mathematics teaching courses that they attended during their university studies influence their teaching practices. This investigation is approached through the case study of Kate, one of the teachers- participants in the research. The analysis refers to data collected through observation and video-recording of Kate's teaching sessions, discussions I had with her before and after the sessions, the interview I had with Kate regarding the "scenario" and the educator's writing. Kate, the teacher of year 2, uses a "scenario" to teach her children how to add a one-digit number to a two-digit number. Kate had experienced this scenario as a student while attending the mathematics teaching methods course in the university. The mathematics teacher educator used this "scenario" in experimental teaching sessions with groups of years 1, 2 and 3 in order to explore children's strategies of adding a one-digit number to a two-digit number. Kate modified this "scenario" and used it to teach a particular method of adding a one-digit number to a two-digit number. In this way she transformed the "scenario" from an area of investigation to a routine, which worked against the aims of developing a rich conceptual understanding of mathematics. Why Kate felt the need to transform an investigative "scenario" to a routine? What happens to the teachers in their way from student-teachers to active teachers?

Research on the pre-university training of first-year students and its influence on the teaching of Mathematics

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Teaching Mathematics in higher education is a complex combination of a proper program, educational methods and ways of students' knowledge assessment. The Department of Mathematics in the Higher Institute of Food and Flavour Industries (HIFFI) is planning to introduce the mathematical assistant CAS DERIVE in the teaching of Mathematics for HIFFI students. In order to achieve better adaptation of that product we should be aware of the mathematical knowledge scope of the newly enrolled students in the Institute and the range of its influence on their grasp of Mathematics. Our research covers four student groups formed on the basis of their majors and the type of secondary school they have come from. We investigated the relations among school-leaving certificate grades, entry test results and ratings from the first year courses of Higher Mathematics Parts I and II. The results enable us to analyze the situation and to prepare more precise and detailed approach to the different groups in developing of our teaching methods. There is a clearly formed tendency for the students from secondary technical schools to show a better ability to cope with the Mathematical Courses in our Institute and to have higher ratings than this from high schools. There are also differences in adaptation and in ratings between students in the technical and technological faculties. Our study and the conclusions we have come permit the differential application of innovative methods in teaching Mathematics using educational software such as the mathematical assistant CAS DERIVE.

New Approach to the Use of Solution Manuals in the Teaching of Higher Mathematics

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We are currently testing a new method of teaching undergraduate mathematics in which we allow the students a free access to the complete solution manual. Our method consists in assigning a very substantial amount of home work problems and allowing the students to consult with the solution manual while doing it. Our philosophy is that the purpose of the home work is not to test the student's knowledge but to give her/him the opportunity to acquire and experience knowledge. The student is thus being exposed to a very large number of examples in which she/he participates actively, with the comfort that if unsuccessful to do the problem alone, there is a resource which can help. Remarkably, we have so far not experienced blind copying from the solution manual, which was our fear to start this program. We attribute this fact to making clear early in the semester that the tests are extensive both in content and level of intellectual challenge. We observe a big jump in the student's motivation, interest in the subject and success on exams. All students in the classes which participate in the study improved their test performance with at least 1.5 letter grade. Additional benefit which we observe is a remarkable increase in the students' self-confidence and study-independence.

Undergraduate Mathematics for Primary School Teachers: The Situation in Portugal

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At the beginning of the 1990s a national reform of the mathematics curriculum took place in Portugal. This was not accompanied by a corresponding reform in the training of primary school teachers. In Portugal teachers are trained in higher education institutions that are officially free to do whatever they believe is appropriate. This leads to wide variation in the training programmes, with some exhibiting a considerable degree of irrelevance (Gomes, Ralha & Hirst, 2001). This is a worrying scenario, because the curricular reform that took place presents new ways of understanding the teaching of mathematics, imposing new challenges on teachers. In Portugal it hasn't been until recently that the scientific community has begun to show an interest in the mathematical training of primary school teachers (APM, 1998). There are very few studies in this area, and they mostly deal with the pedagogical knowledge component of teaching, minimizing the importance of teachers' subject knowledge. In this study we undertake a brief analysis of the pre-service mathematical training for primary teachers currently offered in Portuguese institutions. We shall consider some studies in this area and discuss the possible consequences for the reform of pre-service mathematical education. In particular we pay attention to teachers' subject knowledge of basic mathematics, following the research of Liping Ma (1999).

e-status: a web tool for Learning by Doing Exercises

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The objective of this project is to present a web-based tool which will allow the student to solve an exercise as many times as needed, and always with new data. Students usually see exercises as an essential part of their education. Exercises are undoubtedly a good complement for the "theoretical" lessons: they show how the concepts explained in the classroom are applied in real or simplified cases, and they make easier the comprehension of the exposed ideas, through a typical process of learning by doing (the inductive way). The tool e-status will give the answer to each question by request. Several parameters such as date, problem identifier, number of errors, number of correct answers, time consumed, etc, will be collected and stored in the Students Database, so that this information will be available, not only for consideration of the instructor, but also to provide feedback to the students. Moreover, checks for students' progress will be easily implemented. An exercise will be specified in this way: a) several statements describing the problem, b) a set of arithmetic formulae, each one with its corresponding symbol, c) the available data, d) the questions, each one containing one statement and the answer (some symbol previously defined). At this moment this software is being developed for an introductory course of probability and statistical inference, taught to 2nd year students of Computer Sciences Engineering. Some problem data can be defined as random by the author of the exercise, therefore the tool will be useful to pose any statistical problem in the scope of our courses. The student obtains practice to model diverse situations by means of a (sufficient) number of different exercises, whereas the repetition of the exercise involves the habit of the computation and better understanding of the application of the main concepts. The proposed methodology can be useful to many other courses on a mathematical basis, mainly in engineering studies, whose students make extensive use of problem solving in order to lessen the level of abstraction present in the classroom.

THE IMPROPER INTEGRAL. AN EXPLORATORY STUDY WITH FIRST-YEAR UNIVERSITY STUDENTS

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In this paper we analyse the answers of a group of first-year university Mathematics students to a questionnaire, with the aim of determine the difficulties they have when developing non-rutinary tasks related to improper integrals. Among our research questions, we distinguish the followings: How do students react when they have to face up to tasks of a non-algorithmic type, questions of reasoning and non-rutinary questions in the topic we are involved? In which system of representation do they feel more comfortable? Are they conscious of the paradoxical results they can reach? Are they able to articulate different systems of representation in questions related to improper integrals? Do they establish any relationship between the new knowledge with the previous one, particularly the one related to definite integrals, series and sequences? The questionnaire consisted of nine questions including not only calculus tasks and to determine the convergence of given improper integrals, but intuitive questions and some paradoxical results too (for example, a figure with an infinite longitude which closes the same area than de unit circumference, or an infinite figure with a finite volume). We particularly asked the students to interpret the majority of the results they had reached. Answers given by the students to each of the questions were categorized, what allowed us to establish some partial conclusions of our research. The obtained answers let us determine selection criteria to develop how to interview the chosen students too. From the analysis done we can conclude that there are some difficulties at articulating the different systems of representation, and difficulties to connect and relate this knowledge as a generalization of previous concepts, as the definite integral, series and sequences.

Using technology to improve the curve learning of basic notions in Algebra, Calculus and Geometry

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One of the main problems facing mathematics teachers in scientific and technical disciplines (Physics, Engineering, ...) at universities when receiving first year students is the need of providing them with the capabilities required to understand advanced notions from the early beginning in order to be able of following the initial explanations of teachers talking about Physics, Mechanics, ... : usually first explanations start by the writing down of a differential equation in the blackboard when students hardly understands what a real number is ! The objective of this paper is to report how a proper combination of technology (distance web learning through WebCT plus the Computer Algebra System Maple) and a different way of presenting difficult notions concentrated more on the ideas than in the formalisms have been extremely useful in order to give to the students the capability of understanding the initial explanations of non-math teachers; reduce the gap between the mathematics explained at the secondary school and the mathematics expected to be known by a student when entering at the university; and provide very fast to the students with more solid math foundations. This experience has been organized around a course of 60 hours (27 hours the first month, 21 the second one and 12 the last one) delivered at the early beginning of the first year for Physics students at our university. It consists in ten modules (equations, matrices, derivatives, integrals, differential equations, analytic geometry, etc.) of six hours each with three hours of explanations devoted to motivate and illustrate concepts and techniques plus three hours of practical problems including the using of Maple. The tool used to control the individual progress of each student was WebCT through the realization of several questionnaires containing multiple choice questions trying to identify initial misunderstandings or to detect unexpected difficulties.

Exploiting Technology in Continuous Evaluation

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During the past three years, Technikon Pretoria has incorporated mathematical technology into the syllabi of students in Mathematics on all levels, with the objective of enabling them to apply their knowledge to problems in their various disciplines with confidence. This approach has shown positive results in terms of interdisciplinary collaboration. In this paper we illustrate the application of mathematical technology and systems of nonlinear differential equations, as discussed in third level mathematics classes, to the control of wildlife populations in nature reserves in South Africa. Case I: During the 1970-s the wildebeest population in the Kruger National Park was subjected to a culling program for five years, due to over-utilization of the veld. However, after termination of the culling program, the population did not recover as expected. Students were provided with the necessary data on wildebeest and their primary predator, lion, for a 15-year period. They were then requested to model the problem, explain the dynamics observed, and determine possible solutions and the projected outcomes of the various strategies suggested. Case II: An over-crowding problem of nyala in the Ndumo Game Reserve in KwaZulu-Natal has been detrimental to other species, especially bushbuck, and to vegetation structures in the reserve over the last two decades. Officials have been trying to control the nyala numbers with culling, without success. Students were requested to model the competition as a system of nonlinear differential equations, introducing a cropping term of varying order, and determine possible long-term solutions to the problem. From the solution curves, conclusions were drawn on the effect of different cropping programs on the population dynamics of the two species. The various models for cropping of nyala were evaluated mathematically without actually killing the animals, and the resulting optimum model was implemented at Ndumo in 1998.

A Gentle Introduction to Possible Chaos: A Three Species Model

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The study of systems of nonlinear differential equations and their applications have long been inaccessible to undergraduate students, but with the introduction of mathematical technology we are now able to move beyond the traditional limits of teaching Mathematics on a third semester level. Experimenting with two species models, students are firstly introduced to possible sensitivity of the solutions to small changes in the initial conditions. Three-species models are infamous because of the problem of instability and possible chaos resulting from sensitivity. In this paper we model a real-life situation of three interacting species (lion, wildebeest and zebra in the Kruger National Park, South Africa) and discuss its form, history and limitations. We then suggest how the stability of this model may be improved to such an extent that realistic projections of future population cycles may be possible.

Anamnesis and maieutics as methods to teach and learn

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Platon: ..."our learning is nothing else than recollection (ANAMNESIS)... we must necessarily have learned in some previous time what we now remember. But this is impossible if our soul did not exist somewhere before being born in this human form."(Phaidon, 72 e)"Seeing then that the soul is immortal and has been born many times, and has beheld all things both in this world and in the nether realms, she has acquired knowledge of all and everything;... research and learning are wholly recollection." (Menon 81c)Platon about maieutic (techne tes maieuseos), (midwifery):..."the god compels me to act as midwife... I am not at all a wise person myself... but those who associate with me, although at first some of them seem very ignorant, yet, as our acquaintance advances, all of them to whom the god is gracious, make wonderful progress,...they do this, not because they have learned anything from me, but because they have found in themselves many fair things and have brought them forth. But the delivery is due to the god and me." (Theaetetus 150c) Following Platon all knowledge is within ourselves and it can be brought forth by the help of the teacher and god. Platon does not mean an superficial, external knowledge ("doxa") but an inner, essential knowledge, which is hidden in the depth of our soul. The teacher does not implant ("inform") this knowledge into our brain (or soul) as if we were a "TABULA RASA", but he functions only as a "katalysator", as a "midwife" in connection with god. Platon differs between two kinds of mathematics: the practical and the theoretical mathematics. And it is the theoretical mathematic which brings the teacher and the student to higher insights by the methods of ANAMNESIS AND MAIEUTIC. This mathematic is the best method of "paideia" (education).

The Traveling Salesman Problem, Technology, Cooperative Learning, and Multi-Disciplinary Teaching

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The Traveling Salesman Problem (TSP) is a well-known combinatorial optimization problem and is an introductory example to many time-independent and time-dependent related problems. Their solution requires searches in a multi-dimensional space and combinatorial explosion prevents exhaustive approaches. Consequently, the last thirty years have seen the emergence of many heuristic algorithms offering optimal solutions, i.e. solutions that are very close to the global optimum and are obtained quickly. These algorithms form a rich environment where some approaches are good at solving one specific type of TSPs, while others, perhaps not as speedy, are more robust and adapt to more demanding requirements. This survey of algorithms (together with their implementation using MAPLE) is part of an effort to introduce undergraduate students to the TSP and the different methods of solution available. This is in the context of the development of a special topics course, opened to motivated students in search of research experiences that help them prepare oral presentations at conferences (such as an undergraduate/advanced high-school students research symposia) and write undergraduate research papers. We felt that the students would be more likely to become "engaged" if connections with the real world were to be forthcoming. Several such approaches, for instance use of simulated annealing, neural networks, genetic algorithms, and ant systems are appealing because they attempt to solve complex problems by incorporating in their mode of solution processes observed at work in the world around us. They offer excellent entry points for curious students who are, then, more likely to vigorously investigate well-known algorithms such as the Lin-Kernighan approach for solving the TSP. The topic's variety lends itself to cooperative learning among members of a group, friendly competitive learning across different groups as well as team moderating by different faculty members such as pure mathematicians, applied mathematicians, physicists, computer scientists, and faculty from the engineering school.

Geometrical and figural models in linear algebra

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According to many university teachers, 'geometrical intuition' can help students in their learning and understanding of linear algebra. Fischbein's theory about intuition and intuitive models provided us with a framework that confers a precise meaning to "geometrical intuition", and permits to examine its possible effects on students practices in linear algebra. We study especially the use of geometrical models, stemming from a geometry, and the use of figural models, whose elements are drawings. In France, two main tendencies can be distinguished among linear algebra textbooks : a structural approach, with no references to a geometry and almost no drawings ; and a more geometrical approach, using a geometry, and drawings for linear algebra in dimension two and three. The same tendencies can be observed in the teacher's courses. Linear algebra in dimension two and three appears as a naturally privileged domain to use figural models ; but the interaction with a geometrical model can raise specific difficulties in the students practices. In order to observe such difficulties, and to determine if students use figural models in linear algebra with no intervention of a geometrical model, we proposed a test to students in their first university year. It deals with linear applications of the plane, pointed out by teachers as a topic where figural models can be useful. The task proposed is unfamiliar for the students : for two given sets of vectors of the plane, both represented by a drawing, they were asked to determine if the first can be transformed into the second by a linear application. The student's answers and justifications show many references to the model of geometrical applications of the plane (studied at secondary school). But a figural model is directly associated with linear algebra, and a figural conception of linearity clearly emerges.

Tools for Synchronous Distance Teaching in Geometry

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Internet has opened a new dimension for distance education, where synchronous, long distance interaction and information interchange between students and teachers is relatively easy and affordable. Nevertheless, synchronous communication for distance education courses still occurs mostly through Internet Relay Chats, which are essentially tools for the exchange of text messages. The conventional chat room is inadequate for mathematics, where a special language of symbols, diagrams and text was, from very early times, developed to communicate. The combination of these elements is present even in the early texts of Euclid and Apollonius. To tackle the problem of teaching geometry, using Internet for synchronous communication, our group has developed a suite of three softwares designed to provide an adequate environment. This paper describes some of the characteristics of these tools, and preliminary results of experiments conducted to test how supportive they prove to be in a distance taught course. The first of these, MathChat, is a general-purpose tool for mathematical communication. With it we can provide tools for creating mathematical symbols and equations on the fly, symbolic algebra facilities, and -plotting of functions and surfaces, all integrated with the usual "chat room" facilities.

Tabulae, the second, is a dynamic geometry software integrated into a communication server. With it we can instantly set up a virtual classroom, where each student receives, in real time, each step of a geometrical construction the teacher is realising on his own machine. During the whole process the student is free to modify or add elements of his/her own, and to voice doubts or suggestions. Students can also direct their own work for instant check by the teacher or share it with the "classmates".

Finally there is Mangaba, our 3Dimensional dynamic geometry workhorse. Written in JAVA, it shares most of the communication features available in Tabulae. As an additional feature, it is capable of generating VRML code for any scene the user may construct. The primitives available include a comprehensive repertoire of construction and intersection primitives.

ELECTROCATALYTIC REACTIONS: AN INTERESTING PROBLEM OF NUMERICAL CALCULUS

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Electrochemistry provides interesting problems for applied mathematicians. An example of this is the adsorption of Carbon Dioxide over Platinum surfaces [1]. In fact, this problem was studied in other papers, where several methods of linear algebra, ordinary Differential Equations, Statistics and Numerical Calculus were used [2]. Now, in this paper, we try to show part of the richness of the problem [3], in order to use it in Numerical Calculus courses for Chemical Engineering and other chemical careers. Important concepts as numerical derivatives, and typical processes as fitting curves and determining coefficients numerically [4], can be illustrated in the context of this scientific and technological problem, closely related with other disciplines of these careers. This kind of problems provides a good opportunity for interdisciplinary work, but not only in their solution. In fact, they can be taught in the same way, by a group of teachers of several disciplines. Also, it is possible to propose project works to the students, taking parts of the problem or making small changes in order to motivate them with a real life mathematical and chemical challenge. We discuss results of these and other situations, experimented in the chemistry Faculty at Montevideo, Uruguay by the Mathematical Education research group ([3] and [5]). Taking into account all these experiences, we propose some conclusions and recommendations for this kind of mathematical service courses for chemical students.

**DO THE OUTCOME OF MATHEMATICS POTENTIAL TESTS
AND ACADEMIC PERFORMANCE
RELATE TO EACH OTHER?**

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This paper deals with mathematics potential test in secondary education and the academic performance of pupils measured by grades. It is often claimed that the outcomes of potential tests indicates the pupils' mathematics ability, however, academic performance does not always confirm the results. A series of mathematics potential tests have been given to 10 secondary colleges countrywide. 500 pupils, who have completed the potential tests, have obtained grades on the basis of their class performance in mathematics. The results will be presented in the conference.

About the role of workshops in teaching Mathematics.

Matieva Gulbadan

The practical workshops are very important stage in teaching Mathematics. In order that practical workshops brought the maximal advantage, their contents should be well coordinated with a lecture material: if the student both at lectures, and in practical workshops will get acquainted with the same point of view on concepts investigated by him, will fix in the workshop the theoretical knowledge acquired by him as a result of discussion and analysis of lecture material, and by solving problems, then he not only will acquire this material well and will learn to apply it in practice, but also will receive additional stimulus and it is very important for active study of lectures materials.

It is necessary to emphasize, that only after the students have mastered an investigated material with certain the points of view they can be acquainted, and it is sometimes rather expedient, with other points of view on the same subject. Very harmfully, the realization of the unmatched lectures and practical workshops is especial in the first grade levels when the lecturer and teacher conducting workshop, tell about the same questions from the different points of view, were based on different definitions and in a different sequence of a statement of the separate facts. It can confuse the students, put thus harm to their mathematical education, decrease his efficiency, make process of training more difficult for the students.

The practical workshops on mathematics are collective work , and though at mastering mathematics the large and important role is played independent individual work (man can not learn to think, if he doesn't thinks himself, and skill to think is a basis of possession of mathematics has also collective work, in particular, practical workshops. In workshops take part not more than 15-25 students from one of groups well enough familiar among themselves and with the teacher. It causes specific features of practical seminars. They give a significant positive effect, if on them the atmosphere of goodwill and mutual trust reigns, if the students are in a condition spiritual relaxation, if they do not hesitate neither, nor classmates ask that is not clear, is open, are divided with the teachers and comrades by the reasons.

All is very uneasy it for achieving and only thoughtful self-critical attitude of the teacher to the work, use and another's experience allow to achieve good results to learn to use collective of individuals with different abilities and differences in preparation, with different characters and temperaments for activation of work of an idea of each of them, for creative mastering of an investigated material by each of them. As an example the method "zigzag" within the framework of the project "RWCT" also is offered the brief description of a course of practical workshops on a theme "Canonical frame (heron) of line. Curvature and twisting" (for third course students of Mathematical faculty of university), where this method is used. After the announcement of a theme of workshop some tasks or questions on a theme are offered to the students:

- 1) What is a tangent to the given curve?
- 2) Write the equation of tangent to curve in its any point.
- 3) What is an osculating plane of given curve in its any point?

- 4) Write the equation of an osculating plane of given curve in its any point.
- 5) Write the formula Frene for a space curve.

5-6 minutes of time are given that each of them individually prepared the answers to a question. After that they in pairs discuss the answers within 3-4 minutes. Then the answers of pairs in group are discussed, the teacher writes down results on a board, which are “tool” for the decision of tasks on a theme. if it is planned to decide (solve) four tasks during seminar, the students are grouped in small groups till four and the tasks are offered:

1) To write the equations of an adjoining plane of the line $x = t, y = t^2, z = t^3$

2) To find coordinate vectors of canonical frame of the line

$$x = \cos^3 t, y = \sin^3 t, z = \cos 2t, 0 \leq t \leq 2\pi$$

3) The spiral line is given: $x = a \cos t, y = a \sin t, z = bt$. To write equations of principal normal and binormal in an any point of the line;

4) To calculate curvature and twisting of the line

$$x = t - \sin t, y = 1 - \cos t, z = 4 \sin \frac{t}{2}, -\infty < t < +\infty, t = \pi$$

Each small group refers to as “cooperative” group. Each member of cooperative group manages a task for the decision and subsequent teaching each other (i.e. the first member of group answers for the first task, second – for second, third – for the third, fourth – for the fourth). After that the “expert” groups are created: from each cooperative group the students under same number are invited, for example, all first members of cooperative groups form the first expert group, all second members – second etc. The first expert group works above the decision of the first task, second – second task etc. The members of expert groups solve the task together, plan effective ways of teaching their decision to the comrades on cooperative group, then check whether their comrades have understood.

When all expert groups finish the works, the students come back in the cooperative groups. There each student on turn explains another’s the decision of the task. Tasks of a cooperative team – that each student has mastered of the decisions of all tasks (problems). It is possible then to ask to show the separate members of group the knowledge, having given presentation of the decision of a task, to which they were trained with the comrades on group. It is necessary to note important bringing up aspect of practical workshop. Here teacher has the large opportunity actively to bring up in the student honesty, decency, respect to the surrounding people, love to work. Here student can correctly estimate importance of the help to friend, when that difficult minute will come to him to the aid, role of trust, which are rendered to him by the teacher. Here student can realize, how it is terrible, for example, having deceived somebody, to lose, to deprive itself of friendly dialogue of pleasure of joint work and by that impoverish the life. Here student can understand, that, having lost trust of the surrounding people it is possible to return it back, but for this purpose it is necessary to work much and not by a word, and but to prove by the acts the honesty and unselfishness, basic attitude to the people. The returning of trust to the man should be for him one of the largest holidays in life.

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Linear Differential Equations with constant coefficients: An alternative to the method of undetermined coefficients

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In this paper we present an alternative to the method of undetermined coefficients for obtaining a particular solution of a linear differential equation with constant coefficients. This neither requires any string of rules for the trial solution nor the solution of a system of linear equations. The method only requires differentiation and elementary algebra. The procedure would be illustrated by solving several equations which are amenable to the method of undetermined coefficients. Use of the procedure as a recursive algorithm will also be demonstrated. The technique has been suitably reformulated for the particular solution of the similar linear difference equations with constant coefficients.

Two discourses about the teaching of Mathematics

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The aim of the paper is to show two didactic situations with respect to the teaching of Mathematics. We understand the didactic situation to be the set of implicitly or explicitly established relationships among a group of students, a certain means and an educational agent so that students could grasp a constituted knowledge or a knowledge in the process of constitution. Through an example, we wish to reflect two didactic tendencies, two learning conceptions that can be approached as follows: the first one within the model of behaviourism; the second one, within the frame of the cognitive trends that promote the significant learning as a task of construction and reconstruction of knowledge. The environment of application is the Faculty of Exact Sciences, Engineering and Surveying of the National University of Rosario in two courses of the second year of the Bachelor's Degree in Computing Sciences. In both settings there is one and the same learning aim: the sum of random variables. The learning process with students who have a similar epistemic background is fulfilled, but in each course the way of teaching is different. In one setting, the teacher is positioned as a contingency manager and the learning student is completely passive. In the other setting, the student is a subject who organizes his behaviour interacting with the object of study, regulates this interaction and arranges his outlines in order to capture, incorporate and metabolize that object. Here the teacher constitutes himself as a facilitator of learning and causes learning to develop with less obstacles. Learning is a consequence of a cognitive conflict deriving from the exploring and transforming action exercised over the objects by the student in order to understand these objects by incorporating them within his cognitive structures and by giving them a meaning. We believe that the methodology used shows superior qualities than the one which gives relevance to a receptive and passive learning.

'Writing in a Reformed Differential Equations Class'

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In an attempt to promote the development of understanding over rote memorization, writing in mathematics has received increased attention in recent years. In Calculus, the Rule of Three (based on communicating ideas through algebraic, graphical and numerical means) has been replaced by the Rule of Four in which writing plays a central role. Educators agree that the benefits of writing include the promotion of understanding, and the initiation of the posing of questions. Writing also helps generate meaning, and helps in the retention of content. Various books and research papers have appeared on the subject (MAA Notes (1989), Rose (1989, 1990), Shurle (1991), Porter (1995), Meier and Rishel (1998), and Meel (1999)); yet most of them deal with the skills required to write a good mathematical proof. In this paper, I evaluate the use of writing for analyzing a problem and its solution. The setting is a reformed differential equations class offered at the Lebanese American University. Unlike a traditional ode course where students are provided with a cookbook of methods for solving differential equations, the emphasis in a reformed ode course is placed on the geometry of the solutions and on an analysis of the outcomes. In many instances, students are asked to solve a differential equation by plotting its solution curves without identifying them analytically, and the sketch is to be supplemented by an argument justifying it. In addition, various real life problems (e.g. population growth, predator-prey systems) are modeled and essay questions are asked to analyze the graphs describing these models. Students of this course are regularly interviewed and questionnaires regularly administered. Results show that students first reject the idea, but later rate writing as essential. Furthermore, an improvement in the style and content of the writing exercises is usually noticeable at the end of each semester.

Introducing RME to Junior Secondary Mathematics Teachers in Indonesia

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The study finds its basis on the emergent needs for improving mathematics education in Indonesia which is for a long period experiencing many challenges. From the time when the implementation in schools, many efforts have been done. Since 1977 the government has produced over 900 million copies of textbooks for pupils and teachers, has provided inservice education programme for most of the schools' teachers, and has provided teaching aids for schools (Moegiadi, 1994). A diagnostic survey conducted by Ministry of National Education in 1996 disclosed that many teachers were still using telling method as their teaching approach, and that pupils' performance in mathematics was poor (Suryanto, 1996; Somerset, 1997). This study called IndoMath (Inservice education for Indonesia Mathematics teachers) focuses on the introduction of the Netherlands-based RME (Realistic Mathematics Education) theory to Junior Secondary Mathematics teachers as an effort to improve their competencies. The study has been conducted in Indonesia from 1999 until 2001 involving 50 mathematics teachers. This paper examines the effects on instructional practices that resulted from enhancing teachers' content and pedagogical knowledge of RME using strategies of workshops, classroom practices, and reflections. This study demonstrates that a carefully planned programme of professional development grounded in principles of effective strategies can significantly impact teachers' understanding of RME both theory and practice.

Connections within mathematics - What questions should be asked and what answers should be given?

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Recognizing interconnections within mathematics is one of the main emphasis of the NCTM's standards*: 'Thinking mathematically involves looking for connections· making connections builds mathematical understanding·. In order for the teachers to be able to implement this approach it is necessary that they will possess such a 'meta perception' of mathematics. Working with mathematics teachers, we have identified various concepts that could be used to highlight the interconnections within mathematics, and thus help teachers to develop their awareness to the issue. In our lecture we shall demonstrate three examples of such concepts: straight line, parabola and similarity. · Students first encounter the concept of 'straight line' at the geometry courses. lateron they deal with linear functions and equations. Teachers should discuss with their students questions like: What are the connections between 'all those straight lines'·? What allows us to denominate a function like: $y = 3x + 5$ by the name 'a linear function'·? Whether and why its graph is a straight line? · Similar questions can be raised concerning the concept of 'parabola': Is the graph of a quadratic function a parabola? Is the parabola a graph of a quadratic function? Does a graph of a quadratic function match the definition of the parabola as a locus of points in the plane? · Conventional discussions concerning the concept of 'similarity'often disregard the similarity of curves or polygons that are not triangles. Attention should be given to question like: What are the necessary and sufficient conditions for similarity of two polygons? Is it possible to determine whether or not two parabolas are similar? And so on. We believe that confronting teachers with such questions can contribute to the development of a wider and more complete mathematical knowledge. Only then they will be able to impart this kind of knowledge to their students and see the beauty of the mathematical doing within its consistent rules

Main Theme: Education Research

Secondary Theme: No secondary theme

Using Japanese Industry Principles to model the learning of Undergraduate Mathematics

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The ILUO-principle has its origin in Japanese industry where acquiring basic skills for performing operations are paramount. The concept is that the mastering of any basic skill follows a so-called ILUO pattern. The four letters indicate four different ratings, depending on the level to which a certain skill has been mastered. The lowest rating is an I-rating, followed by an L-rating and then by a U-rating and finally by an O-rating, the highest of the ratings. The reason for the choice of letters is clear from the progression of the four shapes. The APOS model is a constructivistic theory of how learning mathematics might take place and was developed by Dubinsky and others as an attempt to explain Piaget's concept of reflective abstraction. In this talk we compare the two models in the context of learning mathematics at university level.

College Algebra in Context: Redefining the College Algebra

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We have developed a redefined college algebra course, which uses an informal approach, is application driven, is technology-based, and uses real data problems to motivate the skills and concepts of the course. Each major topic contains real data examples and problems, and extended application projects that can be solved by students working collaboratively. Students can take advantage of available technology to solve applied problems that are drawn from real life situations. The students use technology, including graphing calculators, Excel, and Derive, to observe patterns and reach conclusions inductively, to check answers of solved problems, to study function types, and to create models for use in the solution of problems. The course provides the skills and concepts of college algebra in a setting that includes applications from business, economics, biology, and the social sciences. The course was designed to provide the required college algebra skills for students in the Business major, in the Hotel, Restaurant, Tourism Administration major, and for majors in the biological, marine, and social sciences. The real life applications included are the result of collaborations with faculty in Business, in Hotel, Restaurant, Tourism Administration, and in Biology Departments in three Universities. Each mathematical topic of the course is introduced informally with a motivational example that presents a real life setting for that topic. The problem in this example is then solved as the skills needed for its solution are being developed or after the necessary skills have been developed. Some applications provide the models for the data and have students solve related problems, while others require students to develop the models before solving the problems. For some topics, students work in small groups to solve extended application problems and to provide a written report on the results and implications of their study. For other topics, students find appropriate real data in the literature or on the internet, develop a model that fits that data, and use the model to solve problems. Most of the examples and exercises in the course are applied problems.

The Connection Between Old and New Mathematics in Works of Islamic Mathematicians: A Look at the Application of History in the Teaching of Mathematics.

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All over the world, mathematics is known to be a difficult subject. However, over the last fifty years, new tools for understanding mathematics, such as mathematical software like Mathematica and Mathcad, have emerged. Another invaluable tool is the history of mathematics, which has become increasingly important over the last decade. Using the history of mathematics, a mathematics teacher can change the style of his or her teaching. The history of mathematics allows students to understand that mathematics and science is work of all civilizations.

This talk will describe how a large collection of historical materials, made by students and faculty in Iran, can be used to improve the teaching of mathematics. Examples will be given of the history of logarithms. The materials are available on CD..

**Mathematical technology transfer:
Industrial applications and
Educational programmes in Mathematics**

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Mathematical technology is an interdisciplinary area combining applied mathematics, engineering and computer science. Computational technology has made sophisticated mathematical methods viable for practical applications. Models are used to -replace or enhance experiments or laboratory trials. -create virtual and/or visualized images of objects and systems -forecast system behavior and analyze what-if situations -optimize certain values of design parameters -analyze risk factors and failure mechanisms-create imaginary materials and artificial conditions prior to the possible synthesis or construction. -gain understanding of intricate mechanisms and phenomena -perform intelligent analyses on measurement data -manage and control large information systems, networks, data-bases The new view of mathematics should be reflected in educational practices. New kinds of expertise is called for. We should shape the image of an emerging profession, industrial mathematician, computational engineer or symbo numeric analyst? The education should convey the vision about mathematics at work, to display the diversity of application areas, to demonstrate the practical benefits. Possible tools include

- revision of curricula
- educational software environments
- problem seminars and project work
- teachers (re)training

In this talk we discuss the educational challenges, curriculum development and some experiences in the teaching of mathematical modelling. I give a list of real life applications, industrial processes and R&D-questions where mathematical method have a significant role. These examples are meant to carry over the idea of mathematics as a multipurpose environment of problem solving. They are presented in a style which could be used in undergraduate education and teachers training.

Blending technology and pure mathematics: is the hard work worthwhile?

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Incorporating a computing component into an undergraduate pure mathematics course is well-established practice. Reasons given for introducing technology include freeing students from the grind of hand calculations so that they can tackle more realistic problems, exposing students to the possibility of exploratory work, and allowing graphical as well as numerical representations of the mathematics. Although a small number of courses have abandoned lectures and are taught entirely in the laboratory, most still retain the traditional format and present the computing component as a supplement. Integrating the computing work with standard lectures and pen and paper exercises requires a clear understanding of the aims of each type of learning activity. Questions to be considered include: what is an appropriate balance between teaching the students about the software and teaching them mathematics, what do students believe they are learning from computer-based sessions, and are students' perceptions of the purpose of this type of activity markedly different from that of the teacher? Designing a new Matlab-based computer laboratory program for an undergraduate linear algebra course with an enrolment of 850 students presented both a technical challenge and an opportunity to investigate these important questions. Three types of Matlab problems were written: computational, experimental, and graphical/visual. Some of these were designed to illustrate theoretical areas that are conceptually difficult for the average student. The problems are accessed through a graphical user interface. This provides a highly structured work environment in which problems are split into stages and students' work on each stage is monitored electronically. A coded record of their work is generated automatically as each problem is completed. Student reaction, both critical and favourable, is addressed.

Main Theme: Innovative Teaching

Secondary Theme: No secondary theme

The BLK-School project in Baden-Wuerttemberg

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The project „Promoting classroom culture in mathematics“ is Baden-Wuerttemberg’s contribution to the program of the Bund-Länder-Kommission BLK (commission of the German government and the federal states) “Furthering the efficiency of mathematics and science teaching”. This four-year program for the lower secondary level was developed as a meaningful reaction to the less than satisfactory German TIMSS results. The approach of this project focuses on changing the teaching style. The main aim is to develop a holistic design of mathematics teaching integrating comprehension, active participation and long-term productive learning. A report on first experiences of the project in Baden-Wuerttemberg is given. More exactly, I plan the following structure: ·

- TIMSS and the BLK-Project·
- The school project in Baden-Wuerttemberg·
- What has to change? ·
- Our aims ·
- Taking children seriously·
- Productive practise·
- Assessment: Measuring and feedback of gain in competence ·
- Experiences from the project·
- Some problems of realisation ·
- The WUM-inservice teacher education·
- Conclusion

Modelling and Spreadsheet Calculation

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A. The process of modelling is a constant oscillation between various levels of abstraction. This can be divided into the following phases:

- get a grip with the problem (define the key questions)
- formulate a mathematical model
- generate solutions (from the mathematical model)
- validate the model (and if necessary re-formulate the model until it fits with the real-world-context)

B. Especially pupils in lower secondary schools have problems to formulate their ideas or assumptions in a mathematical (algebraically) way. The transfer from the spoken language into the mathematical language becomes a difficulty as well as its not easy for them to generate solutions from the mathematical model. With spreadsheets this effects could be reduced in some fields because it is not urgently necessary to define variables and formulate equations. Furthermore that possibility of intuitive use and the splitting into modules appear as an advantage.

C. We have a classification for models.

Models which process large quantities of data in an elementary fashion.

Models which solve using systematic testing.

Models which are based on iteration and recursion.

The qualitative and quantitative evaluation of data which requires only functional relations.

The simulation of operations from which a mathematical solution model can be deduced.

D. Examples of the classification for models. The gas problem (How fare is it worse to drive for gas?) is an example for the class of models which based on systematic testing. After formulation of the model, the pupils try to solve the task by manipulated the in-data until the out-data fits the problem. The Fermat task (How many piano player does exists in Chicago?) fits in the class of models which based on the evaluation of data. Because there is no “true” answer on the question, the task is to find criterions for evaluation. To check the assumptions and the numbers in the model the using of spreadsheets implements advantages. The financing problem is an example for models which based on iteration and recursion. The make up an financing plan with fix in-data (e.g. time, money, payment) and ask for the interest rate becomes in some cases that one couldn't formulate and solve in an algebraically way. The using of spreadsheets shows different advantages in this case.

E. Conclusions. The earlier one begins with the concept of modelling the better one get these abilities in the course of the school time and the better one recognises mathematics as a part of our world. In the modelling process spreadsheets could be used with all classes of models.

Main Theme: Education Research

Secondary Theme: No secondary theme

Processes of Abstraction in Context

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The common view of abstraction in mathematics is one of decontextualization, focusing on some properties and relations of a set of objects, rather than on the objects themselves. Following Hershkowitz, Schwarz & Dreyfus (2001), we take a notion of abstraction which is strongly depended on context, history of the participants, their interactions, and the tools at their disposal. This model of abstraction involves three epistemic actions: Recognizing, Building-With, and Constructing which are dynamically nested (the RBC model). We present two different learning episodes from Linear Algebra and show how these it within the framework of the RBC model. The first example involves a problem about transformations which requires reflection upon, and eventual formulation of, the notion of linearity. The second activity requires the construction of the notion of a projection in an inner-product space which is not related to the familiar notion of projecting a vector on a plane.

Classifying students' mistakes in Calculus

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Analysis and classification of mistakes has been undertaken in several areas of mathematics, following the pioneering work of Brown and Burton (1978) in arithmetic. In the study described in this paper we used many examples of student errors taken from a complete semester's first year university calculus course and its associated problem sessions and examinations. One can identify a number of distinct types of recurring error, but also some common threads running through these errors. Foremost among these is the tendency for procedural generalisation, which according to Krutetskii (1980) is an inevitable component of student learning. One therefore concludes that such systematic errors are unavoidable, and cannot be eliminated simply by a suitably designed instruction sequence. This emphasises the importance of lecturers being aware of systematic structures involved in classes of errors, and this paper makes a contribution in that direction.

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The Mediational Effects of Texts and Technology in Teacher Preparation

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Teaching at school and at tertiary level may be characterised by three types of practice: direct instruction, mediation and facilitation. The mark of an expert teacher is revealed by his or her ability to move between these various modes of practice, in response to the immediate needs of the students within a classroom, lecture hall or learning environment. This wisdom in practice comes from a clear understanding of the nature of these teaching practices and an awareness of the potential of each to produce the desired responses from the recipient students. This paper suggests that this wisdom or awareness in practice may be developed through learning programmes in which novice teachers experience and reflect on different modes of teaching practice, centred on their subject discipline, within an interactive learning environment. This environment extends the traditional modes of learning through guidance by an expert to include mediated learning with written texts and interactive technology. Furthermore reflective practice is also built in as a necessary component for learning. This reflection refers to the learning of the subject discipline and to the teaching of that discipline. This paper reports on a case study in which prospective teachers were asked to investigate a geometric problem, to reflect on the methods used to construct a solution to the problem and to produce a teaching unit for grade 12 students based on these actions and reflections. Particular attention is paid to the participants use of written texts and computer technology in their resolutions to the geometric problems and as a consequence to their use/abuse of texts and technology in the proposed teaching unit. The results of this investigation raises questions as to the effectiveness of mediational resources in supporting mathematical progress and stimulating creativity and independence in the classroom.

Diagnostic Testing and Follow-Up for First-year Mathematics Students

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Since 1999, all first year students doing the module Mathematics Methods 1 have sat diagnostic tests at the beginning of the academic year. The test results have been used to identify mathematical weaknesses of individual students and of the student group. Staff teaching these students then face the following problems of:

- motivating the students to overcome mathematical weaknesses,
- determining the best availability of resources to assist students to overcome their weaknesses,
- assessing students' progress after diagnostic testing has taken place.

This year at UWE, students completed four diagnostic tests, which covered material in algebra, equation solving, trigonometry and calculus. The content of module Mathematics Methods 1 anticipates a sound knowledge of the material covered in the diagnostic tests. Most students feel threatened by the idea of completing a diagnostic test, particularly at the beginning of the first year after a long gap from any mathematics. Hence to reduce this threat and to ensure that the tests were completed by as many students as possible:

- the tests were made available via the web over an extended period of time,
- students were allowed multiple attempts with the best result counting,
- the test results counted towards 12% of the coursework mark equivalent to 3.6% overall of the final mark in Mathematics Methods 1.

Initially the tests were open for the first two weeks of term and students given three attempts at each test. The results which were then analysed to determine mathematical weaknesses of the group, pointed to serious deficiencies in trigonometry, particularly with understanding of sine and cosine waveforms. The tests were opened again from 8 October to 21 December and students were given an unlimited number of attempts. The results will be analysed to assess students progress.

Entrepreneurial mathematics graduates

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The University of Ulster is developing a strategy for the introduction of "entrepreneurial studies" into all of its programmes. The Learning Outcomes expected of each programme must include the following: - On completion of this programme students will be able to: 1. Demonstrate innovative thinking and creativity. 2. Demonstrate knowledge of future trends in her or his subject area. 3. Identify the steps required to research a market for a business opportunity. 4. Explain the impact of intellectual property rights with respect to new idea generation and product innovation. 5. Describe the component parts of a business plan. 6. Demonstrate familiarity with the range of organizational support available to assist with new enterprise development within UU and the local community. 7. Demonstrate team building ability. 8. Identify the steps required with respect to new company set up and incorporation. 9. Identify the key sources of finance available for business start-up. 10. Communicate new ideas effectively. 11. Demonstrate familiarity with an e-learning environment. The course team for the honours degree in Mathematics and Computing have devised a curriculum which seeks to develop these LOs in students. The curriculum innovations involved include the provision of a module "Entrepreneurship and Modelling". The paper will describe the previous curriculum innovations in the course which were designed to enhance the employability of graduates, and how the current innovations are to be implemented.

A Visual Analysis of Knowledge Networks: Students Discuss Calculus

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This poster describes our findings from a study on the conceptual development of calculus students and emphasizes our attempts to construct visual and schematic diagrams of student understanding and the connectedness of their ideas. In particular, we hope that by introducing graphical and computer-aided methods, we can begin to study student spontaneous reasoning as an emergent phenomenon in its own context and not merely in relation to more official or formal expectations. The importance of the connectivity of student mathematical understanding has been a strong theme in recent work of such varied authors as Li-Ping Ma, Steve Monk and David Tall. In our study, we analyze student understanding of limit and covariational reasoning in terms of their facility with different metaphorical contexts and we attempt to characterize and diagram the connectivity of student ideas using a combination of qualitative analysis and computer-aided text and network analysis. We introduce these techniques in an attempt to acknowledge and capture the role of informal reasoning in the development of rich connected understanding. We believe that the analyses presented are promising first steps towards an analysis of student use of language in reasoning that give evocative approximations of these fleeting but essential mental connections. Our analysis amplifies a careful semantic analysis of local relations between mathematical ideas of students by using computers to analyze the larger patterns that emerge from such fine-scaled analysis, and indeed to constructing visual representations of these larger patterns. The data was collected during the academic year 2000-2001. We performed task-based interviews with 14 students enrolled in a first year calculus course at a major southwestern public university. Each student was interviewed and audiotaped three times during the year for about an hour per session.

A Problem-Based Learning Approach to Introductory Logic

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In [1] Gries claims that teaching students about proof using formal proof methods is superior to teaching them using "semiformal" methods. The approach proposed is one using the calculational predicate logic of the text *A Logical Approach to Discrete Math* [2]. This text has been recommended for an introductory logic course in Trinity College Dublin for over four years. The aims of the course are to develop student skills in the propositional and predicate calculi and to encourage students to exercise these skills in applications that arise in computer science and discrete mathematics. These aims were not reflected in the method of course presentation; too much emphasis was placed on the technical theory involved, and too little on the application of the material. It was noticeable in both lab classes and exams that students tended to avoid questions requiring the very skills that the course is trying to promote. This paper reports on major developmental work done on the course to realign the teaching methods with the course aims. The students work in small groups on substantial problem sequences, supported by the lecturer and a postgraduate student. The students themselves, guided by the problems, construct most of the course theory. However, they also attend a weekly plenary lecture where recent themes are pulled together and coming themes previewed. The increased emphasis on problem solving creates an atmosphere where students engage with the course in a more meaningful and appropriate way. The restructured course is being offered to students for the first time the 2001/2002 academic year.

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Making the Connexion: Utilising Multiple Intelligences to Measure Teaching and Learning Success in Mathematics

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We have all had the experience, at some stage in our own formal education, of knowing a teacher, tutor or lecturer who was a godsend to half the class and a complete irrelevancy to the rest. How is it possible that the same teacher can make perfect sense to some and no sense to others? Obviously, a certain teaching style or mode of delivery will appeal to those to whom it has meaning and be inaccessible to those who do not connect. Why do educators connect with some students and not others? The answer lies in the fact that each student is a unique individual with his or her own learning style who will learn best from a teacher who utilises a suitable teaching style. Is it the role of students to adapt their modes of learning to capitalise on the teacher's offerings or should the teacher be trying to connect with all students by employing a variety of teaching styles? Howard Gardner, in his theory of multiple intelligences, asserts that everyone is intelligent and capable of learning, but that an individual will favour some modes of learning over others. The factors influencing these modes may be genetic, environmental, or experiential, but they are beyond the teacher's control as the favoured learning style is already formed by the time the student walks into the classroom. It is therefore the responsibility of the educator to adapt his or her own preferred teaching style and use a variety of modes in order to make connexions with each student. This paper attempts to analyse various strategies utilised in the teaching, assessment and examination of the Preparatory Mathematics Course at the University of Sydney, Australia, in the context of multiple intelligences. The students were given a questionnaire which aimed to establish:

- their individual preferred learning styles
- which of the multiple intelligences they exhibited
- to what extent the teaching strategies, modes of assessment and examination employed the multiple intelligences
- the suitability of these

Problem-solving sessions with a stress on discovery and the participation of fast-learning students.

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A colleague of mine and I have experimented with an innovative teaching-learning setup in the form of problem-solving sessions of weekly 2-3 hours for classes of 30-40 students to supplement the formal teaching in courses covering the students' first encounter with Probability Theory, Statistics and Set Theory at the second year level. The first problem session consists of an orientation explaining teaching and learning through the discovery method. Problems are exhibited on the board one at a time and the students are asked to attempt to solve them. The teacher goes around among the students to answer queries and suggest methods of approach but not to give the solutions. Fast learning students who get their answers approved join the teacher in acting as guides to other students. The session ends with one or more of the early finishers exhibiting clear and precise solutions. The advantages of this approach are given with some interesting situations such as when different solutions of the same problem arise or when counter examples are given with a discussion thereof.

NT (NEW TECHNOLOGY) HYPOTHESIS

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I teach Mathematics in a undergraduate first-year course at an Economy University; the actual name is "Degree in New Technology and International Economy". The course-subjects are the usual topics you may guess: precalculus, calculus, linear algebra. I think today a big opportunity exists to enhance both the teaching efficiency and the students' interest: we can use and let the students use a Computer Algebra Systems (CAS) or, more generally, a math software like DERIVE, MAPLE, MATHCAD, or a graphic and symbolic calculator as TI-89, TI-92 Plus. My hypothesis is: let us conjecture that all students have at their disposal at all time (during the lesson, studying at home or at university, in every assignment and examination) a math software with the following features: · symbolic and floating-point manipulation · plotting and exploring function graph · capability of defining a function (with as many arguments as they want) · capability of running simple programs Given this hypothesis (I will call it NT Hypothesis) in which way a math course has to change? How should the subjects, the most important topics, the exercises, the problems and the assessment be modified? We would like to present at Creta ICTM-2 Conference a comprehensive description of our work: the project (March-July 2001), the course (September 2001-April 2002) and a first analysis of the results (May-June 2002). We chose to present at the Conference three independent papers (see also the abstracts by G. Osimo and F. Iozzi); each of them takes a rather different point of view.

A limit-free approach to derivatives: report on a classroom project

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From the idea that "... using a graphing calculator to zoom in functions is one of the best ways of seeing local linearity" (Deborah Hughes-Hallet, Andrew M. Gleason et al., in [1, pag. viii]), we developed [2] a complete "route to derivatives", which is particularly suitable for undergraduate teaching. It includes a formal definition of a tangent to a graph, and goes up to the fundamental theorem of calculus. The basic definition (the slope of the tangent) relies on graphic concepts, like screen resolution and pixels' dimensions, so, essentially, it's "limit-free". The fundamental theorem of calculus is presented in a discretized version, showing that the trapezoidal rule for areas and the central difference formula for the tangents' slopes are, in a sense, inverse one to each other. Everything has been implemented on an intermediate graphic calculator (actually the TI-82, 83 and 83+), and carried out in the last three years on a total population of about 750 students of life sciences, pharmacy and chemistry, aged 18-19, from two different universities in Italy. We observed a substantial improvement of low-ranked and middle-ranked students (with respect to previous non technology-based teaching), meaning that a significative percent of E's and D's moved to C. We're referring here to the European Credit Transfer System classification. In this paper we present also of the project a "teacher bank", consisting of TI-GraphLink compatible files containing related programs and data, and more advanced teaching tools as Mathematica notebooks and QuickTime movies, especially generated by TI-83+ screenshots.

[1] D. Hughes-Hallet, A. M. Gleason et al., Calculus, John Wiley & Sons, Inc., International Edition, New York, 1994.

[2] S. Invernizzi, M. Rinaldi and A. Sgarro, Moduli di matematica e statistica (italian) - with CD-ROM, Zanichelli Editore, Bologna, 2000.

Use of a MAPLE - C Interface In Teaching Ordinary Differential Equations

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In this paper, we present a Maple interface to C software for non-autonomous explicit ordinary differential equation initial-value problems using either the standard Runge-Kutta methods or the new Goeken-Johnson algorithms. The methods include the base routines for Runge-Kutta method for orders 2-5 and Goeken-Johnson methods for orders 3-4, and the Maple interface. In the new non-autonomous algorithms developed by Goeken and Johnson, the user utilizes both f and f' . The evaluations of f' , which are done by using Maple, replace the evaluations of f . The advantage of these new methods lies in the fact that fewer evaluations of f are required than in the standard Runge-Kutta methods. The authors have many years of experience in using Maple as a teaching tool. The examples presented in this paper are typically covered in an undergraduate Numerical Analysis course. The C code used in the routines mentioned above is easily accessible to any student who has taken two semesters of C programming. Actually, most students taking an undergraduate Numerical Analysis course have a much broader programming experience.

Collaboration and assessment in a technological framework

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Following one year's experience of lecturing Calculus to undergraduate students at Università Bocconi, Milan, Italy, we have investigated the way students collaborate among themselves and with the lecturers when using an e-learning software (see for more information also the abstracts by M. Impedovo and G. Osimo); then we studied several approaches to the problem of the assessment of students' knowledge. In the first part we have focused on the subjects (which are the preferred topics among students and why), the way the discussions are brought on (which kind of discussions are more popular and how the students discuss the subjects) and the impact of the discussions on the performances of the students (are they related to the way the students are involved in the collaborative environment?). In the second part of this research, three methods to assess students' performances have been compared: a particular mathematical software, the evaluation sections of an e-learning software and a software, developed by the author, specifically designed for lecturers. In the latter case, the technological framework is explained in detail. In particular we discuss: a) the choices made for the interface; b) the intranet set up to guarantee maximum security before, during and after the examination and c) the modularity of the software developed. We consider these aspects interesting by themselves and because the problems they pose are too often neglected.

Do Not Teach Algorithms

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Teaching the standard course "Analysis & Design of Algorithms" at an undergraduate level in a typical Computer Science program has essentially two objectives. The first objective, dealing with analysis, is to familiarize students with existing algorithms. The second one, which is perhaps far more important, is to equip the students with the necessary tools and techniques, and above all the confidence, required in solving a non-textbook problem. This second objective, concerned with the design of algorithms, is essentially a creative effort containing all the ingredients of a thriller: adventure, excitement, challenge, and suspense. Our experience of teaching algorithms indicates that creativity in algorithm design depends, to a large extent, on how we deal with the analysis phase. We stress that while we are familiarizing students with existing algorithms, we should not formally teach anything. Instead we should encourage, rather incite, students to create algorithms themselves using some very fundamental concepts. The objective is that students should experience the tension and excitement of discovery even during the initial phases of understanding existing algorithms. In this paper, we provide a detailed study of a number of graph algorithms that have applications in diverse fields like chemistry, biology, mathematics, engineering, social sciences, and also computer science. We start with a simple algorithm consisting of just four lines of pseudo code. We shall show how this primitive procedure can be used to reinvent a number of existing powerful algorithms in graph theory. With some encouragement from the instructor, the students should develop a keen desire and ability to understand the motives behind, and the procedures followed in order to arrive at innovative solutions. They would learn the ways and means of devising their own algorithms.

Main Theme: Preparation of Teachers

Secondary Theme: Curricula Innovation

The Project "Mathematics History for All" on Teacher Education Program

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The project 'Mathematics History for All (MHA)' aims to develop subject matter knowledge and teaching methods that enable secondary and undergraduate level students to recognize mathematics as a human enterprise. The project, managed by Isoda, M. since 2000, engaged the fifteen participating pre-service or in-service teachers in the master's of education program in the integrated classes of the theory of mathematics education and the seminar of mathematics education. The aim of classes was to study mathematics education research in a practical context. In the theory class, students learned theories of mathematics education first by lecture, then read 'History in Mathematics Education' (Fauvel, J. and Maanen, Jan., 2000). In the seminar class, students (pre- or in-service teachers) read a number of source books on mathematics history with references to the original sources. They also read general articles to develop original subject matter for secondary school students and pre-service teachers using historical texts aimed at recognizing mathematics as a human enterprise. Following the study of the texts, the participants conducted a teaching experiment in which they gave students hermeneutic questions. After the teaching experiment, they analyzed the effects of their teaching experiment on qualitative changes in their classroom students and submitted their papers to the MHA project annual report. Concurrently, they developed their web page that include the files of developed materials for their practice and their research reports, files of used slides and the files of mathematics tools such as Dynamic Geometry Software (Cabri). Finally, they presented their research at the Japan Society of Mathematics Education Annual Meetings. In this paper, we will illustrate the effects of the project on teacher education, using the examples of subject matter knowledge the participants developed through the project, knowledge for the Lesson Study, and their comments how they changed.

The Study of Mathematics Communication on Internet with Palmtop Computer

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In order to develop the palmtop environment for mathematics communication on the Internet as the newest mediational means for mathematics and for analyzing how it works, this study developed and improved Bulletin Board Sites (BBS) based on the screen size of the palmtop computer. Through experimentations with the sites, difficulties coming from the size of the screen are clarified from the perspectives of grounding (Baker et al, 1999) and mediational means (Wertsch 1991). The differences in BBS-s designs strongly influenced the quality of communication. As the pilot study, experiments illustrated that we communicated mathematics well in the palmtop environment if we were accustomed to such an environment. From the study, the following are identified as factors that users must be accustomed to; First, users must be accustomed to ways of communication such as asking for better mathematical explanations, asking to check conditions, confirming what the other side is saying, and greetings. Second, users must be accustomed to ways of using the palmtop computer, such as how to use the BBS with the Internet and how to use tools on it such as Dynamic Geometry Software.

A simple non-trivial student model of a bouncing soccer ball

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When our students have completed a semester of ordinary differential equations, we give small groups a soccer ball and ask them to determine the simplest model describing one bounce of the ball from shoulder height. To start with they play with the ball. However, after a few weeks, they begin to ask penetrating questions and we lead them to a simple model that has a non-trivial twist to the solution. Most accurately a soccer ball should be viewed as a distributed parameter problem that is described by a PDE and which is usually solved using finite-element techniques. The first step in the process of simplifying the model is to reduce it to a (large) set of coupled ODEs. This, however, is still a computationally complex problem to solve. A further simplification would be to model the soccer ball by a single point mass concentrated at the centre of the ball, surrounded by a large number of spring-damper pairs. The next step is to restrict attention to vertical motion of the ball only which enables us to neglect all the spring-damper pairs except for the two pairs directly above and below the point mass. By replacing these two spring-damper pairs by a single spring-damper configuration the simplest possible model, a second order ODE, is obtained. The parameters of this model are the mass M , the spring constant K and the internal frictional coefficient B of the ball.

Understanding Epistemological Diversities in Mathematics Class

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Recently, the notion of community has been increasingly popular in theoretical discourse in mathematics education and became a basic unit for analysis of classroom interaction. In this context, the purpose of this paper is:

- (1) to examine the notion of “mathematics classroom as community” based on the findings from ethnography in mathematics classes in a university mathematics department in the US and
- (2) to identify some educational implications for teaching mathematics.

The data were collected through classroom observation and interviews. The analysis focused on comparing notions of mathematics shared among different groups of mathematicians, i.e., novices and old-timers. Through comparison, I found that there are not only differences but also similarities in their understanding of what mathematics is and that they are intricately related to one another to constitute a practice of mathematics as a whole. The analysis showed that novices were more likely to regard mathematics as a fixed body of skills independent of human beings, while old-timers thought mathematics as a body of perspectives produced through their own engagement; for old-timers, mathematics was a historical and cultural image of themselves. Despite the difference, both groups were concerned with commoditization of mathematics in certain ways. Most novices came to the mathematics class because of credits. Old-timers were under social pressure to be productive in their mathematical research. However, further analysis revealed that this similarity had different social roots and was resolved in different ways which were ultimately characterized in terms of kinds of communal norms about mathematics which each participant was committed to. It is concluded that the notion of mathematics class as community is very complex. The complexity suggests that it is not closed but interacting with outside communities. Each participant in mathematics class is representative of a community that s/he is committed to.

Making relevance relevant in mathematics teacher education

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One of the features of the reform in school mathematics is that school mathematics should be made relevant to the learners. The incorporation of mathematical modelling in school mathematics is one of the ways that is offered to realise the relevance ideal. Ostensibly the inclusion of mathematical modelling will provide school learners opportunities to develop mathematical power i.e. the ability to make sense of the world and of mathematics. A key question in this regard is: How prepared are practising mathematics teachers to incorporate mathematical modelling in their teaching? This preparedness entails that mathematics teachers be knowledgeable with mathematical modelling as content. In this paper this mathematical modelling content is elaborated upon and reports on a study which investigated secondary mathematics teachers' knowledge of mathematical modelling as content in South Africa and Eritrea. The one major finding of this study is that these teachers deem their experience with mathematical modelling as motivational and that they do find mathematical modelling problems dealing with social issues relevant. The second major finding is that in developing mathematical models for social issues teachers utilise very low levels of mathematics which is in essence against the intention of school mathematics reform. It is argued that this disjuncture—the engagement with low level mathematics and the personal expression and experiencing of the modelling as motivational and relevant—that requires attention in mathematics teacher education programmes aimed at assisting teachers to realise the relevance ideal in their teaching. These programmes, it is suggested, should not restrict the mathematical content knowledge to concepts, facts, procedures and proofs but it should also include mathematical modelling as content and at a minimum this would mean that teachers have to experience all the components of the mathematical modelling process.

INTEGRATING REAL MEDICAL STUDIES IN TEACHING BIostatISTICS: A HANDS-ON EXPERIENCE

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This paper describes an innovative way of teaching Biostatistics (or Biostat) at the undergraduate level. Statistics is a fundamental subject in all science or non-science courses. In particular, senior students taking up courses leading to the field of medicine are enrolled in the subject Biostat. However, there is not much difference between the ways Biostat and the fundamental statistics subjects are taught. Their contents (or curricula) are the same, only the case studies are made different. To make this difference strikingly clear to the students, they were asked to do Biostat with medical practitioners. Notably, students experienced the applications of statistics software package which is SPSS® and they encountered and learned statistical analysis tools which are not included in their Biostat curriculum such as: sensitivity and specificity analysis, relative risk estimate, and more. We summarize their studies and the proposed changes to the curriculum of Biostat that their collaborations with medical doctors brought about.

A technology-based approach to teaching mathematical modelling to non-mathematicians

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Many of those who study economics, business and management are not particularly mathematically-minded, considering math-related courses as a hard stuff because of the underlying mathematics that have to be known and successfully applied, frequently without a proper understanding. To help such students develop and utilize mathematical models skilfully and with confidence, a technology-based approach to teaching modelling should be applied since it, due to a built-in procedural knowledge, enables these students to concentrate on conceptual issues, without which modelling simply cannot work. Having in mind ISTE educational technology standards for students (<http://www.iste.org>), such an approach should utilize able software products as tools for communication, research, problem-solving and decision-making, and productivity. To achieve this utilization direction, a course on that informatico-mathematical modelling may be built around a skilful multitasking involving programs such as Internet Explorer, Word, Excel, MathCad and Power Point, requiring students to present their modelling activities in form of Web presentations generated by Word or slides-based presentations produced by Power Point. Such a course is presently being taught at Graduate School of Geoeconomics, Megatrend University of Applied Sciences (<http://www.megatrend-edu.net>), where students engage in modelling activities regarding tasks such as determining selling price, predicting population changes and finding the impact of gross national product on life expectancy. This paper describes the content and realization of this course and summarizes its main cognitive and affective outcomes.

Four critical issues of applying educational technology standards to professional development of mathematics teachers

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To have students adequately prepared for adult citizenship, computer-based technology is to be routinely used at schools and universities. To achieve this end, new approaches to teacher education are to be developed and utilized, which should be based on some suitable educational technology standards like those developed by ISTE (<http://www.iste.org>). Having in mind that computers are, in general, rarely used in mathematics classrooms, such an ET-based approach, enabling these standards to be eventually widely applied, requires the following critical issues to be kept in mind and dealt with in an adequate way.

(1) Many teachers, especially those less-experienced and not so technology-minded, may find 37 indicators of the ISTE standards quite demanding. A solution may be to base teaching practice primarily just upon several basic indicators, being aware of a larger context. So, what may be such indicators?

(2) It has been realized that computer attitudes influence not only the acceptance of computers, but also their use as professional tools or teaching/learning assistants. To have computers widely used in mathematics classrooms, we should help teachers develop positive attitudes towards computers. What may be a promising way to achieve this?

(3) What is the most appropriate software for the teaching/learning of mathematics? Secondary teachers may primarily base their classroom activities on a computer algebra system and a dynamic geometry environment. What should be a proper utilization direction of these or other able programs?

(4) Being aware of rapid developments in educational technology, how to achieve and maintain a critical, balanced and well-designed use of computers in mathematics education? Is Web-based professional development for mathematics teachers an adequate solution? What can be achieved by its use?

This paper examines these questions providing concrete answers that may be used in the design and utilization of an ET-based professional development of mathematics teachers.

What's new in math and in math/ed ?

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The recent period has seen new directions in mathematics at the level of scientific progresses ,results and also at the forefront of education .Examples of these two are respectively :

- strong openings in mathematics in direction of biology ,cognitive sciences ,computer sciences, closer connection with physical sciences and other sciences and technologies
- more "problem-oriented" and concrete application-oriented mathematical education; strong activities in media ,popular books, even theater !

See for example the type of educative material like "For all practical purposes " (Comap, Freeman). As an interesting link between the two new directions, more high-level mathematicians get involved in mathematical education (see recent activities,papers and interviews by H.Bass, M.Gromov, W.Thurston, and many others). What corresponds to deep important tendencies in societal developments and how can we explain these tendencies with the purpose of seeing the future ? How culture (here mathematics) relate with world systems and increase of "money-power" ? Can Universities cope with the real needs and how should they move, for example with respect to e-learning, distance-learning.. How scientific education connects with the major problem of the century :facing the enormous gap in the level of developments of countries in the world? Is it enough to say that mathematics is universal to open doors and create hopes? We will face these various questions, in the country of Euclid , but not only ,the country of Uncle Petros (world-best-seller of Doxiadis about Goldbach's conjecture) . We will rely on our various experiences :

- Writing in french magazines like "La quinzaine littéraire "or Cosinus (magazine for kids with strong mathematical part),
- Creation in 1992 and activities of Pro mathematica which help young russian students to invest in mathematics at home and not emigrate .
- Teaching teachers at Irem-Paris 7 :new activities in the curriculum like TPE,TIPE, interdisciplinary activities and new settings for college students.
- Istron international group on math-education.
- E-learning projects in Paris .

One of our main themes is that we need selection of experiments and strong permanent connections between "think-tanks "easy to gather, program committees , and hard down-to-earth local work ,with interactions in both directions.

Should we be optimistic ?

Should Euclid be proud of Uncle Petros, his distant nephew ?

The Role of the Teacher in a Technology Based Calculus Class

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In this paper I present some teaching techniques and discuss some issues that may be of interest to many technology based math courses. As a working example, I use the approach I take, at the University of South Carolina Aiken, in teaching a multivariable calculus course, based on Calculus and Mathematica (C&M) of Davis, Porta and Uhl. In particular, I discuss how the objectives of the course are set and why I favor C&M over the traditional approach, how I try to alleviate problems related to student weaknesses in computational skills, lack of interest or commitment, and lack of familiarity with Mathematica, and how I evaluate the success of the course. The method I use differs substantially from the method suggested by the authors of C&M. In the presentation I will discuss the following four basic elements of my approach.

(a) Introduction. Each chapter is introduced by a lecture that is close to the C&M approach, using both hand calculations and Mathematica code. The students also receive a handout containing a list of the new concepts and some brief explanations.

(b) Tutoring. The students are required to complete, at least part of each homework assignment, in class, usually in teams of two students. This allows the students to discuss problems among themselves and to ask for the instructor's help when needed.

(c) Feedback. The students are required to study the answers to the homework problems they miss.

(d) Constructive testing. The tests are used as diagnostic tools of student deficiencies. Students who fail a test are required to study further and retest.

This method is quite demanding on the instructor's time. It seems to be quite beneficial for the students, however, because it helps reduce their frustration, excite their interest, and improve their conceptual understanding of calculus.

The Mathematics of Psychology

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Computational psychology is the integration of computing, mathematical modeling, and visualization to solve problems in the behavioral sciences. Computational psychology offers an interdisciplinary approach to psychological research and provides an important tool, alongside theory and experimentation, in the development of knowledge. Beneath the popular conception of psychology is a foundation that is composed of mathematics and that is frequently expressed through computer visualization techniques. Unfortunately, few readily see the connections among these disciplines. Mathematics is used to express various psychological theories so that information can be systematically organized and tested. Mathematics permeates several sub-disciplines within psychology such as learning, intelligence, decision-making, neuronal development, and brain-imaging techniques, while computer visualization is an important tool for understanding complex mental processes. It is within this context that the future of psychology lays. This paper will explore the utility of mathematics to psychology, several applications of mathematics within psychology, and how mathematics and psychology, along with computer science will become even more interdependent in the future. Several examples of mathematical modeling and computer visualization within psychology and the behavioral sciences will be presented along with possible avenues for integrating these models into mathematics and computational science courses.

To write a program = to formulate accurately

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Mathematics required completely accurate formulation. The control of accuracy could, however, be difficult. The control by our own thinking could be unadequate because even when the formulation is not sufficiently correct, we know what we meant to say and forgive ourselves, the unaccuracy we committed. One possibility, how to prevent this is to control the formulations by a program. Thus will be delegate the checking of our expressions to a computational technology, which will carry it out accurately. In my contribution I will therefore introduce several simple programs and demonstrate how to use them for checking accurate formulations of some problems - in particular solutions of examples from secondary-school level Math. To demonstrate this idea I have chosen a graphing calculator (esp. TI83) which offers an easily managed, simple, easily understood programming language, which fulfills the requirements of structural programming. At the same time its' capability is sufficient to enable the solutions of practically all problems dealt with in secondary-school Math. The contribution will include examples of algorithms for simple tasks, as well as examples of more complicated problems, where the solution requires an accurate construction of algorithms for partial tasks. The issue of dealing with the verification of accurate formulation is an integral part of the subject "Computational Technology for Teachers of Math", which is included in the study program for students, prospective teachers of Math in the Faculty of Math. and Physics at the Charles University in Prague, Czech Rep. This subject was included in study program about 5 years ago as part of modernization of this program with respect to increasing use of Computational Technology in the teaching of Math at the secondary school. In addition the contribution also demonstrate non-standard application of the graphing calculators in teaching Math.

Dynamic Geometry Software not only for simple dragging

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Dragging is an important part of dynamic geometry software (DGS), but usually it is used only for the the seeking of intersection- or incidence-properties of geometrical figures. It is shown how especially the process of conjecturing can be improved by two additional features of DGS, which offer the pupil "escape routes" in for him familiar ranges of knowledges. So the risk of "not-finding" of conjectures can be essentially reduced. These additional features are 1. Connecting dragging with "dependence-graphs" and spreadsheets. Familiar ranges are functions and columns of numbers, useful strategies are the looking of invariant sums, products etc. This feature supports functional thinking. 2. Rearranging and dynamization of disassembling and assembling, which requires the breaking of geometrical relations. Familiar range of knowledge are the actions with pictures and a useful strategy is the matching of figures. This feature supports modular thinking. If one can explain the matching, one even gets visual proofs. Examples and short film sequences will demonstrate two features.

Interactive Visualization in Complex Analysis

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Analytic functions of a complex variable exhibit some of the most striking beauty found anywhere -- but in the ages of black-on-white printed textbooks, this facet has been largely inaccessible to all except a few. Needham's recent text "Visual Complex Analysis" clearly demonstrates the power of a visual language as an organizing principle and as a useful tool to develop promising strategies for analytic arguments. But modern technology allows one to go much further: We demonstrate JAVA applets whose ultimate interactivity that transform every learner into an experimenter and researcher! Selected examples include zooming into essential singularities, mappings of the complex plane, winding numbers, and convergence of Laurent series. We report how such tantalizing imagery transformed our own class, where amazing beauty led to inquiry, and a urgent sense of "I want to know how/why that works". We discuss and contrast CAS worksheets with model JAVA applets: On one side the user may modify and change everything -- but the algebraic-symbolic language of CAS worksheets usually requires a nontrivial "manual". On the other side, well-designed JAVA applets ideally require no instructions at all. Moreover, by using the "mouse" for input, and a graphic language for output, they take advantage of tactile, kinesthetic and visual pathways that arguably have been much underutilized in mathematics teaching in recent centuries.

Imaginative Deployment of Computer Algebra in the Undergraduate Mathematics Curriculum

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Much of the author's recent experience is attempting to teach Mathematics primarily to undergraduate students following degree programmes in Electronics or Audio Technology. Increasingly, it is found that although such students may be able to perform mechanistic steps such as obtaining a simple derivative, or evaluating a straightforward definite integral, they have little idea as to what these quantities mean. Very few (if any?) would know that these results are connected to a limiting process. Unless the student's understanding of basic calculus is strengthened, they have little chance of subsequently dealing with the solution of differential equations or the construction of Fourier series. This paper shows how imaginative deployment of computer algebra (DERIVE) can substantially assist the understanding of calculus and its applications in the aforementioned areas. In particular, the paper will demonstrate the advantages of using computer algebra as an on-line teaching aid in the classroom compared with using traditional methods of teaching topics such as solving differential equations.

TOWARDS A 'NON EUCLIDEAN WORLD' FOR THE STUDENTS

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The axiomatized geometry that has been taught to greek students at high school (a modern version of Euclid's Elements) is their only contact with geometry as the science of space, and also with the notion of axiomatic system. Thus a deeper comprehension of this double nature of school geometry seems almost impossible for the students. During the last years in Greece a reference to a noneuclidean geometries has also been introduced in geometry textbooks under the form of historical remark. This remark seems to create more confusion and usually is faced with mistrust. In this way students begin their university studies without understanding the axiomatic system concept. On the other hand Euclidean Geometry, represents for them the absolute geometry of space, in Bolyai's words. Compulsory Geometry courses in Greece are usually restricted to Analytic and classical Differential geometry, while the optional courses differ from department to department both in content and extension. Therefore the proposal of a geometry course seems reasonable which first treats the arguments on Euclid's fifth axiom and efforts to prove it, then continues with the discovery of Noneuclidean Geometries and ends with the axiomatic basis of Elliptic or Hyperbolic geometry and the systematic study of their models. Such a course until now seems hardly to have called the attention of curriculum makers of the mathematics departments, at least in Greece. It will be better that such a course follows a Differential geometry course, because in that way the students will have been taught some surface theory and will be familiar with the idea of Gaussian curvature. Moreover, Elliptic geometry will be a better choice for a natural passage from Euclidean to non Euclidean geometry, as the surface of the earth will be a familiar model to students. Besides, this model has been already successfully used in a teaching experience called "Mecca" at the Freudenthal Institute for students of secondary education.

Increasing retention of underrepresented students through cooperative learning workshops: looking back and looking forward

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This talk focuses on a workshop class model with the aim of recruitment and retention of underrepresented students. In the United States, in fields such as science and mathematics, this refers to increasing the population of female, African-American, Latino, and Native American students. We consider workshops first designed by Uri Treisman at the University of California, Berkeley, called the "Professional Development Program" and modeled elsewhere as "Emerging Scholars Programs." Today these programs are found not only in mathematics but also in other fields such as physics and biology. As an introduction, we briefly describe the research that motivated the original calculus workshops. Then we look at how these workshops run at Berkeley: how students are recruited, what the classes consist of, and how students who have taken courses in the program are involved as assistants and mentors in following years. Since calculus is a requirement for so many other disciplines, the original workshops and Emerging Scholars Programs focused primarily on calculus. However, we also discuss subsequent work done with upper division math classes, to support students later in their university careers. Time permitting, we look at the presenter's experience with courses and enrichment programs taught elsewhere, using ideas from the Berkeley model. Finally, we touch on the impact of political change in California on programs such as Berkeley's. Throughout, the emphasis will be on concrete examples. We also present anecdotal evidence for the strengthened relationship between teacher and student such workshops provide and the positive effect this has on the retention of underrepresented students.

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Main Theme: Others

Secondary Theme:

Application of Landau theorem on estimates of derivatives to definition of Schwartz space

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We give an elementary proof for the equivalence of the original definition of rapidly decreasing functions belonging to the Schwartz space and our better characterization for the Schwartz space \mathcal{S} . The new proof is based on the elementary Landau inequality concerning the estimates of derivatives.

Understanding Secondary Mathematics: A Role for Manipulatives in the Mathematical Preparation of Secondary Teachers

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This study describes the gains in mathematical understanding that secondary teacher candidates self-report from their experience of modeling mathematics with manipulatives. Theoretical Framework. A performance conception of understanding undergirds the study based on a reconstruction of Skemp (1978) using a levels of understanding rubric (Perkins and Simmons, 1988). Following Kinach (2002), the levels of understanding rubric identifies five types of subject-matter knowledge common to all disciplines: information, conceptual, problem solving, epistemological, and inquiry. Data Analysis. The present qualitative study explores emerging mathematical understanding in the narrative journal reflections of 33 college students following their completion of a Secondary Mathematics Teaching course. The research questions are: 1. What topic-specific mathematical understandings do mathematics majors in the secondary teacher education program self-report in this reflection format? 2. Are there knowledge-type generalizations in the self reports of learning? There were 165 instances of self-reported growth in mathematical understanding from learning to teach with manipulatives. Using a qualitative design, data was first coded by type of manipulative reported, and sub-coded for types (or levels) of mathematical understanding gained. Results. It is often suggested that manipulatives are useful only in primary mathematics teaching. This study challenges that notion, showing that manipulatives offer even undergraduate mathematics majors insights into secondary mathematics. Preliminary findings identify attribute blocks, algebra tiles, and the geosphere as promising manipulatives for modeling secondary mathematics topics to deepen prospective mathematics teachers' understanding of secondary mathematics. This research recommends advanced studies of secondary mathematics (Ma 1999), an emphasis on representational thinking, and the need for a theory of referents (Steinbring 1998) in the preparation of secondary mathematics teachers.

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The experiential use of personal technology

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Technology can serve a number of important roles in mathematics education. In this paper, we distinguish computational roles, influential roles and experiential roles, with a focus on the relevance of these for graphics calculators in particular. In a computational role, the main emphasis is on producing an answer to an applied mathematical question. An influential role encourages some reconsideration of the balance of the curriculum, suggesting possible inclusions, exclusions or changes in emphasis consistent with student access to technology. The experiential role is argued to be of most importance and is concerned with the provision of learning experiences for students, especially those that are difficult to provide without access to personal technology. Examples of promising uses of technology for experiential purposes are described in the paper, drawn from a number of aspects of the early undergraduate curriculum. These include elementary probability and data analysis, the study of functions, equation solving, iterative methods, differential calculus and limiting processes. An analysis of published student text material suggests that greater emphasis on experiential uses of graphics calculators is desirable.

A Project in Euclidean Geometry

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One of the most effective instructional approaches in teaching Mathematics is project work, which, in an earlier paper, I connected with active learning, see Klaoudatos (1998). And this approach is going to be more interesting for the students, if the project has been developed in collaboration with them. At the same time, these kinds of projects include 'dangers' for teacher because of unexpected demands that might be found within. In this presentation, I will describe such a project, which had been created in a problem solving class during the year 2000, first semester. Through successive generalizations of a simple geometric task, the students developed the following problem: 'In an ABC triangle, D is a point on BC from which we construct segments that form equal angles at the sides AB, AC, at the points I, K respectively. Which is the position of D so that the length of IK will be minimum?'. The problem is expressed in terms of classical Euclidean geometry so that, at first glance, there is no evidence of the hidden difficulties. The presentation will consist of two parts. In the first, I will describe the situation context, Wedege (1999), in which the project took place. The second part will be the solution of the problem and the role that the computer had in the development of the solution.

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Linear Transformations and Eigenvectors with Cabri II Via Maple V.

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Teaching Linear Algebra to the beginners raises many cognitive problems related to the three thinking modes intertwined: geometric, computational (with matrices) and algebraic (Symbolic). These difficulties were first described by the authors: Defence, T. Dreyfus, J. Hillel, A. Sierpiska, & S. Khatcherian. In this presentation, we first study linear transformations in R^2 and R^3 and their eigenvectors in the Maple V environment. Here the mode is only symbolic and computational. To bring the geometric mode, students can be shown Maple animations. However during such animations Maple takes the role of a ·moviemaker· and prevents students from participating as actors. Then we use the Cabri microworld where Maple animations can be rendered with the two Cabri's functions ·Locus· and ·Animation·. In this micro-world, students can produce easily their own ·movies·, change transformations, vectors and run their own explorations. We claim that students performing with Cabri will enhance their geometric and as well conceptual understanding of Linear Algebra and also Cabri enables to link the three thinking modes.

A numerical investigation of the Gibbs' Phenomenon for series of Orthogonal Polynomials.

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Three classes of orthogonal polynomials, Fourier-Bessel, Legendre and Hermite- are investigated and the Gibbs' phenomenon at a jump discontinuity for the corresponding orthogonal polynomial series expansions is investigated numerically using Mathematica. Dealing with specific examples, the unexpected observation is that the Gibbs' constant that arises for each class of the above polynomials appears to be the same as that for the Fourier series. Although a numerical investigation is not absolutely conclusive the findings suggest further investigation suitable for undergraduate research projects or small group investigations.

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Main Theme: Technology

Secondary Theme: Distance Learning

Teaching Calculus with Digital Libraries

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A few years ago I was shown some Java applets that were of sufficient educational interest to cause me to begin a modest search for material I could use in my elementary calculus class. The search was frustrating—a lot of widely scattered material and much more chaff than wheat, but some of the good material showed real promise. This led, with a lot of help from my friends, to a successful proposal to the National Science Foundation's Digital Libraries Initiative for the Math Forum along with the Mathematical Association of America to create the Journal of Online Mathematics and its Applications, <http://joma.org>. Part of the goal of JOMA is to search out and peer review mathlets, applets and other interactive web-based teaching tools for mathematics. The MAA and Math Forum soon received a further grant to create the MathDL site, <http://mathdl.org>, for which JOMA is the cornerstone. In this talk I'll examine the advantages to using digital libraries for finding calculus resources and how this material can be effectively used for teaching calculus. In addition to JOMA, a number of other digital libraries now contain mathlets and more extensive teaching material. These include Merlot and iLumina. Such libraries will be surveyed, they will be contrasted with some other sites devoted to teaching calculus, and a crystal ball will be consulted to foretell the future. I have taught calculus using these resources and will be doing so this fall. My talk will update this rapidly changing area through June, 2002. Gene Klotz The Math Forum, <http://mathforum.org>

Using Counter Examples to Enhance Students' Conceptual Understanding in Engineering Undergraduate Mathematics: A Parallel Study

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This paper addresses a practical issue encountered by many lecturers teaching first-year university engineering mathematics. A big proportion of students seems to be able to find correct solutions to test and exam questions using familiar steps and procedures. Yet they lack deep conceptual understanding of the underlying theorems and sometimes have misconceptions. In order to eliminate misconceptions and for deeper understanding of the concepts involved, the students were given the incorrect mathematical statements and were asked to construct counter examples to prove that the statements were wrong. They had enough knowledge to do that. However, for most of the students that kind of activity was very challenging and even created psychological discomfort and conflict for a number of reasons. In this study, practice was selected as the basis for the research framework and, it was decided 'to follow conventional wisdom as understood by the people who are stakeholders in the practice' (Zevenbergen R, Begg A, 1999). The theoretical framework was based on Piaget's notion of cognitive conflict (Piaget, 1985). 127 students from two universities were questioned regarding their attitudes towards the method of using counter examples for eliminating misconceptions and deeper conceptual understanding. The vast majority of the students (96% in the German group and 84% in the New Zealand group) reported that the method was very effective. Many of the students made positive comments that using counter examples helped them to eliminate misconceptions, prevent mistakes in future, understand concepts better, and develop logical and critical thinking.

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USE OF TECHNOLOGY IN PRE-SERVICE TEACHER TRAINING

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Use of technology in pre-service teacher training, for preparing students to become mathematics teacher, has different aspects: - Using technology as a support for pre-service teacher training (doing the mathematics the future teachers need with the help of computers or hand held technology)- Showing the examples of using technology in the classroom- Teaching the future teachers how to implement the use of technology for teaching (to ensure a reasonable pedagogical use of technology). Preparing the future teacher for the didactically appropriate use of technology in their teaching has to be one of the main issues in today's mathematics and science teacher training. Theory and practice of this item will be discussed in this session. We give some examples for - Technology supporting the problem solving and- Technology supporting the introduction of a concept and discuss the respective teaching goals and pedagogical competencies the teacher should have.

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Main Theme: Education Research

Secondary Theme: Technology

Mathematical education. Relationship with an industrial activity

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USSR had a high mathematical standard on all levels of education. In the process of reforms (90-s) has occurred a reduction of industrial activity (about 50% down). This influenced the level of the mathematical education in the high school. The influence at the different educating levels was different. The Influence also depended on the type the universities (high-level, middle-level universities). The influence occurred both through teachers, and through students. Detailed data and their analysis will be presented in the report.

Electronic access to Literature in Mathematics Education

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There has been a substantial increase in publications dealing with results of research in mathematical education in general and in particular on experiments in various countries, new pedagogical concepts and insights, topics, and innovative ways of teaching. One of the features of the growth is the increasing number of conference proceedings (this new conference is an example) being published. The penetration of calculators and computers in education led to the creation of whole new areas of research. Another aspect is the expansion of journals in this field in both number and page count. Journals are of great importance for everyone interested in national developments as well as for an international exchange of ideas. About 400 journals on mathematics education and /or computer science education serve worldwide as channels for scientific communication. (see an overview in <http://www.fiz-karlsruhe.de/fiz/publications/zdm/zdmzs.html>) Educational professionals like other scientists are thus faced with the problem of how to extract from a vast pool of potential information those items which they need for their own work. The purpose of this paper is to provide an insight into how to cope with this flood of information. The reader is given some information on the international services which may help him keep up to date with the current progress in elementary mathematics and mathematical education: abstracting journals and on-line databases. The abstracting service Zentralblatt für Didaktik der Mathematik (ZDM) and its bibliographic mathematics education database MATHDI (MATHematics Didactics) possibly complemented by other ones which are presented in this paper, enable specialists in mathematics education to keep up with the literature in their subject by providing them with a manageable source of information on current developments, controversies and advances, selected from virtually the whole of the international literature. In addition ZDM/MATHDI assist in maximizing the use of the time scholars have available for reading. They spend their available reading time scanning core journals and can then use abstracting services covering their field to identify other papers. In addition online databases can be scanned to highlight trends in research. Mathematics and mathematics education, like other subjects, suffer fashions and a given topic may be an active research area for a time and may then be neglected temporarily. Some of those topics are now "The Impact of Computer or Calculator Technology on Mathematics Education" "Statistics Education", "Distance Learning Technology". By identifying the annual total of articles published in the past five years one can see an increasing interest in these general subjects. The search and retrieval in MATHDI is exemplified by identifying literature proving such trends in mathematical education and mathematics education research.

Successful Interdisciplinary Teaching: Making One Plus One Equal One

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In the last decade, the call for interdisciplinary learning has risen from a suggestion to an exhortation. Integrating mathematics and the sciences has never been problematic (as one student told me, "Physics is math."), but linking mathematics to epistemologically more distant disciplines has a spotty history. Interdisciplinary undergraduate mathematics courses are often considered "fluff," a sop to students unwilling to do the hard work of learning calculus. This presentation uses extensive quantitative and qualitative evaluation data from ten new mathematics and humanities courses at Dartmouth College to argue that this kind of interdisciplinary learning has pedagogical value for students and instructors. Linking literature, art, and music with topics like elementary group theory, number theory and non-Euclidean geometry offers students who lack a practical or intellectual interest in calculus a more congenial option for continuing their mathematics learning. For previously unsuccessful math students, they offer a fresh start with novel pedagogy and material. For enthusiastic mathematicians, they provide a new perspective on a favorite subject. For all, they expose a breadth and richness of mathematics usually withheld from introductory students and thereby sustain their interest in mathematics. Interdisciplinary courses are not easy to teach. The principal challenges are presenting non-trivial math to a diverse audience and creating a genuinely interdisciplinary approach. Comparing student outcomes with faculty interviews and classroom observations, we suggest strategies for designing and teaching such courses. Successful course developers began at the point of interdisciplinary intersection and sought relevant material (rather than seeking connections among familiar topics), acquired student-level competence in the other discipline, and made patent (thereby surmounting) their imbedded discipline-specific linguistic, epistemological-and consequently pedagogical-differences. This takes more time than conventional teaching, but faculty found that the benefits to their students, the collegiality, and their own intellectual and pedagogical growth well repaid the effort.

Teaching vectors with technology

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We want to present to other teachers our experience using the technology by teaching vectors to secondary school students. In our presentation we will introduce materials that cover teaching of: - vector calculus (multiplication by a scalar, addition) and their properties by Cabri- some applications (the median centre, Menelaus' Theorem, ...) using Cabri II- vectors in 3 dimensions (straight line, plane) and their visualization using Derive 5- some tasks where utility files in Derive 5 can be applied - other technology means (Internet, Java, ...) in teaching this subject. We will focus how this approach corresponds to the ultimate goal of teaching - understanding the theme not just following the recipes.

TEACHING PERIODIC DECIMALS IN TERTIARY EDUCATION

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Periodic decimals (pd) constitute a fundamental representation of rational numbers. Additionally, their study leads to the understanding of basic characteristics of the decimal system and to a more profound comprehension of the operation of division. Furthermore, the a-priori analysis of the properties of the pd indicates that this area is adequate for learning activities, in which students may work by combining the use of inductive and deductive methods. This claim is supported by the historical development of Number Theory. However, only a very limited part of the Mathematics curriculum in elementary and secondary education is devoted to this area. Often, this is also true for the pre-service undergraduate studies of (elementary and secondary) schoolteachers of Mathematics in Mathematics and Education Departments. In the first part of this work, we present results of an empirical study of the knowledge, which (elementary and secondary) schoolteachers of Mathematics and students of Departments of Education in Greece have on pd. Our results point out that the properties of pd are largely unknown both to the teachers and to the students who have been asked. For example, 96% of 207 students of Departments of Education don't know that in all non-terminating divisions, the quotient is a pd. In the second part of this work, we give an outline of a teaching approach concerning pd, which has been applied in the Department of Education of the University of Crete. In the context of this teaching approach, students become able to discover the basic properties of pd and to prove many of them, by combining the use of inductive and deductive methods. Apart from a significant improvement of the students' knowledge concerning the pd and the decimal system, we have also observed an important improvement of their understanding of inductive methods and of the fruitfulness to combine such methods with deductive methods, in order to study problems in Arithmetic and Number Theory.

Undergraduate students' projects with special needs pupils

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Undergraduate students' projects with special needs pupils Abstract of the paper The paper will concentrate on the topic of undergraduate students' projects that were elaborated within the course of didactics of mathematics as a part of full time study for future special needs teachers. Most of the students do not like mathematics because they do not understand it. A common reason for these students to study at the university comes from their need to help special needs children or people. To achieve this goal means that they also have to help children to understand mathematics which means that they must improve their own mathematical understanding during their course at university. Most of them attending the course have already worked with special needs children. A project was set for them which involved doing experiments with special needs pupils. The students had to describe their experiments and then to analyse these together with any work produced to determine the pupil's thinking processes in solving a mathematical problem. A tutor has subsequently analysed the student's analysis of their experimental work, concentrating on the following aspects: how did the students work with the pupils?; how confident are the students to analyse their pupils' work?; how can the student's attitude towards mathematics and understanding of mathematics be improved by their analysing of pupil's work? One project will be analysed from these points of view. The mathematics, which these students are asked to do in school, is not conventional classroom mathematics but non-traditional problems and environments, e.g. making buildings from dices and then the children having to sum the visible dots on them, some tasks involve tetraminoes or pentaminoes, orientation in the plane, making buildings from cubes and recording their characteristics, addition triangles, triads, patterns, combinatorial problems.

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Main Theme: Mathematics And Other Disciplines

Secondary Theme: Mathematics And Other Disciplines

A Genetic Approach to Axiomatics

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The genetic method is often regarded as a counter-current to the New Math and its exaggeration of formal and axiomatic mathematics. As a consequence axiomatics has been considerably reduced (nearly deleted) in school mathematics, whereas university mathematics is mostly still presented in a rigid deductive way. This discrepancy leads many freshmen to lots of difficulties as we all know. In this lecture I will propose a synthesis between genetic and axiomatic method. In particular the axiomatic method is not only a method but also an interesting and very important subject of teaching and research itself: a milestone in the development of mathematics (Euclid), its philosophical background (Aristoteles), its purpose (Zenon), its consequences (construction with compass and ruler). Axiomatics as a model for representation of topics of mathematics (and other sciences) up to now, axiomatics as a destination of a process, not a starting point. Examples (mainly from the history of mathematics) how to cope with axiomatics at school and at university will be discussed in this lecture.

**USING INTELLIGENT ALGORITHMS TO GUIDE A BEST
SOLUTION EXPLANATION MODEL FOR AN INTELLIGENT
TUTORING SYSTEM IN ALGEBRA MANIPULATIONS**

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Over the past twenty years, there has been a range of learning techniques developed for intelligent tutoring systems to improve the student's learning process. These include error explanations and learning strategy instantiation. The aim of these techniques is to correct student misunderstandings. However, the student may answer the question correctly in inefficient and complex ways. In this paper, we present a new 'optimal solution' based feedback model which is centered around our model-tracing reasoning mechanism. The latter has been proposed to provide our intelligent algebra tutoring system with the best feedback generation to improve the student's algebra manipulation skills. The paper will start by a brief description of the architecture of our intelligent algebra tutoring system. This will be followed by a detailed presentation of the organisation of the student model component, which will be illustrated by an evaluative case study. Finally, the paper will draw some general conclusions and present a description of some further work.

Misconceptions in learning Linear Algebra

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The purpose of the research, reported in this paper, was to identify misconceptions in the study of Linear Algebra (LA). The main assumption of the research is that, generally speaking, mistakes are not random, but are results of misconceptions. We studied the following questions:

1. What are the difficulties that students have in understanding the concepts and in solving problems in LA?
2. What are the sources of mistakes that students of LA make and can these mistakes be categorized?
3. What are the misconceptions in learning LA?
4. How can the difficulties, mistakes and misconceptions be treated by a suitable learning process?

The research was performed in two stages. In the first stage we went over 470 midterm and final tests of LA students in the Technion-Israel Institute of Technology. Based on these tests we identified 11 categories mistakes and 5 main sources of difficulties. In the second stage 16 students, 10 from the Technion and 6 from the University of Haifa, went through a sequence of 15 clinical interviews. This was followed by theoretical analysis and a discussion of possible treatment.

The Central Limit Theorem in Teacher Training - a Possibility

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The central limit theorem plays a decisive role in probability theory. Therefore the math-teacher-student should get acquainted to it. However it is almost impossible to give a formal proof. Therefore we propose a 'convince instead of proof'-strategy. For this purpose a vast amount of digital data is at our disposal by means of the internet. The student can encode these data by very simple programs arbitrarily into number sequences. Then a few lines of code will yield standardized partial sums and sequence graphs of the transformed data will demonstrate the convergence towards the normal distribution.

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Main Theme: Innovative Teaching

Secondary Theme: Education Research

The New Idea of Math Teaching in Poland

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The present project is about an innovative idea of teaching methods and educational research. It shows my conclusions based on researches carried out in math-informatics classes in polish graduate school. This concept is "How to teach mathematics with computer technology", especially with programs like CABRI and MuPAD. This work is a base to elaborate a concept on mathematical education after reform in a polish graduate school in a next school year 2001/2002.

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Main Theme: Technology

Secondary Theme: Preparation of Teachers

Intra- and inter-personal variations of teaching Mathematics with technology in pre-service teacher education

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At undergraduate level, individual differences in pre-service teacher education are often ignored. Based on one IT teacher training course at CITE, this paper endeavors to trace out some secondary Mathematics teachers' inter- and intra-pair (or group) learning differences in Hong Kong. The object of their learning was on how to integrate various sorts of technology (like data-logging devices, dynamic geometry software, graphing calculators and spreadsheets) into daytime lessons through phenomenographic analysis. Meantime, the multi-faceted roles of IT mentors were traced out after step-wise implementation with both qualitative and quantitative evaluations. By linking with some theoretical accounts of collaborative knowledge building cyber discourse, didactical power relationships between teachers and the tutors were portrayed. Lastly, a conceptual model for addressing key components in IT undergraduate teacher education program was generated.

Aim - A Parable in Dissemination

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AIM is an initiative in interactive mathematics which exploits the power of the computer algebra package Maple V in an extremely flexible way that can be applied to a variety of curricula. The authors present the results of a project concerned with replacing parts of core first year materials at the University of Birmingham, UK.

Most software for computer-based assessment has limited use in mathematics. Common problems are:

- Poor display of mathematical expressions. (despite MathML and plug-ins like IBM TechExplorer)

- Restricted choice of question types,

- Failure to recognise mathematically equivalent solutions,

- Difficulty of assigning partial credit,

- Inability to test students' creativity (eg give an example of a function which satisfies XXX but does not satisfy YYY)

Effective integration of computer algebra has made it possible to address these issues. The ability to monitor students' progress in more detail has allowed us to provide individual students with tailored advice on suitable additional learning opportunities (e.g. the use of appropriate learning packages) and to efficiently mount support activities (e.g. targeted small group sessions). This has enhanced and made more focussed support for our students. Pilot studies have been very encouraging. Students find the software easy to use (97% agree/strongly agree), like the immediate feedback (100% agree/strongly agree), and find it helpful (87% agree/strongly agree). Our paper:

- outlines the genesis and nature of AIM,

- reports and elaborates on the above results,

- offers an indication of the range of applicability of this shareware - from widening participation to honing advanced specialist skills.

Of particular interest is a parallel study which explores the factors which determine whether an innovation is likely to be easily transferable. We look to distil principles of value to innovators in the learning and teaching of mathematics.

Pre-service and in-service teacher of Mathematics training in teaching with the use of Computers

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In this paper we propose a program of pre-service and in-service teacher of Mathematics training in teaching with the use of computer software programs. The program consists of a) the presentation of the most characteristic theories of learning, teaching methods and models and models of using computers in the teaching-learning environment and b) the training of teachers in the use of computer software programs that are being chosen as appropriate to offer more than the traditional instruction. Moreover we present the results of the application of the program in the Mathematics Department of the University of Athens during the academic years 1999-2000 and 2000-2001. The computer software program used in the application is Mathematica® in point of the possibilities it offers in the negotiation of mathematical subjects in Secondary Mathematics Education. This paper studies the evaluation of the use of the program by the pre-service and in-service teachers that participated in the research in point of the aims, the operation and the use of the program in the teaching-learning process of Secondary Mathematics Education. Moreover it studies the change in the aspect of in-service teachers in point of the use of computer software programs in their lesson and the teaching approaches they propose as more effective for the teaching of specific mathematical subjects.

Using Computer Spreadsheets to Teach Undergraduates Numerical Modeling Techniques Applied to Environmental Geoscience Problems

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The quantitative nature of the Earth Sciences and Environmental Sciences continues to become increasingly important in addressing current issues facing scientists and society. Spreadsheets offer a powerful tool to teach students the many quantitative relations found throughout the Geosciences. Their advantages are numerous including their ubiquitous nature in many institutions of higher learning and student's prior exposure to their functionality in the pre-college environment. These attributes ease the implementation of performing exercises focusing more on the numerical techniques of importance. Secondly, they provide instantaneous graphical feedback to assist students in the visualization of the results of numerical calculations. Thirdly, most computer spreadsheets provide powerful intrinsic functions or iterative capabilities to allow for the solution of partial differential equations and many other complex mathematical relations. It is this functionality that has been employed in the creation of educational modules within the Computational Geology and Environmental Sciences. We have developed a series of exercises related to the solution of the Laplace partial differential equation using a finite difference technique to model groundwater flow within various geologic settings. Spreadsheets allow students to explore the importance of geologic boundary conditions, parameters, and initial conditions in the simulation of the groundwater and contaminant movement in a local aquifer system. These active learning projects allow the students to understand the importance of the mathematics in the application to a complex Geoscience problem and how spreadsheets can be used as an advanced learning tool in the math and sciences.

**Calculation of areas:
discussion of a mathematical-historical
problem that exposes the students' conceptions of Proofs**

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Already in the eighties researches have shown that many students do not quite understand the essence and significance of mathematical proof, although they are generally capable of performing the ceremony of proof. This concept fits Sir Arthur Eddington's concept, that as far as they are concerned Proof is the idol before whom the pure mathematician tortures himself. In a study conducted by Fishbein & Kedem (1992) students received the proof of a mathematical statement and were then asked to state whether further concrete examples are required in order to establish its truth. Their study main finding showed that although most students claimed that they had understood the presented proof, they felt that they should examine further examples in order to consider whether it is true or not. This study is another attempt to check students' concepts about the essence and significance of mathematical proofs. This inquiry was performed in an environment of a mathematical-historical discussion. The StudyWithin the framework of the course which deals with the development of mathematics, the students - elementary school preservice mathematics teachers - were exposed to some problems taken from the Egyptian mathematics. In a two-hours meeting they were shown a way to calculate the area of a quadrilateral as it appears in the Rhind Papyrus (Eves, 1982, p. 14) . The analysis of the students' responses will constitute the basis of this presentation. The Egyptian algorithm for finding the area of the general quadrilateral is to take the product of the arithmetic means of the opposite sides.

On the Shift of "Archetypa - Pattern" in Geometry

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It is widely accepted that the process of demonstration firstly appears as a dominant intellectual activity in connection with geometry and that the fundamental results which analyzed and presented under this new line are directly connected with the names of Theles and Pythagoras. By the requirements of logical deduction, these first results were incorporated in what today is referred as Plane Euclidean Geometry, in an anti-empirical and anti-visual manner. It is remarkable that, in this setting all these results were not strictly independent to each other. Furthermore, their nature and range easily overcome the frame of Euclidean Geometry. Indeed, there is a tracking diffusion of them in many branches of today mathematics. Apart of the historical and epistemological points of view, it seems interesting to examine results like The Pythagorean Theorem and thales theory of similarity and proportion, in the cognitive term of "prototypes", in the sense of tall and others. In this respect, we propose a scheme of their incorporation as abstract mathematical results in Geometry as follows: how an archetypical pattern permits an early geometrical understanding, achieves its standard geometric form and finally shifts through the development and reformulations of the new theories. In the sequel, we discuss in detail some certain mathematical examples directly related to these central themes which clarify the above proposed scheme and indicate that, a science relies upon Deduction in a stage of its progress, assimilates results, invents tools and sets prerequisites that overcome the pure logic.

INFORMATION-ACTIVITIES POINT OF VIEW AS THE POSSIBLE BASIS OF HIGHER MATHEMATICS

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The course of higher mathematics in the higher educational establishment must be aimed at these functions:

- 1) the formation of mathematical knowledge and skills to apply it;
- 2) the formation of methodology of solving of the applied problems.

The first and the second functions can be carried out in general on the basis of certain point of view and the leading role of the development of student's mentality. For the effective teaching of higher mathematics we must take into consideration the double nature of mentality, that may be formed as search of values and personal sense of life, and as information process, which is determined such as the perception, keeping and remarking of information. In the abstract meaning the remarking of information by all means is regulation and compression of it to determinate aim or the given problem, by definite language or code. For professional activity this is the selection of information from the "noise", lowering the entropy according with the interests, orientations and possibilities of the specialist's personality. The factor of integration of the variety of different given training of methods and application for development of thinking of the future specialist concerning its double nature, can be information-activity point of view to professional training and development of the student's personality. Its main idea is the organization of the educational- professional activity, in which the compression of the training information takes place by regulating it, by imitation of professional activity according to the subject, the nature of motives, knowledge and actions of their application.

How to find the internal angle of a regular polygon: Strategies of pre-service teachers

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The task of finding regular polygon internal angle can be explored by students from middle school to college and beyond. This task can be investigated in many different ways from which it is possible to learn more about various properties of basic geometrical shapes such as triangles, quadrangles, and regular polygons. The study presented here is aimed at characterizing strategies of pre-service teachers that were asked to find the internal angle of a given regular polygon. Forty two pre-service mathematics teachers attended a two-hour workshop that focused on regular polygons. Each participant was asked to find the internal angle of a regular pentagon and then to generalize it to an n -side regular polygon. At the first part of the workshop participants worked individually, but then convened for a full class discussion followed by further investigation of the task in a computerized environment. The computerized environment investigation was based on the results of the individual work, and additional strategies that were suggested during the discussion. During the individual work, eight different solving strategies were used for the case of the regular pentagon, but only three of these approaches were used to generalize to the n -side case. The classroom discussion yielded additional solving strategies, most of these were generalized into the n -side case. In the paper we describe the different solving strategies and discuss the contribution of the whole class discussion to the learning process. Since the different solving strategies all shared some common features, we suggest that this task could promote mathematical generalization as well as abstraction.

Name, Arts, Mathematics, and Technology

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For some years the author has been experimenting with an activity based on designs formed by her students' names. In order to form the design, we transform the student's name into points on the coordinate plane, then we connect points to form a closed polygon, then we subject this basic polygon to three 90-degree rotations. The result is an individualized, frequently interesting and complex polygonal design. Students can then, by coloring in regions, obtain interesting and often quite beautiful designs. One can then ask mathematical questions about these figures, how many pairs of parallel or perpendicular lines there are, and what are the areas of the various regions, and so on. Students seem to enjoy this activity, and the fact that the activity automatically yields individualized projects seems to enhance the students' interest. When the lines involved happen to pass through points of the graph paper grid, it is comparatively easy to find their slopes and thus determine properties such as being perpendicular. Areas of squares and other quadrilaterals can be computed readily if their vertices lie on grid points. There will be some quadrilaterals whose vertices do not lie on grid points and we must learn how to solve pairs of linear equations to find these vertices. There may be many quadrilateral figures in a given design, and so we encourage the use of calculators to keep the computational labor from being excessive. We find this project has been helpful to students about to enter calculus, because it affords an amusing and motivated review of the important pre-calculus notions. It also is good for prospective teachers because it gives a way to vertically integrate parts of the curriculum. The author has been working with this pedagogical device for several years and new ideas still seem to be coming up.

Exams and Computer Algebra Systems

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The aim of this presentation is to tackle the following question : how to assess whether a student is able to use a Computer Algebra System (Maple in my presentation) as a tool for doing Mathematics? In the French system of CPGE (undergraduate level), CAS (Computer Algebra System) is used as a mathematical aid. The ability of students to use CAS as a tool in a real mathematical activity is relatively easy to test if you are tutoring them in the context of their research projects. It is not the same in exams. On these occasions, the question is chosen by the examiner and the examinee has just an hour to tackle it. In our presentation, using examples from French exams in Maple, we will endeavour to show the various pitfalls to avoid and how an examiner can become able to assess the ability of students to use CAS as a mathematical tool.

Teachers estimate the arithmetic skills of their students when they enter the First Grade of Primary School

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Nowadays, in the area of contemporary teaching, the role of the teacher in the class has changed. Teaching is based on students and a great emphasis is given to the communication between teachers and students; the preliminary knowledge of children is taken under serious consideration for the establishment of a new knowledge. Teachers have to know this pre-established knowledge and estimate the abilities of their students, so that they will be able to organize their teaching according to this knowledge. In the present paper we investigated the predictions and estimations of Greek teachers about the arithmetic skills of their students when they enter the first grade of Primary School. In the first stage of the research, teachers were interviewed and asked to estimate their students' abilities in enumeration, addition and subtraction, writing numbers and solving problems. In the second stage, teachers themselves tested their students, one after the other, in the above-mentioned processes. After completing this test and gathering all the answers, teachers were interviewed again and this time they were asked to evaluate their assumptions about children's knowledge. The final results of our research show that the teachers' predictions about their students' mathematic abilities, in some cases are away from reality. For example, teachers underestimate their students' abilities in writing numbers, solving simple problems of addition and subtraction, etc. It seems that this perception is enforced by the instructions of the Greek analytical program, which ignores what students already know before they enter school.

Graphics calculators for younger pupils

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There are different positions about learning mathematics by using graphics calculators in math lessons for younger pupils (aged 10-12 years). The poster wants to show some ideas about the support of this calculators in such math lessons. So they can help in solving problems by using proportional relations, in working with data and in the geometry to draw, change and move figures. To use graphics calculators for this problems means at the same time to work with coordinates, to use the different possibilities in representation of relations (equation, table, graph) and to work heuristic experimentally.

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Main Theme: Mathematics And Other Disciplines

Secondary Theme: Curricula Innovation

Mathematics Education for Software Engineers: It should be radically different!

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Software engineering is a young engineering discipline which is different in many aspects from the classical engineering fields. For me the most distinguishing point is the kind of mathematics that serves the respective fields well. By giving examples I will try to show that classical, calculus based mathematics is of no help for defining central notions in software engineering, like "abstract data type". Thus, mathematics education for software engineering students should be radically different from the traditional curricula for science and engineering students. In particular, the changes to be made go far beyond putting more emphasis on discrete mathematics as done in many math curricula for computer science students. I will report on our introductory mathematics course that we teach at the Polytechnic University of Upper Austria for several years now. The whole first year is dedicated to teach "The Language and Methods of Mathematics". I will also report on experiments with using the THEOREMA language and system in the lab exercises for this course, both about highlights and problems. THEOREMA is being developed by Bruno Buchberger and his team at Risc-Linz and aims at combining general predicate logic proof methods and special proof methods in one coherent system. An important observation is that students are in no way prepared for this kind of mathematics after high school. Since computers and information technologies gain more and more importance in our lives, the ability to develop software with mathematical rigour will be a crucial asset for the competitiveness of the software industry of any country in the future. This implies that changes in the high school mathematics curricula towards usability for software engineering should be considered as well.

Of course R^3 is blue!
Developing an approach to turn a mathematics course
into a mathematics education course

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This study had as its objective to investigate the possibility of offering a mathematics-content course (in this case Linear Algebra) that is adequate to the professional development of mathematics educators (teachers, teacher educators and researchers); for reasons we will present and discuss, we started with the assumption that traditional content courses (in many cases the same as those presented to future mathematics researchers) were not adequate. The study consisted in the analysis of the transcriptions of videotaped lessons and other protocols collected at a four-months Linear Algebra course, taught to postgraduate students in a mathematics education postgraduate program in Brazil. We will focus on the presentation and discussion of the processes generated by the students' attempts to solve a mathematical problem, particularly on those relating to the production of meaning for the notion of space, and how the approach we took as professors (for instance, only to intervene to call their attention to recurrent statements or to divergences in the whole-group discussions) opened up a 'magic window' to the meanings they were producing for the notions involved, despite several Linear Algebra textbooks being available to them at all times. We will also argue that such a reading of the meaning production processes not only produces a very useful material for reflection during the course, but that it is in fact a necessary condition if we want content courses to be mathematics education courses that actually contribute to the professional development of our mathematics education students (teachers, teacher educators and researchers).

Supplementary Technology Implementation for Teaching

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Probability and Theoretical Statistics are courses for junior or senior students in the Department of Mathematical sciences at the University of South Dakota. These two courses contain higher level concepts in mathematics, such as convergence almost surely, convergence in measure, transformations, ...etc. Probability and Theoretical Statistics are usually offered in two consecutive semesters at University of South Dakota. The traditional way in teaching of Probability and Theoretical Statistics is showing the proofs of all the concepts from these topics. After students finish taking these two courses, students usually have less ability to know the linkage between the theoretical concepts and natural phenomenon that presented through data behaviors. To fulfill this gap, I propose incorporating technology for teaching these two courses. A computer algebra system (Maple software) and Web CT page are used to develop lab sections merging into traditional teaching as a new pedagogy of teaching these two courses in University of South Dakota. First, I assembled all the concepts and compiled into twenty topics for these two courses. Each topic contains introduction, goal of study, main body of study including theoretical results, data applications or implementations, and simulations. Then I used Microsoft word to type the contents of topics and used Maple software to create related processes for all the topics. The Maple software processes contain the theoretical developments, data applications or implementations, and simulations. All of works are presented through Web CT page of University of South Dakota. At present time this Web CT lab had been done. Starting next spring semester, I will teach Theoretical Statistics using this Web CT as supplementary. All the math major students who finished Probability will be expected to take Theoretical Statistics. Students taking Theoretical Statistics will be grouped with 4 students in each and each group will be assigned a different topic as a class project each time. Students will be allowed two weeks to finish project after they have been instructed with the assigned group project. Students will be able to use computer lab and view the topic related to the assigned project through Web CT page. Then students will use Maple software to finish the group project. Each group will be assigned four projects during a semester. Students are required to write and turn group reports for the projects within the semester. A survey from students will be used to evaluate the effect of this Web CT lab during the spring 2002 and the results will be presented along with this technology work in the conference.

**IMPROVED COMPUTER SOFTWARE FOR THE TEACHING
OF ORDINARY DIFFERENTIAL EQUATIONS:
The ODE Toolkit in the Classroom Setting**

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Ordinary Differential Equations are a very important tool in modern mathematics. As such, the subject is often introduced early in an undergraduate's education, before the student has developed the mathematical sophistication to understand the nuances of differential equations and their solutions. Because of this, teaching an introduction to ODEs presents special challenges. We feel that the use of graphical techniques presents the material in a manner which is more accessible to the average first or second year undergraduate. Combining this approach with the use of online computer software enables students to get a firm grasp on how differential equations can be used to solve real world problems. We present ODEToolkit, a powerful free online ordinary differential equations solver based at Harvey Mudd College. We will demonstrate the various features of the Toolkit, including solving first and second order equations as well as arbitrarily large systems of differential equations. We will conclude with a discussion of how to integrate ODEToolkit into an introductory ODEs course.

Developing College Students' Views on Mathematical Thinking in a Historical Approach, Problem-Based Calculus Course

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Mathematical problem solving alone is not enough for developing one's mathematical thinking (Schoenfeld). A missing component is developing students' mathematical perspective. Many educational researchers propose problem-based curricula to improve students' views of mathematical thinking. Meanwhile, scholars like Rickey and Siu, advocate using historical problems to attain this end. Nevertheless, the two ideas have rarely been empirically investigated in a single study. This paper therefore plans to report findings of an in-progress empirical study regarding effects of a historical approach, problem-based curriculum to foster Taiwanese college students' views of mathematical thinking. Our study consists of three stages. During the initial phase, 44 engineering majors' views on mathematical thinking are tabulated by an open-ended questionnaire (developed in four stages) and follow-up interviews. Students then receive an 18-week historical approach, problem-based calculus course in which mathematical concepts are problematizing to challenge their intuition-based empirical beliefs in doing mathematics, comprehend the necessity of rigorizing mathematical ideas, appreciate alternative strategies for attacking identical problems, and elicit intrinsic reflection on mathematics in general. Several historical problems serve to reach the goal-e.g., Liu Hui and Archimedes' derivation of the area of a circle and volume of a solid, Zu and Cavalieri's indivisible principle, Fermat, Descartes, and Barrow's approaches to finding the slope of tangent lines, early misunderstanding of infinite series. Near the end of the semester, participants answer the identical questionnaire and are interviewed to pinpoint what shift, if any, their views on mathematical thinking have undergone. Special attention will also be given to students' views about the different fashion of ancient Chinese and Greek mathematical thinking. The present study is still in progress, slated to conclude by the end of 2001. In sum, the paper aims to provide empirical as well as theoretical perspectives regarding the issue of integrating historical problems into college mathematics.

Mathematics and technology

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How will teaching of mathematics look like in the future? Opinions range from those who think that there is no need for any changes and that lecture model with chalkboard is completely suitable, to those who believe that majority of lecturing will be given by some sort of computers. Due to very slow adoption of some new possibilities sometimes looks like the ones who favor to some extent the first group are in majority. But looking historically, technology always had a significant impact on the way mathematics is taught. Logarithmic calculators, tables, calculators, ... are just few examples. Nowadays, in the age of informatics the possible influence of this technology is enormous. How teachers of mathematics, already overwhelmed with their "usual" work, cope with all this possibilities? In the talk we will describe and evaluate some possibilities, brought to us with information age, teachers of mathematics can use. We will focus mainly on secondary school teachers, but most of the possibilities presented can be used also in primary education and also at college level. Various computer algebra systems, programming languages, Internet and related media, handheld computers, graphic calculators, symbolic calculators, various programs, multimedia are just some of possibilities we will tackle.

Visualisation, mind maps, relaxation, confidence and the outside school tutor: A case study

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Research suggests that as students progress through schooling, her/his perception of the difficulty of mathematics increases. Confidence is an important ingredient of success in any of life's pursuits. In an academic subject such as mathematics this is even more prevalent and therefore, the difficulty that is perceived with mathematics could be mistaken with a decline in confidence in the subject. In this study, I have examined the role of the outside school tutor (tutor) in mathematics and the effect that the tutor has on the level of confidence of the student. Parallels will be drawn with the relationships between the sporting coach and sportsman and the academic coach and the student. The study consists of a number of ongoing questionnaires presented to the students in the study, their school mathematics teachers, parents and the tutor. A structured weekly journal was kept by students outlining their perceptions of the effect that the tutoring process had on their confidence in mathematics as well as observations of this by the tutor. The tutor used techniques such as relaxation methods, affirmations, memory maps as well as traditional teaching methods. The study hoped to show that students will improve their understanding and thus performance in mathematics with the help of the tutor, not only in an academic sense but also in building confidence generally.

The following outcomes were achieved :

- a) The use of alternative methods of teaching such as relaxation, visualisation and memory maps have a positive effect on the confidence of students.
- b) Outside school tutoring, in a small group situation, has a positive effect on a student's confidence in mathematics.
- c) Positive affirmations and positive talk increase students' awareness of their confidence and ability in maths.

In addition, the study suggested skills and strategies that can be used the generally in the classroom to increase the students' confidence in mathematics. A more detailed look at each of the methods used and how they can be brought into classroom practice will be discussing in the paper.

Interdisciplinarity Projects in the Arts and Science Programme at McMaster University

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Arts and Science Programme at McMaster University (Hamilton, Ontario, Canada) is an interdisciplinary program that offers students an opportunity "to use their university years to further their intellectual growth through a study of significant achievements in both arts and sciences." The main goal of the program is to give students an understanding of sciences, arts, and technology, to help them develop skills in communication, in qualitative and quantitative reasoning, and to help them become critical and independent thinkers. I have been teaching mathematics in the Arts and Science Programme for the last five years. In my presentation, I plan to outline and to discuss two interdisciplinary projects that I have been involved with: * "Mathematics and Culture and Society" project - links mathematics and informal logic and writing courses* "Science Inquiry" course - uses the powers of mathematics, physics and chemistry to investigate questions in biology Major part of my presentation will focus on various issues that I have been faced with, such as: * Do students learn mathematics better in the context of interdisciplinarity? * If so, what is (acceptable) evidence? * What do students really learn in interdisciplinary courses? * Can interdisciplinarity help us strike the right balance between skills development and knowledge acquisition?

Educational aspects involved in the representation of trigonometric series functions in CAS environments

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Many researchers have investigated the role played by the external representations of mathematical concepts into the students' learning processes (Duval, 1999; Moreno, 1999, among others). In the ·function· case, Janvier (1987) classifies the representations in four categories, according to the different cognitive processes that they involve: verbal, tabular, graphic and symbolic representations. If we consider computational environments to carry out this kind of representations some new features must be analyzed: - On the one hand, these environments impose new representation and transformation rules derived of their internal computational design, non necessarily founded on mathematical requests; - On the other hand, they increase some representation capacities, allowing to handle some manipulations impossible to carry out by hand. In this work, we analyze these features in the case of functions defined by means of trigonometric series in CAS environments as Maple, Mathematica or TI-92 Plus Calculator. We analyze different forms to define these functions in the computational system and their respective consequences on the operations we can realize on them. In this sense, we remark that the use of the infinite summation symbol is problematic, as it does not provide the numerical values expected when we perform the traditional substitution. The explicit use of the limit concept is required in order to relate symbolic and tabular representations. Concerning the graphical representation, we focus in the capacity of these environments to capture the infinite process involved in the definition of trigonometric series functions by means of the graphical analysis of the finite sums (Hitt, 1997). This idea allows us to handle, in a graphical context, the concepts of convergence, uniform convergence or periodicity. We propose this approach in an educational context, assuming that in the process of concepts formation, the manipulation of the operational conception helps to acquire later the structural one (Sfard, 1991).

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Main Theme: Distance Learning

Secondary Theme: Mathematics And Other Disciplines

Distance learning courses on numerical methods with access to software libraries

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An approach to development and technology of realization of distance learning courses for mathematics subjects is suggested. Courses 'Methods of optimization' and 'Elements of calculus' are described. These courses are used in the Sumy State University (SSU) for teaching the full-time and correspondent students and require usage of software. For computer practical lessons in the distance courses we have provided description of libraries of numerical methods programs, organization of access to them, adaptation of the programs to the user. Let's have a look at the opportunities of the course 'Methods of optimization'. The course includes 4 stages of learning: studying of theoretical material; self-control of received knowledge; mastering practical skills; evaluation of received knowledge and skills. Lectures are designed as a tree structure of text documents. Testing is organized both on each topic and on the course as whole. Evaluation procedure is possible through an e-mail or on-line answer. For fulfillment of laboratory tasks we have provided an access to programs of numerical methods. Software was written in C++, Pascal and Object Pascal by students of SSU. Moreover, original texts of programs are well documented and directions on self-programming of numerical methods of optimization are given. System of access to library files is offered. The library of programs is divided into blocks and can be accessed in the distance courses as archive files. Students can copy these files to his computer, unpack them and start working with programs immediately. The course has a chapter with examples of applied engineering, economical tasks and their solutions. Features of applications such as Maple, Excel and scientific Fortran-library are described. The distance course is designed for a russian-speaking audience and is open for free usage on the following address: <http://dl.sumdu.edu.ua/mo/index.html>.

Calculus Courses at the Computer Science Faculty, the University of Indonesia

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Students of the Faculty of Computer Science at The University of Indonesia are among the top high school graduates. Their average entrance test score is the highest at the university. However, two years ago a report showed that for several semesters around twenty percents of them failed in calculus courses. The responses to questionnaires given by students said that the students have lack of enthusiasm, they questioned about the importance of calculus for their subsequent work in computer science, they found that calculus is difficult and less challenging, and they thought that calculus is just a list of rules and formulas need to be memorized. This paper presents a new approach in teaching calculus given in the last three semesters, its effects, and the obstacles. The approach is devoted to give students strong background in calculus and greater capacity to use the methods and hence better prepared to complete their degree in computer science. The effort to achieve the goals includes promoting student centered learning, building a bridge between calculus and computer science, giving computer related examples, using Maple for calculus projects, and using computer science terms to explain some calculus concepts.

Learning evaluation with respect to conditions of admittance in a freshman Mathematics course for engineering majors

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Summary: The Universidad Nacional del Litoral (UNL), located in the city of Santa Fe, Argentina, offers a college preparatory course in several areas named: Program of Articulation at Distance (PROARDIS). It is called so because of the distance teaching modality adopted, which is imparted through the Satellital Net of Remote Classrooms. The different Schools had, in the year 2000, the option of demanding it as precondition to register in the first course of the respective area. However, starting this year, the approval of this course is a prerequisite. Consequently in the School of Chemical Engineering (FIQ), it was not demanded as requirement for the first mathematics course: Matemática A (Math-A), in the year 2000 but it was so in the year 2001. The purpose of this work is to asses the influence of this preparatory course in the overall student's performance. Freshmen students in five different majors in the FIQ were considered for this study. Databases that picked up the experience of both years were used and analyzed. Some of the conclusions arrived at are of mayor importance in helping several academic decision-making processes of the Institution which will undoubtedly have great repercussion in the career of most of the students of the University. Also, for both years, contingency charts were draw and studied. In these charts each student is related the yield, with regard to the elected career, his middle-school, type of this school (public or private). The purpose was to try to obtain conclusions which will allow to determine a priori possibilities of success for each entering student.

A Four-Year Undergraduate Degree Program In Mathematics

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The purpose of this article is to describe innovations carried out in the program of studies of the ·Licenciatura en Matemaica Aplicada· in the School of Chemical Engineering of the Universidad Nacional del Litoral. In Argentina this degree required five years of studies. A particular feature of this career is its applied character, which previously to the reformulation already satisfied the recomendations of the Society for Industrial and Applied Mathematics, being unique in our Country in this respect. The reformation was carried out taking in consideration curricular guidelines of the National Mathematics Organization (Unión Matematica Argentina, UMA), as well as its articulation with the Teachers Preparation Program in Mathematics (·Profesorado en Matematica) which, oddly, is imparted in the UNL in a different School. We also included the creation of a Masters degree and the integration of both careers with an already existing PhD Program in Mathematics. Their innovative character resides in the transformation of the subjects in modules of smaller extension and the introduction of transversal workshops (·Talleres·) for the integration of contents. As a consequence, together with optional courses, a larger ductility is attained, making it possible to incorporate new topics and giving the possibility of developping different study programs adjusted to the student's particular interests.

Using Computer Algebra to encourage a deep learning approach to calculus

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The underlying concepts and proofs of introductory calculus involve difficult and abstract ideas that present a mountainous obstacle to many students. A tempting solution for lecturers is to focus the teaching at this stage on techniques. This may have the advantage of ensuring acceptable pass rates but helps neither the students nor the teaching staff in the long term. Computer algebra systems offer both an opportunity and a challenge to present new approaches that assist students to develop better understanding of the basic concepts. They can be used to change the emphasis of learning and teaching of calculus away from techniques and routine symbolic manipulation towards higher level cognitive skills that focus on concepts and problem solving. Two of the key indicators of deep learning and conceptual understanding are the ability to transfer knowledge learned in one task to another task and the ability to move between different representations of mathematical objects. Computer algebra systems are multiple representation systems, that is, they have the ability to facilitate graphical, algebraic and numerical approaches to a topic. The author will describe how carefully structured worksheets are used with Derive to ask questions then let the students provide the answers in such a way that they can construct their own knowledge. This allows learners to discover rules, to make and test conjectures and to explore the relationship between different representations of functions and other mathematical objects using a blend of visual, symbolic and computational approaches. Students enjoy the power and versatility of computer algebra and are encouraged to become reflective, deep learners.

The Faulty Ways of Thinking of Freshmen on Learning Mathematics are distinguished from Ways of Understanding

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The ways of thinking of freshmen are important, because it can be used for filtering and interpreting the lecture. The way of understanding influences how well the student's understand mathematical content as well as the quality of their ways of thinking. If the students' way of thinking are good, then they can catch the mathematical concepts well. The faulty ways of thinking are caused by many things such as :

- symbolic logic reasoning
- lack of the basis intellectual curiosity of wondering why a state assertion is true or false, helping student elicit, and difficulty in recognizing the intellectual need,
- in-effective concept of definitions, inadequate the problem-solving.

Even though ways of thinking are difficult to relinquish, they are not unalterable. More than 25 years in teaching mathematics, I feel there are many students have faulty ways of thinking, I feel that there must be something wrong. And I found that it must be one of the above cases. So the sources are the student and the teacher vice versa. Both students and teacher should know what their roles are. In my paper I would like to present some faulty ways of thinking based on my experiences and observations in teaching especially linear algebra and calculus.

Design and implementation of an industrial mathematics degree course

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We present the proposal and one year experimentation of computer science course designed for students of mathematics after the 2001 revolution of the Italian university structure where the duration of a degree course is passing from an unique level of 4 years to a two level (3+2 years) being intended an early entrance in the job market after the first level. Our purpose is to prepare professionals for the limited (but increasing and exclusive) niche market of mathematics applications in industry and in the services. We introduced a set of new teaching modules for the students of the first year of undergraduate level either in Mathematics and Mathematics and Computational Informatics degree programs. The use of computer starts from the very beginning of the first year. It deals with a general laboratory 32 hours introduction (in the first semester) followed by two parallel 46 hours courses (in the second semester) in Numerical Calculus and Introduction to Computer Science. It considers and is co-ordinated (and not overlapped) with European level initiatives such as the European Computer Driving License which is complementary. The first year courses starts giving a short glimpse of mathematical models and applications running on computers as the target to be reached after 3 years; then a course follows where the description of the basics of the Information and Communication Technology is given applied to mathematics. In the meantime a supplemental 20 hours lab introduces Mathematica. In the second and third years there will be four more courses dealing with numerical analysis and mathematical modelling (plus image processing and computer graphics as elective courses) and with mathematical algorithm analysis and programming. We set up www tools to support the teaching and we had the opportunity in the year 2000 to open a new laboratory suited for the new curriculum.

Developing Open and Flexible Computing Environments for Teaching Mathematics and Science

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Most of the software packages designed for teaching mathematics offer a limited set of tools, utilities and procedures. This means that teachers quite often have to use a number of different computer programs in order to teach different topics. This situation makes our teaching very inefficient. We need to teach students how to use each tool. We need also to teach different, environment related, strategies of solving problems, and quite often a completely different philosophy of a computer package. A dream situation is a computer package where the teacher or even the student has an opportunity to modify the teaching resources offered by the program, add his own procedures, functions and operations, or even build his own mathematical domains. Such a computer package would give teachers a great flexibility in developing their own teaching environments and remove limitations in developing strategies for teaching various topics. In my paper I am going to show how teachers can develop their own teaching environments with the use of MuPAD - a Computer Algebra System from the University of Paderborn in Germany. I am going to show step by step how teachers can build their own libraries, add and integrate them with MuPAD resources and finally use them while teaching with a standalone or online installation of MuPAD. I am going to point out and discuss advantages of such approach.

Using Technology to Integrate Constructivism and Visualisation in Mathematics Education

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This paper provides a discussion of the pros and cons of instructivism and constructivism in the mathematics classroom, and endeavours to show why the latter is a preferable methodology to the former when considering the effective use of technology to enhance visualisation. The adoption of a constructivist approach to the teaching and learning of Mathematics has highlighted a shift from teacher dominance. Visually stimulating computer environments can allow students to become immersed in their own knowledge construction. However, it is not a trivial matter how to utilise this considerable technological capability most effectively for educational benefit, emphasising the importance of a teaching and learning methodology. It is necessary to encourage more exploratory approaches to learning, where students can be the initiators and controllers of their own learning. There is much empirical evidence that this approach significantly improves the understanding of higher order concepts. Knowledge is built up from personal experiences, and making these experiences more dynamic will assist in the development of cognitive structures. Computer-based attractive environments with visually compelling displays, together with facilities for interaction, can provide the setting for more dynamic, powerful experiences. These environments are filled with stimuli which encourage rich constructions by students. The integration of constructivism and visualisation can encourage the reformulation of conceptual structures and the development of higher order skills. Having reviewed and examined the effectiveness of previous work by authors such as Tall, Dubinsky, von Glasersfeld, etc., and different constructivist perspectives, consideration is given to the best way to employ constructivism in teaching and learning with computer-based visualisation. The effectiveness of this approach is evaluated, and students' experiences are discussed in terms of the enhancement of mathematical skills via the constructive use of visual software.

Elements for teaching Game Theory

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Game Theory should be included in undergraduate programs for many professions, specially for economics, business administration, industrial engineering and obviously mathematics and statistics. It becomes indispensable in a globalized and technified society to know theoretic points of view to decide conflict situations. Game Theory gives a nice opportunity to university lecturers to carry out the essential role of stimulating in our future professionals the attitude of observing, analyzing and theorizing as a way to build a better world. Moreover, it is highly formative to know basic results of a theory developed in XX century and to use elements of probabilities to examine multiperson decision problems. In the teaching learning processes of mathematics we should take care how and when to present the rigorous formalization of the concepts and the use of specific techniques because it is very important to stimulate both an intuitive approach to the concepts that we are introducing and a creative use of the previous knowledge of our students. When we teach Game Theory we have a nice opportunity to apply these criterions through the collaborative learning and solving problems according to the following sequence: understanding (includes organization of the information and representation), intuitive approach to a solution, solution (or intents to solve) using previous knowledge, intuitive introduction of new concepts or theorems related with the problem, solution (or intents to solve) using the new concepts or theorems, formal and rigorous presentation of the new concepts or theorems, formal solution of the problem, search of other ways to solve, explorations modifying the problem and a deep study of the theoretical aspects using the intuition and the formalization. With this didactical propose, I made it easy for my students to understand the concepts of Game Theory, specially Nash equilibrium and mixed strategies for non zero sum games and their applications.

The Influence of the Family in the Learning of Mathematics

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With the theme 'the influence of the family in the learning of mathematics' as a starting point, I raised two questions to guide the research. One of them deals with the superior status of formal mathematical speech. Exercising formal mathematical speech signifies 'being specialised labour', and therefore being part of a group of people that has a higher economic status (with different levels within the profession) than that of groups of people that do not have the qualification. In this way, I understand that the hierarchical positions of work are also determined as a function of this status, which produced the differentiation between social positions and income. The other question is whether or not the family participates in the qualification of the workforce. I collected data related to three students in the fifth grade in a public school in the city of Rio Claro. Since I was interested in the influence of the family in the constitution of codes of prestige and discredit in the process of students' academic performance, I selected students who were classified by the school as excellent, good, and poor, respectively. I concluded that the family is responsible for the introduction of the codes of ideological recognition, which may or may not coincide with the codes adopted for bureaucratized instances, instances of the production of knowledge, such as the School. The recognition of these signs (culture) is going to say what can be learned as codes that can be deciphered. However, the production of meanings that occurs in the family makes the differentiation in the labour force commodities, once their qualification depends on access to texts that can or cannot be read deciphered, dialogues that can or cannot be experienced. Thus, such commodities assume use-value and exchange-value beginning with the attribution of sign-value, which is exactly what guides differentiation in academic evaluation, and which depends on the insertion of the 'subjectification' proposed by the family. Thus, the subjects who share in the informal speech, which coincides with the formal in the diverse fields of valid knowledge, will have greater advantage over those who do not share it.

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Mathematics Vocation as Academic Recognition

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In this work, I investigated the power relationships that constitute the person while agent social that follow the scientific mathematical practice, and that he is seen, many times, as founded in the creation capacity of the individual's "endowed" with vocation. Considering that the mathematical production has its defined contours, we can affirm that the mathematician, he will be called like this if he is recognised by the scientific community that classifies him. In this case, what we call mathematical vocation it is the recognition produced by the mathematical community. Professional researchers in Brazil exist in institutions such as universities, research centres, and others - places where they exercise their practices, which in Althusser's sense (1980), will be consonant with the ideology that permeates these apparatuses. Whether or not these professionals are in agreement with the policy exercised is not exactly of interest, but rather the fact that all of them, independently of what they believe, carry out the work as a duty to be fulfilled (Weber, 1944). I analysed an exam that involved two important Brazil History of Mathematics names. This exam happened in University of São Paulo in 1933, it was to decide somebody in the most important teacher position in Mathematics Department. In the process of recognition, there is bestowment of prestige (Baudrillard, 1972) upon new members who are initiating their careers; the rituals (examinations, celebrations) mark their steps, the norms provide the limits, taking into account (curriculum vitae) the adaptation to the discipline (conduct) established by the institutions to which the candidates belong (schools, universities, among others), and then the authorised opinions are produced, that is, a conclusion is arrived at concerning the candidate. Those who were able to remain in the institution and utilise the prestige it provided demonstrated, above all, that they were able to submit to the discipline (Foucault, 1977), that they were able to allow themselves to be classified, fit in. Therefore, it becomes "natural" to attribute mathematical vocation to some through the recognition provided by the community, rendering the one attributed with mathematical vocation captive from the first moment to the existing articulation of the subject, thus constituted by those who are in a position to allow entry into the order of discourse.

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A study about the development of knowledge in fifth to eighth graders subject to the same didactical intervention involving ordering relations

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Iglori and Maranhao checked, in a late study, that fifth graders restricted the meanings of the relations "come before than" and "not come after than" in solving problems. For these students, "come before than" meant "come immediately before than" and, in the ordering relation "not come after than" (equivalent to "come before or together with"), they did not admit "come together with" as an ordering. The authors also checked if it was possible to see an improvement in the knowledge of students subject to a didactic sequence based on the Didactical Situations Theory by Guy Brousseau (1997). To this end, questions about enunciations of the following kind were proposed: A teacher wanted to know the order of arrival of her students. They informed her but she could not figure out the exact order of their arrival. Give the possible orders of arrival according to the statements students gave to her. Maria said that she came to school before Eni. Eni said that came before Bia. Rita could not remember the arrival of her mates, but she was sure she came after Eni. In this study, the order of presentation of characters in the problems· enunciation was not questioned as a possible didactic variable. Therefore, this article presents the results for the following questions: 1. Did the fifth to eighth graders (10 to 14) have the same problems diagnosed in the late research? And if they had, was the evolution of a restricted conception to a broader one different for each grade for the same didactic intervention? 2. Was the order of characters· presentation in the problems· enunciations a didactic variable? The problems presented were adapted so as to boost analyses that answered these questions. The didactical intervention was triple phased dated a week apart: a pre-test, a class and a post-test.

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The Role of Physics in Students Conceptualizations of Calculus Concepts: Implications of Research on Teaching Practice

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This paper discusses the implications of research on undergraduate calculus learning for calculus teaching practice. In particular, this paper addresses the challenges of using research results and adapting instructional materials in diverse classroom settings, and aligning research-based conceptions of teaching with practice. In a previous research study, the author investigated students' use of physics experiences and concepts as they construct calculus concepts in an interdisciplinary calculus and physics course. The results of this study suggest that students frequently draw upon physics experiences and concepts as they develop understandings of average rate of change, but that students less frequently make use of physics experiences as they develop understandings of derivative and antiderivative. In addition, other researchers have alluded to the importance of prior physics experiences on students' conceptualization of the average rate of change concept (e.g. Nemirovsky & Noble, 1997). However, implications of these research results for calculus teaching practice have received little attention. The combination of the author's research findings and those of other researchers suggest four major implications for calculus teaching practice. Research results:

1. Modify pre-existing instructional design theory.
2. Influence the design of classroom activities and development of learning sequences. In particular, research provides information about the potential mismatch between the experiences students bring with them to the classroom and teachers' assumptions about students' past experiences.
3. Influence how the teacher conceptualizes the role of the students in the classroom community.
4. Alter the role of technology in the classroom.

The results of the author's previous research as well as other research results are discussed relative to these four outcomes for undergraduate calculus teaching practice.

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Numerical Calculus and Analytical Chemistry: An example of interdisciplinary teaching

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Analytical Chemistry is an almost unexplored source of real · life problems for Numerical Calculus courses in chemical careers [1]. In this paper, we discuss one of these problems: the pH determination of a weak monoprotic acid aqueous solution [2]. From the mathematical viewpoint, this problem led us to solve very difficult algebraic equations. In several cases is possible to obtain an algebraic exact solution, but in other situations the algebraic approach is not useful. So, if we want to generalise our methods, we need a numeric approximate solution. We analyse several algorithms from well known methods as Newton-Raphson, Regula Falsi, Bisection and others [3]. We also study a couple of methods, developed specially for this kind of problems. The variety of situations, and the mathematical and chemical richness of them, suggests proposing an interdisciplinary work in research and teaching. This can be carried out by a group of both Analytical Chemistry and Numerical Calculus teachers. In the same way possible to use these problems for students project · work, with interesting advantages. We comment here, some important results, strongly related with this style of teaching ([4] and [5]). Finally, we suggest some recommendations for these mathematical service courses in chemical careers.

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Main Theme: Curricula Innovation

Secondary Theme: Innovative Teaching

Precalculus with Internet-based Parallel Review

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An innovative precalculus course with an integrated Internet-based component will be described.

The course has:

- (1) an Internet-based just in time review component,
- (2) Internet-based weekly tutorials, practice, and testing,
- (3) is designed for science and engineering students,
- (4) integrates the study of functions of two variables and other basic three dimensional ideas, and
- (5) incorporates the use of symbolic algebra systems and other innovative pedagogy.

The study of multivariable functions is traditionally postponed until multivariable calculus. However, with the aid of a set of manipulatives that we have developed to aid in the visualization of three dimensions, multivariable topics are being effectively incorporated into the precalculus curriculum. This aims to build in our students early on in their college careers the three-dimensional geometric intuition that is necessary very early in the engineering curriculum. The Internet component of the project allows the establishment of a weekly practice, tutorial, and quiz system that helps students review the pre-requisites for upcoming material and review the material just covered in class. This component consists of a large and highly organized data bank of questions, a set of accompanying tutorials, and the software necessary for generating and administering quizzes on-line. The Internet component is being designed to facilitate its implementation in a wide variety of institutions. Interested faculty will be able to easily edit, contribute to, and adjust the data bank of questions to suit their needs.

Developing WALLIS: a web-based system to enhance mathematics teaching

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Mathematics coursework is required for most science and engineering degrees. However, reports in the field of Mathematics Education address the growing deficiency in mathematical skills amongst science and engineering students. This problem and the need for action are now widely acknowledged. Although long-term joint efforts from governments, schools, universities, and pedagogical institutes are addressing this problem, universities take their own steps to --- at least --- alleviate it within their own immediate sphere of influence. This poster presents our work on integrating IT in the teaching of non-specialist mathematics courses. Based on our previous separate work we are now developing a more generic tutoring environment that facilitates techniques from computational mathematics and artificial intelligence. The prototype is involved in an authoring and tutoring system that allows members of staff to design their own material for dissemination through it. This system permits students to work on it independently and in their own time as an extra support in their conventional studies. It is hoped that the interaction with the system will eliminate misconceptions that they may have from high-school and should increase their motivation to study. The application comprises of existing web-based material enhanced with interactive parts that can also provide tailored feedback according to the students' actions. The tutoring contents of the system are driven from both the students' actions and their understanding of the subject. Following methodology for designing Intelligent Tutoring Systems we are collaborating with students, lecturers, and developers. Thorough observations of lectures and real tutoring situations provide valuable information for the design of the system and the needs, problems, and misconceptions that students may have in specific subjects. Direct feedback from the students and the lecturers together with loops of observations, changes, and development will result in a more solid and effective system. Our prototype is soon to be pilot-tested on vector algebra and integration.

Interactive Classrooms and the Teaching of Mathematics

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This paper will explain how group (audience) response systems in interactive classrooms have been used to activate the teaching and learning of mathematics. Students input their answers to questions posed in class via individual handsets and can see live feedback on their collective results immediately. By enabling the delivery of Socratic teaching by questioning, this emerging technology helps support the role of the teacher as an "inspirer-for-the-enquirer" rather than "sage-on-the-stage" or as "guide-on-the-inside" rather than "guide-on-the-outside". Students become more involved during classes and more motivated to study after classes. Two examples illustrating the benefits of group response systems will be presented, giving delegates an opportunity to try out the Personal Response System for themselves. The first example is a second year Multivariable Calculus course where computer based learning, incorporating computer algebra, have been used to provide the main study materials. The second example is taken from teaching mathematics to computer science students, where it is necessary to demonstrate the relevance of the subject. In this case the use of group response systems during revision classes has been especially valuable. On-line and locally delivered computer assisted assessment (CAA) are used for both of these classes. By practising assessments together during classes, students are able to see how they are progressing in relation to others and the lecturer can tell which parts of the course need to be reviewed. Evaluation of the system by students themselves shows that interactive classrooms are enjoyable to use and help to improve their learning.

Interactive Visualization Software for Calculus

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A variety of new generation visualization tools have been developed to illustrate concepts and applications in calculus. We will demonstrate some of these computer tools and talk about how they can be used in the appropriate courses. Hu Hohn of the Massachusetts College of Art has done the graphics design on the tools. All of the tools are designed to be interactive in that graphs and models are linked to the action of sliders and the cursor. The tools were developed because the ideas that they illustrate lend themselves to interactive visual presentations that are far more effective than the usual static ones. The topics range through function families, curve fitting, tangents, differentiation, integration, level curves, simple differential equations and an unusual representation of linear regression. Although the majority of tools are applicable to calculus in general, a subset includes applications from economics with such topics as optimization, consumers surplus and indifference curves.

Main Theme: Technology

Secondary Theme: Innovative Teaching

The Birthday Problem: The Electronic Spreadsheet in a Gentle Introduction to Discrete Mathematics

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In elementary statistics classes, a common example is the "Birthday Problem": In a group of people, what is the probability that at least two people have the same birthday? This naturally leads to the question "On average, how many queries are necessary to find a match or determine that one does not exist?" The paper presents a recursive model that can be examined with an electronic spreadsheet. A similar analysis is used to find the probability distribution for the random variable N representing the number of queries required to find a match or determine that one does not exist. Also included are the results of exploring these models with an electronic spreadsheet and the results of a computer simulation that validates the mathematical analysis. This problem is related to more practical applications. For example, what are the implications that size of a secondary key has in a database? What are the security implications of the large, but finite, number of physical keys available for a lock design? The paper gives other examples from the areas of finance, probability, and analysis of algorithms that illustrate how an electronic spreadsheet can be used in a gentle introduction to discrete mathematics.

Some points connected with teaching undergraduate mathematics in technical universities

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In technical universities priority should be given to practical lessons and not to theoretical lectures, in proportion 2:1 or 4:2. In student learning assessment not only his taking an exam should be considered but mainly his efforts and results during the whole semester, as well as his activities at practical lessons, attendance and doing homeworks. This is why distribution of points 10+20+70 (exam) does not give a stimulant to a student for learning hard during the semester. We propose the way of student knowledge assessment which had been successfully used in Georgian Technical University since 1991, namely: 1. The semester material is divided into two equivalent parts; each of them is evaluated at the microexam by 45 points (35 points for practical writing + 10 points for theory). 2. Teacher may add up to 10 points to those mentioned above, conforming to student's attendance and diligence. 3. If a student obtains at least 40 points at both practical writings, he/she is allowed not to go in for an exam. 4. If a student gets less than 40 points or he/she wants to improve his/her result, he/she may take an exam, at which one can obtain not more than 70 points. So, from the expounded above, you can see that the main stress is made on student's skill to apply his theoretical knowledge in practical way, and this faculty is very important for future engineer.

On the need of Teaching Finite Fields at the Undergraduate Level

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Finite fields are mainly Pure Mathematics. However, they have a large spectrum of applications, motivating algebraic, geometric and computational problems with varying difficulty degree: easy applications to Signal Processing, Discrete Fourier Transforms, Cyclic Convolutions; applications to Algebraic Coding Theory; hard applications to Cryptography and Data Security; the Discrete Logarithm Problem and the Diffie-Hellman Problem. There is a contradistinction between textbook and real applications of Mathematics. A 'distance' is defined between the mathematical tool and the application, which is natural in the real case and artificial in the textbook case. Finite fields are involved in both types of applications (cryptography is a real application, puzzles are textbook (or educational software) applications) and are applied to High Technology (Data Security, e-Commerce). It is natural in their teaching to use modern interactive tools, which are web-related. Recent advancement in software, hardware, networks and the Internet, has created fast-growing demands at the undergraduate level. The creation of an Internet Interactive Mathematics Server (IIMS) with the use of JavaScript or Java, could lead to interactive process on the server, making a sophisticated computation tool, dedicated to Engineering Education and training. For an ordinary user, IIMS will be web sited just like any other, on which users can consult interactive textbooks, answer mathematical exercises, be self-taught in real application of mathematics on a user-friendly graphic environment, make computations, draw curves or surfaces, or play math games, by clicking on buttons, just like on any other web page. They can interact with the server by playing games or explore intelligent processes (media cryptography, sending 'requests' back to the server which computes the request and sends back the result). A finite field course can be offered, either as an autonomous course in a Mathematics Department, or as a link of a chain since it supposes other obligatory, or compulsory courses (General Algebra, Number theory, Galois Theory). It can be included in an undergraduate program in Computer Science, Electrical or Industrial Engineering.

STUDYING THE EVOLUTION OF STUDENTS' THINKING ABOUT VARIATION THROUGH USE OF THE TRANSFORMATIVE AND CONJECTURE-DRIVEN RESEARCH DESIGN

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The paper describes the experiences gained from adopting the transformative and conjecture-driven research design (Confrey and Lanchance, 1999) in a study which examined introductory statistics students' understanding of variation and its relevance to statistics. It discusses the many possibilities that this design, which sees research and practice as interwoven and advocates curriculum construction based on an ongoing process of development and feedback, offered for systematically researching students' conceptual change, and contrasts them to the limitations of the prevailing methodology employed by researchers examining conceptions of data and chance of taking snapshots of students' thought processes in order to catalogue their misconceptions. The rich insights into the evolution of students' thinking of the role of variation that the study managed to provide by employing a variety of quantitative and qualitative data gathering techniques throughout the course are discussed. Finally, the paper points out the benefits of the conjecture-driven design's approach of relaxing some of the constraints of typical classrooms while letting others remain in force, and makes the argument that such an approach which utilizes both theory and common core classroom conditions has a much greater potential for wide-scale implementation than the lately commonly observed research practice of studying 'best practices' in the hope that new classroom practices will evolve from these exemplary practices.

**General Requirements and Methodological Foundations for Course
of Higher Mathematics for Geology students in
Peoples' Friendship University Of Russia**

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Different level of mathematical knowledge and skills and their orientation for professional needs is required for students of various specialities. Course of mathematics for non-mathematical specialities became more application-orientated. The great attention is paid for using of modern computer technologies for numerical problems' solution. But it's necessary for a student to realize the basics of mathematical science, mathematical language and to develop the ability of modeling. The paper presents general requirements for Higher Mathematics course for speciality "Geology", based on the analysis of particular educational process, curriculums, contents of geological courses. Didactical principle of "intersubjectness" should be taken into consideration for the development of the course. Intersubject relations allow to concentrate attention on the main goals of educational process, to apply knowledge and skills from various fields, to stimulate educational activity of students. Mathematical skills are formed in the process of problem solution. Problems with intersubject content were offered for using in the course of Higher Mathematics. For example: Problem 1 (subject "Geophysics"). Equation of rocks structure deformation model with one flexible section surface is $v_1/y_1=kq_1$. The problem is offered in topic "Differential equations of first order -- homogeneous differential equations". The problem is directed to transfer of text-based problem to mathematical one. Mathematical part of the problem results in applying of table integrals. The paper presents description of a number of classified problems with methodical recommendation of their using in educational process.

Main Theme: Innovative Teaching
Secondary Theme: Mathematics And Other Disciplines

Project work in CPGE(France)

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In France, a large majority of students enter engineering schools through the system of CPGE. Some 50000 students undergo their first 2 or 3 years of tertiary education in this centralized structure, then take competitive examination to gain entry to engineering schools. The teaching system is the same as in secondary school : a class of about 40 students, sometimes divided in subgroups, works with one teacher for each subject. In mathematics, this teacher teaches one class 16-20 hours a week. One of the main criticism of this system is that he doesn't favour openness of mind, autonomy, initiative, team work. As an answer, an important reform took place in 1995 . It aims to reduce the importance of mathematics itself as well as relating it more closely to other subjects, and to develop a spirit of initiative among students. Among the innovations, 2 hours a week are consecrated to an interdisciplinary project work. This work is presented for the final assessment. I shall present some commentaries about this innovation, from the point of view of students, teachers, and examiners.

Object Teaching of Graph Algorithms

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The base concepts and theorems of the Graph Theory and related Graph Algorithms are taught in the frame of the subject Discrete Mathematics at our university, University of Hradec Kralove, Czech republic. The Graph Theory is a wonderful, practical discipline, often little as puzzles. The good understanding of graph algorithms develops the logical thinking of students very much. Therefore we focus properly on these problems. When explaining algorithms we put emphasis on mutual relations between individual algorithms. When students make sense of the concepts tree and spanning tree we start to speak about the well-known optimisation problem, the minimum spanning tree problem. We show them three classical algorithms (Borůvka, Jarník (Prim) and Kruskal algorithms) and also for comparison one dual algorithms (dual Kruskal algorithm). All methods we describe as an edge colouring process. On the base of Jarník's solution of the mentioned problem we continue our lectures with description of other algorithms. First we show the relationship of the Jarník's method to Dijkstra's algorithms for finding the shortest path. Then we speak about Breadth-First Search and Depth-First Search. And on the base of these searching algorithms we discuss several other graph algorithms. In the article we would like to show the relationships among mentioned graph algorithms more deeply and at the conference we will introduce in the environment Delphi prepared visual presentations which we use in the lectures as a very nice complement for illustration all above-mentioned algorithms. We use them also as the study materials for students who study by combined form (face to face and distance education).

Multiple Choice Testing

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Multiple choice testing does not reflect the true level of a student's knowledge and does not require the student to think. In Ukraine, some years ago, we became aware this testing methodology and, at a conference, we discussed such tests. United to a man, we refused this testing as worthless. Students must not guess answers. They should show their work and find answers. Students must know definitions, basic theorems, formulas, and applications. Students should be taught how to prove mathematical statements and apply such knowledge to solve problems. Correct testing is very important because it shows the real level of a student's knowledge. Incorrect testing is not only useless; it is harmful to students and teachers. For Ukraine high school graduation, teachers receive Department of Education prepared mathematics and literature tests thirty minutes before the examination. The mathematics geometrical problem requires the student to make drawings, strictly explain every step of solutions, apply knowledge of trigonometry, state theorems or formulas used, and write the answer. Some proofs of mathematical statements are required. There is no place to guess an answer. Some mathematics testing is in oral form. The student must be able to answer questions using mathematical language. Good testing requires the student to think. Instructors need to prepare their students for real mathematics testing not guesswork testing. Good test grading requires the instructor to see the real knowledge of the student. Multiple choice testing allows the student to pick a correct answer without understanding. Multiple choice testing allows the instructor to count the correct answers without seeing the real knowledge of the student. Multiple choice testing should be discarded like a worn out shoe.

"Ratio": Raising Teachers' Awareness of Children's Thinking

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We believe that the problem of teacher preparation is one of development of what Shulman (1987) calls pedagogical content knowledge rather than subject knowledge per se. In previous work, we have found that even experienced teachers may not be aware of the misconceptions that learners tend to exhibit, or at what stage of development and in what areas of the curriculum these are likely to be manifested. This pedagogical knowledge is, we believe, important to teachers' mental models of their learners, and hence their teaching effectiveness (Williams & Ryan 2000, Hadjidemetriou & Williams, 2001). In this paper, we present a diagnostic instrument which reveals children's thinking in the field of 'ratio': a topic which is difficult to teach and learn in the middle school years (Hart 1984, Tourniaire and Pulos 1985). This has been constructed using items from previous literature (CSMS 1984, Kaput and West 1994, Lamon 1993, etc). Using grade 6,7,8 children, we produce a developmental scale of these items which reveals how they compare in difficulty and in the diagnostic errors they generate. Our instrument contains two versions, one with 'models', thought to be of service to children and one without. These models involve pictures, tables or double number lines, which can be used in modelling ratio problems. We compare the difficulty of the parallel items for the children. We also present some data on trainee teachers' pedagogical content knowledge with respect to children's thinking in this topic. We use the same items which form the children's diagnostic instrument, but we ask the students to predict the children's errors and likely explanations. We present these base-line data for children's and teachers' knowledge. The next stage of the research will investigate their use in teaching and in teacher education.

Main Theme: Technology

Secondary Theme: Innovative Teaching

A novel approach to the engineering mathematics curriculum

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At Harvey Mudd College, we have successfully implemented a major revision of the conventional engineering and science mathematics curriculum. This revision accomplished several objectives, including improved coordination of mathematics with client disciplines, creation of more opportunities for reinforcement of key concepts, decompartmentalization of mathematical ideas, introduction of probability and statistics into the foundation courses, and improved use of applications. Our traditional four-semester sequence (through ODEs) has been replaced by a sequence of eight half-semester modules arranged in a carefully-coordinated sequence. Assessment has shown that students have improved retention, and improved understanding of the connections between different mathematical topics and between mathematics and applications. The curricular design and implementation was accomplished as part of the NSF-Sponsored Project Intermath.

Main Theme: Technology

Secondary Theme: Innovative Teaching

High-tech and Low-tech in teaching differential equations

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The differential equations course provides many opportunities to make effective use of computer technology and other teaching strategies to illustrate concepts and applications. In this talk we will show how the software package ODE Architect (J.Wiley & Sons) can be used to enhance the course, and extend the range of student experiences. In particular, we will illustrate how the availability of powerful numeric solvers, coupled with graphic animation tools, can be used to more effectively introduce realistic mathematical modeling applications to the ODE course. Beyond the use of high-technology, we will also show how simple physical demonstrations can be use as a highly effective device to enhance the delivery and impact of course material.

Calculators as a Tool to Develop Number Sense for Prospective Elementary Teachers

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Calculators are common, in school classrooms at all levels. Yet, while the technology keeps improving, few teachers do more than simply allow students to "check their work", which often leads to an over-reliance on the calculator for basic skills. The majority of the students in my math for prospective elementary teacher courses are dependent on their calculators for basic calculations. They are unaware of how to use calculators as a tool for developing number sense. Since most elementary teachers did not see their own teachers use calculators appropriately, and because most teachers "teach the way they are taught", it is essential to address this issue in both their mathematics content and methods courses. The two "Guess my Number" estimation/mental math games that we'll investigate during this presentation encourage students to think about number relationships, patterns, and operations, instead of computational skills. They would not be practical without a calculator to do the frequent computations. Since different models of calculators have different operating systems, bring your own calculator to determine how it would work with these activities.

CAL2000-A System for generating multiple choice questions on Mathematics and delivering them by Internet

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In the first semester 2000/2001 we led an experiment at IST, Lisbon-Portugal, during which we used the CAL2000 system on 400 students of Linear Algebra (we are using it this year for the second time). The results of this experiment were very impressive. During the whole semester students were highly motivated and kept asking specific questions about the subject matter. The CAL2000 system allows one to write model multiple-choice questions, and generates random instances of these questions, thus producing individual web-based exercise lists for students to solve and can be used for assessment, and as a training basis for first-year undergraduates. During the last two years, we created a database of multiple-choice questions on Linear Algebra. The text of each model question depends on parameters and is written in the Mathematica programming language [1]. The program in Mathematica randomly determines the parameters, and also determines the right answer. In this way we are able to get different instances of the same model questions. We do this for five to eight model questions, on the same subject and with different difficulty levels, creating individual exercise lists that are posted afterwards in a web page. It is possible to use the multiple-choice questions generated in this way in a more network-like interactive way. The main purpose of the interactive quizzes is then not evaluation, but to teach the student how to find and correlate knowledge by solving exercises. The system CAL2000 has further possibilities to cover other subjects. For instance, we are already constructing a data base of multiple-choice questions on Calculus II for first-year undergraduates. We also plan to create a database of interactive quizzes on Linear Algebra and Calculus II. In addition, we are currently improving the interface with the teacher in many ways: selecting the questions for a test in an easier way, entering student data and entering student scores in the grade book.

Main Theme: Preparation of Teachers

Secondary Theme: Curricula Innovation

Enhancing mathematics teacher programs and responding to the shortage of mathematics teachers

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Through the Department of Mathematics the author has spearheaded many innovative courses and programs to improve the mathematics education of future teachers at all levels. This work has been recognized by a joint appointment to the Brock Faculty of Education. As co-chair of the Mathematics Education Forum of the Fields Institute for Research in the Mathematical Sciences, he has motivated strategies to address the shortage of mathematics teachers in Ontario. This presentation will consider the following: Too many middle school teachers in Ontario show a lack of understanding of and enthusiasm for mathematics. In 1990 the Mathematics Department, with the collaboration of other Science Departments and the Faculty of Education, instituted a unique program for middle school teachers. To teach at the secondary level in Ontario an individual must present two subjects, a first teachable (a minimum of six university courses) and a second teachable (minimum of three university courses). Half of the teachers in Ontario teach mathematics with a second teachable qualification and with mathematical experiences gained in Service Courses. The Department of Mathematics has reviewed its programs and opened appropriate courses to students wanting mathematics as a second teachable. Teacher education in Ontario is principally consecutive, namely, teacher candidates apply to a Faculty of Education after a first degree. There are no mathematics requirements to qualify for elementary school teaching in Ontario. The author has instituted a mathematics course for future elementary teachers who did not complete their highschool mathematics. This course is now required by the Brock Faculty of Education. Ontario is facing a shortage of mathematics teachers. For three years, the Mathematics Education Forum of the Fields Institute has been developing strategies to address this concern. It is hoped that the sharing of these developments will help others to implement changes within their own educational systems.

To teaching of new technology of design of some problems for ordinary differential equations in high schools

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The technology of design of two-point boundary value problems for ordinary differential equations, containing also a boundary layer effects is elaborated, using [1]. The proposed methods essentially refine and enlarged a class of algorithms for solving aforesaid problems. From these methods follow also classical methods, including methods of Collatz, Henrici, Marchuk, Schroder, Tikhonov-Samarsky, finite elements and exponentially fitted methods. Then the program part is realized in the form of package of applied programs consisting of control program and modules. For fulfilling of this work we followed the manual [2] with its software that was kind given to us by Gilbert. Some parts of this technology are systematically inculcated on teaching process and not only on the basic courses and also for student's course and diploma works at the Javakhishvili Tbilisi State University, Vekua Institute of Applied Mathematics, University of Delaware. The contents of the report besides the scientific sides presents an effective manual, realizing purposes, which are stipulated by the conference.

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Using Web-Based Interactive Graphics to Enhance Understanding of Parametric Equations: Lessons Learned

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Like many instructors, we have increasingly employed Web-based resources both for in-class demonstrations and for use by students on out-of-class assignments. Web-based interactive resources have the potential to enrich learning in ways that print resources lack. Furthermore, web-based interactive resources offer several advantages over other technological devices: they can be more flexible than graphing calculators, need not require students to learn new software, and (in principle) can be run on any networked computer using any Web browser. As an example, many standard college algebra and calculus courses introduce curves given by parametric equations $x(t)$ and $y(t)$. Students are often puzzled by the relationship between the parametric equations and the resulting curve. Graphing calculator features such as TRACE provide students some ability to explore relationships. In addition, graphing calculators can be used to examine the graph of x as a function t and the graph of y as a function of t . However, it is unwieldy on graphing calculators to display all three graphs (x , y , and the curve given parametrically by x and y) simultaneously to examine relationships graphically. Web-based interactive graphics provide the possibility of displaying all three graphs simultaneously, on separate sets of axes, with standard features such as TRACE still available. Use of such a tool has the potential to enhance not only understanding of parametric equations, but also the ability to analyze graphs and relationships more generally. In the process of employing such Web-based interactive graphics, we have learned some lessons about issues such as: "How do students interact with Web-based interactive graphics?" and "What kinds of activities facilitate learning with such graphics?" In this paper, we will show examples of Web-based interactive graphics that we have used, like <http://www.math.ou.edu/~tjmurphy/Teaching/2433/2433.0102/2433.html> and we will offer experience-driven recommendations for future implementation and development.

Stiff Differential Equation: an Introduction for Undergraduate Students

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Stiff systems of differential equations occur in all branches of science and are particularly prevalent in modeling chemical processes arising from chemical engineering problems. General purpose software packages for the numerical solution of such systems have been available for more than a decade, however, not all of these give satisfactory results nor are they readily available to a general user. Recent programming advances and generally available user-friendly software such as MATLAB and Mathematica now put the solution software in the grasp of the undergraduate student. This paper discusses stiff systems and provides an overview of where they arise and how the systems have been solved and with what software. In addition, we discuss three standard examples and solve them using MATLAB and Mathematica, giving step-by-step procedures, so that an undergraduate student can implement the software on other problems simply by modifying the commands to suit. The graphics capabilities of the software packages are exploited to discuss and display features of each example.

Developing a pedagogic discourse in the teaching of undergraduate mathematics: on tutors' uses of generic examples and other techniques

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This paper reports from a research project at Oxford in the UK that focused

- (a) on university mathematics teachers' conceptualisations of first-year undergraduate teaching related to observation of their teaching; and
- (b) on issues relating the conceptualisations to mathematics as a discipline.

This research builds on a qualitative study of learning difficulties of first year undergraduates in their encounter with the abstractions of advanced mathematics within a tutorial-based pedagogy. Six tutors' responses to and interpretations of such difficulties were studied in semi-structured interviews conducted during an 8-week university term and following minimally-participant observation of their tutorials. We describe a 4-stage spectrum of pedagogical development (SPD) that emerged from the analysis of the tutors'

1. conceptualisations of the students' difficulties;
2. descriptive accounts of the strategies they employ in order to facilitate their students' overcoming of these difficulties; and,
3. self-evaluative reflective accounts regarding their teaching practices.

We then exemplify the third and fourth stages of SPD with regard to (2) through a discussion of characteristic examples from the interview data. In these stages the tutors' strategies begin to resemble less a traditional induction process and more a process of facilitating the students' construction of mathematical meaning. In our discussion we employ tools from sociocultural, enactivist and constructivist theories on the teaching and learning of mathematics. In particular, the data used here exemplify certain tutor strategies such as: encouraging the students' use of rich and evocative verbal descriptions of mathematical concepts, properties and relationships; using generic examples and offering genetic decompositions to create and reinforce concept images of newly introduced concepts; highlighting the transferability of a technique rather than dwelling on mastering its execution; employing empathetic methods (pretend ignorance of sophisticated methods) to achieve consideration of students' needs. Overall we propose SPD as a useful pedagogic descriptor of undergraduate mathematics teaching.

Main Theme: Education Research
Secondary Theme: No secondary theme

The Use of Concept Maps to Compare Pre-service Elementary Teachers from Canada and Lebanon on their Understanding of how Children Learn Mathematics: A Cross-Cultural Study

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Concept maps are used to assess the organization of learning by tapping into learners' cognitive structure and understand specific educational concepts in the subject matter (Novak, 1984). The use of concept maps has been limited in their assessment of pre-service elementary mathematics education students. This study seeks to compare pre-service elementary Canadian (n=29) and Lebanese (n=29) schoolteachers on their understanding of how children learn mathematics through concept maps. While aptitude and abilities have been used to measure domain specific differences among mathematics university students, little work on process skills has sought to understand pedagogical differences among schoolteachers of different cultures. The study leads us to judge the quality and effectiveness of teacher-training programs with the assumption that a skilled teacher who understands how students learn mathematics is a prerequisite to effective teaching (Schroeder and Lester, 1989). The maps will be scored through the structural scoring method described by Novak and Gowin (1984) who consider higher levels of structure within the concept maps. Points are given based on the number of hierarchical levels and cross-links identified on the maps. Raters will score the maps to establish the construct validity and reliability of the scoring procedure. Through identifying cross-cultural differences in mathematics education, this study aids educators and policy makers towards a better planning and pedagogical perspective that bridge cross-cultural epistemological gaps in mathematics education. We further hope that the study will orient a universal mathematics education program that supports the individual as a critical and reflective thinker, actively constructing mathematics by juggling, and relating to previous and new ideas on a way to build meaning to understanding (Brown, Collins, & Duguid, 1989; Wittrock, 1990).

VISUM: Virtual Seminar fo Education in Mathematics

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VISUM - the (VI)rtual (S)ystem for Ed(U)cation in (M)athematics is a fast growing knowledge base containing material about teaching mathematics (URL: <http://visum2.uni-muenster.de>). The project is funded by German ministry of science and education with 1.6 Million Euro and will cover mathematical as well a didactical content for the education of student teachers from primary to higher secondary level. Mainly written in German, the content will be translated into English in the coming years. When preparing knowledge for presentation in an internet based multimedia system, special methods are needed to avoid - for instance - the "lost in hyperspace"-problem. The designer's answer to this problem is the so called object oriented theme analysis which is based on ideas coming from computer science, but adapted to the analysis of didactical knowledge about certain topics (e.g., arithmetic in the primary level, working aids, ...)The lecture will present this method, give a survey of the constructivist background of the system and the role of media (video, audio, ...) in the system, and show examples. The VISUM software creates a navigation platform which can be used as an authoring tool by universities worldwide - at the time of the conference there will be an online authoring tool for this purpose available. So, the lecture will contain an invitation to take part in the creation of a world wide network for teacher education in the field of mathematics.

Integrating Mathematical Knowledge and Pedagogy in Teacher Preparation Programs

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It is often assumed that because the topics covered in K-12 mathematics are so basic, they should be easy to teach. However, a number of mathematicians and mathematics educators researchers have recognize the special nature of the mathematical knowledge needed for K-12 teaching and its implications for the mathematical preparation of teachers. By teaching mathematical contents alone, the needs of pre-service teachers are not necessarily met. Knowledge is not something that can be handed down on authority. The greatest obstacle some of these students may need to overcome is a poor attitude toward mathematics. Perhaps one of the most important outcomes of a pre-service mathematics education course is to instill a sense of confidence in preparing elementary education majors. Confidence comes with the successful completion and understanding of material and concepts. Self confidence built on success is perhaps the most important objective of a good mathematics education curriculum. Pre-service teachers must have a better appreciation of how mathematics has practical applications in every day life. They can no longer afford to learn mathematics as a set of rules to be memorized with no utility in the real world. Reform oriented teaching requires that teachers have a deeper knowledge of the pedagogy as well as the subject matter. Teachers must be able to engage students in hands-on activities by using materials such as pattern blocks, base ten blocks, geoboards and technology to help them to understand the meaning behind the formulas. The presenter will bring examples that she uses in her mathematics content courses for undergraduate pre-service teachers to help students to develop problem solving skills, conceptual understanding, and knowledge about how to connect mathematics with real life situations.

"All of a sudden they got it": Understanding preservice teachers' perceptions of what it means to know (in) math

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In a recent study at the University of Regina, preservice teachers were asked questions about their internship experiences of teaching mathematics. One question in the study focussed on asking preservice teachers to recall their most meaningful experiences in the mathematics classroom during their internship, to which many responded with stories of how their students all of a sudden just "got" a concept and how this could even be visually detected. It is interesting to note the comparisons between their responses to this question about meaningful experiences and their responses to other questions concerning their images of math as a subject, their attitudes toward math, and their perceptions of what it means to know (in) math. Factors other than ability influence students' approaches to challenges, their persistence (or withdrawal) when facing difficulties, and how they use cognitive skills. This paper explores goal theory and achievement motivation as a perspective for examining the issue of what it means to know (in) math. The question of the role of the teacher in how students focus their efforts in mathematics classrooms, or in setting the classroom climate, is also of significance to this discussion. This paper presents implications for the changing needs of teacher education programs, including the contexts of mathematics education courses as well as critical issues in curriculum development and implementation in general.

**A study of classroom processes related to the production of meaning for 'function':
the context of Real Analysis vs the context of Dual Vector Spaces**

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Usually we expect our students to produce meaning for functions in Real Analysis as 'a correspondence between sets of real numbers'. In Algebra we generally start with function as 'a particular subset of a cartesian product', but when working with dual vector spaces we expect them to understand functions as elements of the base set of an algebraic structure. While our teaching experience had already confirmed the results of previous studies showing that students remain attached to the 'analytical' understanding of function, we decided to conduct a study that could further our understanding of this process. This study happened in the context of a regular Algebra course (undergraduate mathematics degree) particularly the section on duality of vector spaces. The data we will present and discuss come from transcriptions of lessons and from tests applied during the course. The theoretical support comes from:

- (i) EP ('Ensenhanza Problemica, in Spanish), a didactical model developed in the former Soviet Union during the second half of the 20th century, based in the historical-cultural theory of Vygotsky, and which provides us with a set of categories that allows us to organise in a dynamical way professor-students interaction; and,
- (ii) the Theoretical Model of Semantic Fields, developed by R. Lins (2001), an epistemological model that allows us to 'read' the processes of meaning production as they happen, 'on the fly'.

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The Development of Young Childrens' Algebraic Reasoning .

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Mathematics is a language ,the language is a life Mathematics is a language of communication, reasoning and modeling.Algebra is a logical language of mathematics . It is the language of mathematical reasoning .The key of algebra learning is the children construction of algebraic reasoning which refers to the sense /understandin/ modeling of the algebraic concepts : function,Equation,quantity,inequality,graphing,...., all possiple patterns /structures. The fundamental approaches to constructing of algebraic reasoning are the patterning and diagrams. The goal of the paper is to provide a model of young children development in algebraic reasoning ,as a cognitive framework .This model consists of four stage: Intuitive (non logical),induction(semi logical), abstract (semi logical) ,and formal (logical). The paper emphasis the sequential development of the children algebraic reasoning on three - dimensional standard : Conceptual ,Logical ,Application .The paper presents multiple examples of children use of the patterns and diagrams to develop this kind of reasoning.

Web-based Teaching and Learning with math-kit

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In this article we present the concepts and first results of math-kit, which is developed at the universities of Bayreuth, Hagen, Hamburg and Paderborn. The research and development is part of the programm 'Zukunftsinvestitionsprogram' sponsored by the German government to introduce new media in university teaching. Math-kit is a web-based construction kit which provides professors and students with multimedia support for central topics in undergraduate mathematics. Moreover, math-kit is intended to close the gaps between the education of mathematics students and the training in technical disciplines such as computer science or mechanical engineering. Math-kit tools combine mathematical algorithms with examples from other disciplines and vice versa. It contains elements for motivation, exploration, applications and visualization. Professors can use math-kit to compose individual teaching units. Its elements can be combined in several ways to support different learning objectives. From the technical point of view math-kit is based on the Sharable Content Object Reference (SCORM) standard and uses XML as implementation language. The atomic elements of math-kit are called assets, small highly interactive components like Java applets. A learning unit can be built from assets, complete courses consist of different learning units. Other technical highlights of math-kit are the accessibility via the web and the possibility to use the computer algebra system MuPAD as mathematical engine. In contrast to other systems the elements of math-kit can not only perform numerical computations. With the help of MuPAD symbolic computations are also possible. The mathematical power of MuPAD can be used through the web without forcing the user to learn its complex programming language. An outline of the system structure and some examples will be given.

Project on critical thinking development using technology

Tatiana Oleinik

This paper presents the results of special courses given to undergraduate teacher students with mathematics and computer science as subjects. A general purpose of our study is to understand the possibilities of technology for the realization of the ideas, which we want to address in our project on the development of critical thinking. A basic point of our theoretical framework is to mould teachers' competencies as reflective decision-makers and lifelong learners. It is evident that today the problem of changing a person's social level demands special attention: only a widely educated person with SOFT SKILLS is able to flexibly restructure the direction and essence of his or her activity. It is evident that for this it is not enough to have broad information on the existing level of professional knowledge and to know optimal ways how to use it. There is no doubt that in order to reach the up-to-date level of professional activities, essentially new features of personality are to be formed, e.g.:

Ability to have constructive dialogues with partners, to defend one's own point of view, irrespective of other opinions, and to recognize its imperfection when opponents present sufficient arguments;

Openness to new information and to non-standard solutions of open problems, acceptance thereon, formulation of original ideas and rational selection from competing ideas;

Ability to comprehensively and deeply analyze and understand new information; to reveal inner features and causes of different phenomena; to distrust any directly perceived fact, to take a cautious attitude towards any first-hand perception of events;

Orientation towards self-diagnosis in respect to the degree of formation of different abilities and skills by comparing one's own results with extraneous standards.

According to this, the development of a master teacher is based on processes of reflection, research and change. Students have to learn new teaching techniques that will question how they currently teach and even what they currently believe about teaching and learning. Using the critical thinking strategies from class they will do some research on their own actions. Teachers have to reflect critically about school practice and students' learning, question conventional rules and carry out their research. This is very important for posing questions, analyzing and at last understanding of what is happening. In other words, master teachers have to find clear and fruitful ways to characterize and carefully consider possibilities or alternatives for action and thoughtful assessment of choices made and implemented. Hence the main goals of the project are

to develop open, collaborative, collegial, long term relations between teachers of various subjects which will expand the understanding of teaching and learning for all students and lead to a free flow of ideas between peoples;

to increase the capacity of students to think critically, engage in critical reflection, take responsibility for their own learning, form independent opinions, and show respect for the opinions of others;

to present practical methods of teaching based on philosophically consistent and theoretically sound ideas;to place teaching within a comprehensive instructional framework which guides instructional decision-making;
to empower the faculty to take responsibility for becoming model teachers who are able to reflect on students' thinking and learning and refine methods based upon those reflections.

**ASSESSMENT OF MATHEMATICS TEACHERS AND ITS EFFECTS
ON STUDENTS PERFORMANCE IN SELECTED UNIVERSITIES IN
AFRICA.**

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This study critically examined the level of preparation of Mathematics teachers and its effects on students performance in some randomly selected Universities in West and East Africa. Two set of questionnaire were administered, one for the teachers and the other for the Mathematics students. This include information on teaching aids/ lesson guide and arrangement of content of the topic to be taught as well as the evaluation techniques adopted. The students' questionnaire dealt with time for tutorials and practical, students attitudes, textbooks and other information technology materials available to aid teaching and learning processes. The statistical analysis results shows that majority of the teachers sampled depends solely on their old lecture notes and textbooks for teaching and failed to update it with current materials and new method of teaching Mathematics involving Computers and softwares. It was discovered that both the teachers and the students were adversely affected by the economic factors which has tremendous negative effects on research, teaching and learning processes. The paper conclude by recommending some economic policies that can enhance teaching, learning and research activities in the University system.

Racing to Keep Up

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Preparing secondary mathematics teachers for today's technology rich classroom environment is a continually evolving process. Classroom technology options have expanded from the once innovative graphing calculators and data-collection devices to include more all-inclusive software packages, video clips, and digital images. Coupling the increased number of technological options with the stronger technological background of today's students significantly influences how pre-service teachers are prepared. It is no longer sufficient to concentrate only on developing mathematics content knowledge. As new instructional tools emerge, active engagement of students in learning and doing mathematics through the use of real-world contexts becomes a larger and more realistic focus for teacher preparation efforts. Today's mathematics teachers are expected to demonstrate the ability to incorporate a variety of instructional strategies and tools as well as multiple assessment techniques in their teaching. How pre-service teachers acquire the knowledge and skills to meet such performance expectations has motivated teacher preparation programs to rethink and restructure how individuals are now prepared for teaching mathematics. As the needs of today's teachers change, so must teacher preparation programs. Specifically, this paper will focus on the interplay among instruction, assessment, and technology in light of current teacher preparation realities. A synopsis of tools and techniques employed and experience gained will be presented.

"THALIS" A REPRESENTATION SYSTEM FOR UTILIZATION IN TEACHING AND LEARNING FRACTIONS

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In this paper we present an attempt to promote the development of teachers' own intuitive mathematical knowledge of rational numbers (1) with the use of a new teaching tool which we call Thalís. For the construction of this tool we have taken under consideration children's difficulties to produce adequate intuitive models to represent rational numbers and operations with them. To teach concepts of rational numbers we usually use representation systems which are either not self-consistent, since they are capable of producing contradictory situations or self-consistent but over-specific(2) since they are capable of producing multiple representations of a problem's solution. This paper recommends a new representation system for the teaching of rational numbers which has the form of a natural transformer. This system does not allow for any misconceptions, since, as it will be discussed, it is a model of the field of rational numbers. Moreover, since it is a natural transformer, it permits authentic measurement activities and ratio computations in school contexts. With this new system an improvement is expected in:

1. Children's ability to experiment
2. Teachers' ability to plan constructivistic activities for the teaching of rational numbers.

The paper presentation structure for Thalís will be the following:

1. Representation systems of rational numbers
2. Informal presentation of Thalís
3. Examples of Thalís use in representing some operations of rational numbers
4. Discussion about Thalís being a model of the field of rational numbers.
5. A teaching script with the use of Thalís

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E-learning in Mathematics undergraduate courses (an Italian experience)

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E-learning in Mathematics undergraduate courses(an Italian experience) The teaching of mathematics in Italian Universities is undergoing a period of deep transformations, partly due to general reasons and partly to national ones. The strongest drives are probably:

- 1) a recent reform of the Italian University system, which leaves to each single University more autonomy and decisional power than in the past;
- 2) the deep changes occurred in the past decades all over the world in the perception of the relations between mathematics and its applications;
- 3) technological innovations, and the major changes they imply both in teaching methods and in the mathematical contents we teach.

In this context, in March 2001 a group of ten mathematicians and computer experts working in Universit¹ Bocconi in Milan - a well-known business University - started a project focused on the integration of heavy e-learning technologies into the traditional structure of Mathematics courses for undergraduates. We would like to present at Creta ICTM-2 Conference a comprehensive description of our work: the project (March-July 2001), the courses (September 2001-April 2002) and a first analysis of the results (May-June 2002). We chose to present at the Conference three independent papers (see also the abstracts by M. Impedovo and F. Iozzi); each of them takes a rather different point of view. The first part of my paper deals with a general description of the whole project, following the above framework. The second part analyses the parts of the project in which I was more deeply involved:

- 1) a complete e-learning course, specifically dedicated to students with poor performances in mathematics (approximately 100 students);
- 2) a basic e-learning course, to be furtherly developed next year and to be dedicated, presumably, to all first-year students in Bocconi University (approximately 2500 students).

Main Theme: Mathematics And Other Disciplines

Secondary Theme: Technology

An Introduction to Symbolic Math Guide: an Innovative Way of Teaching and Learning Algebra Using TI-89 and TI-92+ Graphing Calculators

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In this presentation, we will introduce Texas Instruments' Symbolic Math Guide (SMG) - a Computer Algebra System (CAS) freely available for users of TI-89 and TI-92 Plus graphing calculators. SMG was written to address concerns regarding the "default" symbolic manipulation software included with TI calculators - in particular, the "default" software's use of non-standard algebraic notation and tendency to skip intermediate algebraic steps. As they use SMG, students select transformations to apply to algebraic expressions. Because the application requires students to focus more attention on the conceptual underpinnings of algebraic simplification and equation solving, we propose that SMG helps students learn how to manipulate symbols more efficiently and more effectively. While exploring algebraic examples in both "default" and SMG-based environments, we highlight SMG's faithfulness to the mathematics and mathematical notation found in school textbooks. After specific features of SMG are introduced, the advantages and disadvantages of SMG will be discussed. SMG can be utilized with both secondary school students and college students in remedial classes. Two possible uses of SMG will be presented: simplifying expressions with powers and simplifying radical expressions.

From The Inca's Quipus to The Yupana The Inca's Computer

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The search of what is the Ethnomathematics and his true validity in this new Century is a challenge that impels us to modernize ancestral knowledge that, even today in day they continue having validity in the praxis of the people, in his daily life. That incessant and substantial search, until arriving in retrospective form to the origins of the prehispanic cultures, to the incario and, there we find the "Yupana" and the Incan Quipu" as the usefull instruments in the countable practice of the Incario. (Graf. 1) What is the Quipu and What is the Yupana? "Quipu" in Quechua means knot and, particular way was used in the incario to name to the knotted strings whose utility was the serving like mnemonic records that represented quantities. In what concerns him, we don't seek to make a study rigorous of what was or it is, we limit ourselves from to general way to know their origin, construction, chromatism, categories, utility that he lent. The "Yupana" is a board to carry out bills, its name also comes from the Quechua. With the Incan "yupana", we will make a detailed study, seeing their origin, their relationship with the quipu and the positive aplication that can give us at the present time. We will try to see three main topics: a first part on the nature of the quipu; the second, the study of the Yupana and third, their application by means of examples and practical exercises adjusted more to real situations of the learning of the arithmetic operation.

"First Ethnogeometry for the Ethnomathematics"

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In this last decade the Ethnomathematics has been presented, like a new current of the mathematical knowledge, trying to rescue the values that the people and its culture have. This current is view for some with certain scepticism and for others as the new alternative for the learning of the Mathematical one.

WHAT is Ethnomathematics? Although there is a long list of authors that they try to give an exact definition, we will make it according with the concept of. D'Ambrosio, The ETNHOMATEMATICS is in his Natural and Cultural environment: Explain, Teach, Understand, Manage with Arts, Tecnic. Barton says: "The different forms of mathematical that are characteristic of the cultural groups, we call them of Ethnomatematic and from our vision Ethnomatematics is not Mathematic, it is the mathematical of the people like the group of mathematical, practical and theoretical, produced or assimilated and effective knowledge in his respective sociocultural context that supposes the processes of to count, to classify, to order, to calculate, to measure, to organize the space and the time, to estimate and to infer."

WHAT is THE ETHNOGEOMETRY? It is the Study and knowledge of the Geometry under the cultural aspect of the nations comparing their likeness of cultural or social anthropology and of the civilization knots that it characterizes them." We also take the semiological sense of the concept, they refer to the town, to people of our days, to us a daily practice of the geometric application is made now, therefore in almost all its chores. The Etnogeometry can create"... a bridge between the Mathematical one and the ideas (concepts and practical) of other Cultures.

Using Interactive Digital Video and Motion Analysis to bridge Abstract Mathematical Notions with Concrete Everyday Experiences

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In an attempt to offer a means for better visualization and conceptualization of abstract mathematical notions, we investigated how the analysis of motion contained in a digital video performed in a special computer software environment, can help students increase their understanding on specific topics. Previous research on Digital Interactive Video Technologies (DIVT) was limited to the domain of kinematics and graph interpretation in particular. It was the conviction and in some cases the conclusion of those researchers that students would benefit more from the study of everyday motion as presented in a video, rather than in simulation software. We believe that this is particularly true in the case of mathematics teaching, where students often have difficulty in perceiving the meaning behind an algebraic or graphical representation. Pre-service teachers need to gain a profound understanding on such abstract concepts, as those are usually the ones they have more difficulty teaching. This pilot study is part of a full-scale research that aims to

- 1) extend the field of investigation using Digital Video Technologies as a connecting link for the Integration of Mathematics and Science,
- 2) investigate how different dynamic software environments that offer advanced visualization options affect students' learning of mathematics.

This paper is the report of the first part of the pilot-study, where the main aspects of teaching with the aid of DIVT were investigated. Five pre-service teachers participated in this study, which consists of two parts, one without and one with DIVT support. The analysis of data gathered indicates that being able to manipulate the reference frame in the environment of the DIVT software and notice how it affects coordinates, graphs and equations of motion, had the greatest impact on the pre-service teachers' understanding on this subject.

Developmental Mathematics.

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The American faith in egalitarianism has resulted in open admission policies to colleges and universities. As a result to that, almost all higher education institutions, save community colleges, require the results of national examinations, which include mathematics, for admission. Yet, the fact is that most of them use such scores as a point of reference rather than as a definitive criterion for admission to their programs. American higher education institutions have developed special programs, as a rule called 'general education,' which include at least one course in mathematics, that all their students must take and successfully complete regardless of their academic field of studies. But as a result of the practice of open admission, a significantly increasing number of students is unable to master even the lowest collegiate mathematics course. As a result of this situation colleges and universities have initiated programs in mathematics that are known by the collective term 'developmental mathematics.' Students who are not able to go directly into collegiate level mathematics are channeled to the developmental programs. These programs are not identical from institution to institution but certain common concepts and practices run through all of them. Besides the on campus developmental programs, higher education institutions are attempting to meet this problem by distance learning techniques, in two ways. One, by offering electronically assistance to small and rural high schools which, because of either financial resources or personnel problems, are unable to offer a strong mathematics curriculum. The second way is to provide electronically transmitted developmental courses to adults planning to enter university education (as nontraditional students). To that end, state governments are increasingly moving to assist higher education institutions in developing electronically transmitted programs for distance learning. An example of that is the fact that Northern State University has been selected by the governor, and funded by the legislature, of South Dakota to be the state's public e-learning institutional center.

UNCLE PETER AND HIS STEREOTYPING NEPHEW/NIECE:

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Number theory is not offered as an autonomous course in the mathematics department of Patras university. Some Number Theory elements are included in algebra courses. Students learn to prove formally given propositions, but they are hardly asked to check or to refute given statements by producing a counterexample. To see to what degree students could reach this target, a story - problem inspired by a best seller titled «Uncle Peter and Goldbach's Conjecture» was given to them as a test in mathematical problem solving. In this story, Uncle Peter states the following conjecture: Let A be any even number greater than 2, and let p_A be the greater prime number less than $A-1$. Then the number $q_A = A - p_A$ is also prime. Students should check this conjecture and propose a method of testing similar cases. We analyzed the answers of 40 students (2nd- 4th year). Some of them were interviewed after the test. 16 of the 40 students (40%) examine no numerical example at all, while 11 of these 16 students tried to prove or disprove the conjecture by using irrelevant stereotyped methods taught at school (such as mathematical induction or reduction at absurdum); 22 students (55%) examined one, two or a whole series of non-critical examples, while 15 of them also tried to use stereotyped methods; one student reached a counterexample without reporting it, while at the same time this student was also stereotyping. Only one student reported a counterexample and used it to refute the conjecture. Thus there seems to be a tentative stability in the proportion of stereotyping students among those who do not examine any example at all (11/16) as compared to the proportion of stereotyping students among those who do examine non-critical examples (15/22). Another astonishing result is that the proportion of students who are unable to choose between examining examples and using stereotyped methods seems to increase with the number of examples used.

INTEGRATING SYMMETRIES OF POLYHEDRA IN THE CONTEXT OF REAL SPACE EXPERIENCE:

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During Antiquity and Renaissance and more dramatically with Kepler and his Platonist view of the solar system polyhedra have appeared in multiple studies of real space. Many artists, philosophers and technicians have introduced polyhedral symmetry in their constructions. However, such kinds of symmetry and related subjects have hardly been included in geometry curricula. According to a classification due to the didactician Guy Brousseau, spatial situations belong to three main kinds of ·conceptions· of real space: these are the macro-space, the meso-space and micro-space ([1], p.70). Although polyhedra are firstly met in micro-space experience of the child and then in meso-space experience with buildings, cars etc, modern scientific practice decontextualizes and ·extends· this process by using polyhedra as a direct link between the ·micro· and the ·macro· level (this is particularly evident in the theoretical method of approximation of smooth surfaces by convex polyhedra in modern topological geometry). To search for all symmetries of a given polyhedron P means to try to determine the group of all rigid motions of Euclidean space that leave P setwise fixed. This task is contextualized among others (starting from the easier task of determining the symmetries of a polygone) in a project addressed to final year students of the department of mathematics at Patras University as an innovation in the teaching of geometry. Another task of the same project is to review some of the vast existing literature on the relation between art and symmetries of polyhedra. Students are encouraged to construct and use concrete models of their own and they are interviewed in several phases of the project. Also their 2-dimensional representations of (3-dimensional) rigid motions are selected and analysed. A sample of students' work will be discussed during our presentation.

[1] G. Brousseau, ·Les propriétés didactiques de la géométrie élémentaire: l'étude de l'espace et de la géométrie·. Proceedings of 2nd Colloquium on Didactics of Mathematics, Rethymnon, 2000.

Computer Science students need adequate mathematical background

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With the rapid development of Computer Science, the role of Mathematics in the undergraduate CS programs tends to get smaller and less important. Students are getting minimal mathematical background these days. This is becoming a topic of major concern of a big army of computer science educators. Different studies in the field of Computer Science Education establish how important mathematical thinking in computer science is, and how it affects the problem solving ability of the future programmer. Our goal in this paper is to show the necessity of a deeper mathematical background in the study of basic theory of Computer Science. The lack of adequate mathematical knowledge makes computer science majors incapable to read and understand commonly used textbooks, define concepts, express their thoughts precisely and unambiguously. There is an increasing fear of any text containing math notation and formulae. All this brings to a growing hostility of students not only towards Mathematics, but also towards theoretical courses of Computer Science. In this paper we'll assess the significance of students' mathematical background characteristics on their ability to understand and appreciate theoretical aspects of Computer Science, particularly the Theory of Computing. We'll introduce statistical data, presenting the relationship between the grade in Theory of Computing and the number of math courses taken before enrolling into it. We'll also introduce a set of recurring mistakes in tests and discuss the reasons for students' poor performance, linking it to the insufficient amount and ineffective distribution of mathematical courses in the CS curriculum. This study was motivated by observations made throughout years of teaching in two very different Computer Science Programs - heavily math flavored program of former Soviet Union's Higher Educational system, and almost math-free program of the California State University system.

The Interactive Learning of Geometry

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Solving problems in geometry in a plane using congruencies such as translation, rotation, central symmetry or reflection belongs to the most difficult parts of basic and secondary school mathematics. To make this problem easier interactive manipulation with geometrical figures can be introduced. In this article we present a text, which has been developed at the Pedagogical Faculty, University of South Bohemia with the co-operation of trainee teachers of mathematics. The text is written for users who do not possess a computer but is especially suitable for those who are able to take advantage of interactive geometrical software - Cabri, Cinderella etc. There are two ways to use the text: One way is classical and is as follows. Each example starts with the formulation of a problem. The situation is drawn and analysed. Afterwards the solution is described and the sequence of steps of the construction is given. Finally the whole construction is performed and the figure is drawn. The other way of using the text begins with clicking on the logo for Cabri (or Cinderella) which is placed in the corner of each example. We get the same situation as in the previous case, but now the diagram is interactive. We are able to move objects and, for example, change the initial position of given points or lines. Clicking on the analysis icon gives a drawn solution. Moving and clicking with the cursor in help mode gradually reveals step by step a solution of the problem. Such a process has a number of advantages. Very useful is the continuous help, which encourages students to think about the problem. Another important aspect has students watching continuous transformations e.g. turning a line by 60 degrees, leading to a better understanding of the problem and its solution. This text is currently being tested both by our students at the university and by students and their teachers at several secondary schools.

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Main Theme: Distance Learning

Secondary Theme: Education Research

Teaching an Online Statistics Course

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This presentation will discuss the statistics course that I have taught online for the past four years. I will discuss how I facilitate student communication and interaction in an online environment. I will also discuss the structure of the course and the ways in which I and the students communicate mathematical ideas and formulas. I will summarize with the positive and negative aspects of teaching online.

Integrating Web-based Maple with a First Year Calculus and Linear Algebra Course

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This paper reports on extensive work carried out over the last eight years at the University of Queensland to adapt Maple to each of the topics of a first year Calculus and Linear Algebra and the results of this implementation. The course has about a thousand students mainly engineering and science students with a few from biological science or arts. Most students start with little if any CAS skills, though some have used Derive or graphics calculators at school. Each topic in the course is introduced by discussion until the analytical background is established. Once this has been covered and digested Maple applications are illustrated on the computer in the lectures and then students work through similar ideas and extensions in their next lab tutorial. Each student has a one hour computer lab every week. From week two students are introduced to Maple and they can work through the twelve tutorials at their own rate, though one a week is recommended. The tutorials are on the web and students can download them. Week one provides an introduction to Maple followed by introductions to arithmetic, algebra and calculus so that, by week five, students have some understanding of Maple commands and syntax. The next tutorials take students through Taylor and Maclaurin series and their uses in approximating p and e and sine, cosine and log functions. Tutorial 7 is a tutor marked test which allows students to judge their progress. The last tutorials cover numerical integration and then Linear Algebra, including vectors, matrices, linear independence, Leslie matrices and the start of programming and finally eigenvalues and eigenvectors. Projects include practical applications to numerical approximations and using Leslie matrices for predicting changes in populations and dominating eigenvalues to estimate asymptotic distributions. This paper tries to evaluate the advantages and disadvantages of such an approach.

Accumulation and Numerical Integration in Calculus Courses

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Technology in the hands of students has allowed calculus courses to introduce many topics from numerical analysis. One example is numerical integration. In addition, many calculus courses can now consider the discrete analog for calculus with continuous functions. Digital collection devices in the lab recover a discrete sample of some unknown continuous process. The discrete analog for integration is generally an accumulation. This talk will present some accumulation and numerical integration techniques that have made their way into popular U.S. calculus textbooks. We will briefly discuss what might be the appropriate balance between the algorithm and implementation details and the derivation and explanation of theoretical error bounds. One of the perhaps unintended consequences of the infusion of numerical analysis topics earlier into the curriculum has been the decline in enrollments for later courses concentrating on numerical analysis. This introduction to numerical integration in calculus will be followed only by applications of these techniques in later science and engineering courses for most students. Thus it is extremely important that instructors mention and demonstrate more accurate algorithms that are now commonly available on calculators and within mathematical applications on computers. Most texts still seem to emphasize small hand computations rather than larger, more accurate computations using available technology. Less important than detailed error analysis with theoretical error bounds is the repeated computation of the same integral with different step sizes or methods to give a computable error estimate. Experimentation with repeated computations can motivate the adaptive integration algorithms now commonly used. With discrete data, the step size may be fixed but you can still try different methods. Specific examples of how to implement many of these numerical techniques will be demonstrated on the TI-89 graphing calculator.

Teaching for Understanding for Teaching: Addressing the Challenge

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We propose to present findings from two TE mathematics classes that exclusively use a model of a 'mathematical situation' and a set of physical actions to reason about all students' mathematical doings. This constructivist approach 'forces' prospective teachers to understand their doings and to develop a reasoning attitude towards teaching and learning. A unique system of reinforcements, grading and assessment methods support the learning experience. Students are constantly engaged in verbally explaining each step of their doings and their motivation for doing it, while using only non-formal mathematical language. They are constantly required to relate various representations and doings across situations and across concepts, while investigating thoroughly a few situations rather than studying many repetitive situations. Simple whole number situations are used to establish a solid basis of understanding and to scaffold students' mathematical doings in more complicated situations. The students are constantly encouraged to confront their mis-doings and their fears, as well as to maintain high level of awareness of their thinking, doings and of their feelings. To illustrate: The students are encouraged to use JOIN instead of add, EXCHANGE instead of finding common denominator, to say CORRESPOND instead of compare, to use words like INCLUDE, BREAKDOWN, TIE (x to $f(x)$) etc.. For obvious reasons we can bring here only very simple examples that cannot fully exhibit the potential of our approach. We found this approach to be very efficient particularly in promoting students' understanding of fraction-s situations. Students that first resisted the teacher 'extremist' initiative eventually 'discovered' that "It's so simple, I can't believe I did not understand it before". However this approach calls for persistence in implementation in order to be effective, as "It's too hard" for the students, and for the most part they prefer "just tell me what to do and I'll do it".

Interactive animated problem sets in learning functions and binary operations

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A viable understanding of the concepts "function" and "binary operation" constitute a necessary background for learning undergraduate mathematics. Therefore, we have tried to "implicitly force" the students to rethink the definitions, especially the role of sets; the domains and the range/image. Here we use computer based exercises, which completely differ from conventional teaching and also from the usual ways to use e.g. CAS to visualize functions. The students faced with "playing" with the mathematical objects, like in e.g. Geometer's Sketchpad or Cabri, but they only have very restricted tools. Tools may be dragging points, controlling an animation with buttons or mixing these two, using draggable segments for measuring or receiving hints or warnings as response to their actions. The pedagogical background comes from the work of Shlomo Vinner & co. concerning "concept images" and concept definitions, combined with the trinity verbal-symbolical-graphical representations of functions. In October 2001 our freshmen had a three hour computer lab where we used an html-form containing Javasketchpad applet based interactive animations combined with problems targeted to clarify different aspects of the function definition. In some problems we asked whether a shown dynamical graphical representation describes a function, and if not, why. In many problems the domain and range were asked for, and in some the function rule itself was asked to be explained using a symbolical formula and/or own words. The html-form sent the students answers as email to us immediately they had finished their work. There was also a paper-and-pencil pre-test and a post-test including a detailed student evaluation of one particularly problematic applet construction. We shall discuss the outcomes of this experiment and a previous one concerning binary operations in linear algebra, especially in the light of distance education.

Main Theme: Preparation of Teachers

Secondary Theme: No secondary theme

Didactical deductive geometry for teachers

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In the autumn 2001 Department of Mathematics at the University of Joensuu started a special study unity called cum laude of didactical mathematics. Purpose is to develop the mathematics instruction aimed at preservice subject teachers and elementary teachers didactically towards the needs of school teaching. Especially we aim at developing such instruction that will help students to transfer the university mathematics to practical teaching work in comprehensive school and in high school. I am responsible for the course deductive geometry and I'm doing research, for my Ph.D thesis, related to this geometry course. The course will be carried out first time next spring 2002, during march and april. My aim is to create a computer-aided sosiokonstruktivistic learning environment and to research how students' know-how concerning mathematics teaching develops in this environment. In my presentation (paper) I'm going to tell about ideas which I use as a framework while planning that course, about research questions, research methods and in the summer I can also tell about first experiences and results.

Solving process of a narrative combinatorial problem: an exploratory study

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Problem solving is one of the main goals of the learning process as it concerns knowledge in action. It is regarded as the search of possibilities, evidences and goals, involving the production of inferences, arguments and strategies to validate or refute a statement. It is related to the way in which the student models the situation and applies or creates solving strategies. Formal and informal reasoning are activated in this process. Formal reasoning is generally associated to well-defined situations, where all the relevant data are given. It is based on logical inferences where the initial premises imply implicitly a conclusion. Informal reasoning, generally associated to 'open' situations, is not restricted by logical operations as it may include inferential processes (developed, sustained and evaluated by a system of beliefs or by common sense). In the present article, we discuss problem solving processes that are involved when students solve a combinatorial situation, written in a narrative style, and they have to recognize the relevant data and to decide possible solving strategies. These aspects were analysed in a sample of 70 students, with ages between 15 and 53 years old, that attended different Mathematics courses. Fourteen aspects were considered as variables, among them: data comprehension, information processing, completeness, explicit representations as support, searching criteria, solving features, formalization level, formalization tools, answer contents, answer type. Information has been obtained through the application of multivariate statistical techniques. This study leads to the construction of a typology attending to solving processes features. The classification analysis of the protocols allowed the identification of four classes, with almost the same number of constituents, which are identified as: bewildered, rough and ordered, heuristic and formal and tidy. They are discussed in terms of the two main bias that emerged through the study

Mathematics in a Liberal Arts Education

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The ancient Greeks understood the Liberal Arts to be Mathematics, viz. their Quadrivium: arithmetic, applied arithmetic (harmony), geometry, and applied geometry (astronomy). These "arts" are characterized by the signal fact that, unlike, say politics, their content can be *learned* --recall that the Greek root of the word 'mathematics' translates roughly as "that which is learnable". Mathematics is thus paradigmatic of all learning, and hence, central to any liberal education. But, alas, this view of Mathematics and the Liberal Arts has fallen on hard times. This fall is owing, at least in part, to the seduction of the self-assertive passion aroused by the spectacular success of mathematics as a utensil, an implement, in the service of Science. But it is also owing to our inability to deal constructively with the unavoidable tension between the inexorable press of politics on education (from Meno asking Socrates "can virtue be taught" to "education for citizenship" in our own time--to say nothing of programs, from the NSF to the DOD, that reward achievement in mathematics based *exclusively* on mathematics' ability to serve political aims) against the backdrop of freedom a Liberal Arts education requires. In my talk I will explore ways to respond constructively to this tension, chiefly through the lens of Mathematics viewed as central to a Liberal Arts education. The ICTM2, *in Greece*, seems like the ideal opportunity to present this paper.

Problem Solving and Technology as Resources in Calculus Teaching: Some initial learning outcomes and methodological suggestions

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In this work I present some experiences obtained in teaching Mathematics at the Faculty of Mathematical Physique Sciences at the Autonomous University of Puebla. The main methodology of the course was the problem solving based on the notebook of interactive geometry "Cabri Geometre II". The author adopts this methodology due to his experience as a trainer of Mathematical Olympiad students and to previous researches. Both have shown to be useful in developing students capacity for understanding of mathematical concepts and facts. On the other hand, I used versatility of the Cabri Geometre II software to elaborate of the dynamical constructions that allow students to visualise and understand the concepts of variable, function, limit and derived with much more clarity. Also this software is a powerful tool to observe the concepts like objects in the plane that move with regard to the time and interpret the functions as equations of movement, and analyze the limits as places to which approximate the object, and derivative as the speed and the second derivative as the acceleration, etc. The following methodological actions are considered:

1. We present to the students a problem and we request them it execute an action.
2. In the second step we use the software to simulate the situation with the purpose that the student understands better the situation.
3. The following step consists of considering different variants or modifications of the problem no too far from the original with the idea that the students observes invariant or patterns of behaviour or imagine any new trace which helps to visualise a possible way to the solution. It allows to find the appropriate strategy. Here we took advantage of auxiliary construction in order to determine which data are free and fixed (independent variable) and which other data may vary (dependent variable). We certainly also begin to introduce the concept of function.
4. The following step, consists of execute the strategy that the students have conceived as appropriate. It is an individual task.
5. The obtained solution is confronted with the solution that comes from the simulation. To do this, we appeal to the Cabri so that, indicates us graphically in the Cartesian plane, the geometric place that describes the dependent variable with regard to the independent variable. That is, the geometrical laces are compared.
6. In the last step, the students have to think about problems which involve the idea of approximation of the dependent variable values with regard to the approximation to any relevant independent variable values. This last step leads some student group to consider real situations (like, some kind of motions) in which the geometric place describes the position of objects with regard a particular frame of reference.

Teachers Training for Using Dynamic Geometry Packages in Teaching Geometry

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The article is dedicated to the questions of using Dynamic geometry software in training course for future math teachers in Kharkov State Pedagogical University. The development of information technologies cardinally changes requirements for teachers' education. The application of educational software packages allows coming close to the research work but it requires additional efforts and adequate teacher's qualification. The process of using such software brings up the following issues:

- 1) Learner's adaptation to the dynamic geometry environment, development of necessary skills and habits;
- 2) Comparative analysis of Euclidean geometry and its computer interpretation;
- 3) Evaluation of package capabilities for secondary school geometry course;
- 4) Methodological questions of students' worksheets implementation;
- 5) Selection and exploration of provoking, open-ended research problems;
- 6) Creation and analysis of geometrical models for constructing macros plays an important role in the development of algorithmic reasoning.

Educational Reforms and the Competencies of the Mathematics Teacher

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Under the context of the Educational Reform in Chile, throughout the past years, the scarce emphasis in learning of a higher type has been recognised while critically diagnosing mathematics curriculum. Specifically, at different educational levels there are no studies and assessment that indicate what professional competencies mathematics teachers have or should have, so as to get a better quantitative and qualitative improvement of students' higher learning achievement in varied areas of mathematics, and adapt to current scientific and technological changes. These changes are generally led by the teacher who acts as an agent developing varied didactic actions so as to reconstruct and recontextualise the mathematical knowledge in the classroom. In this context of Educational Reform, this research concentrates on the structure of knowledge so as to define frameworks of competencies for mathematics teachers that consider the knowledge/content of and about mathematics; the didactic/know-how of pedagogy, of teaching and learning and evaluation; the being/transversal with respect to values; the evolving/knowledge-being of what is adaptable (Poblete, Diaz, 2001). The frameworks of competencies, in turn, are characterised by dimensions connected to the quality of the teacher's professional competence, which are associated to indicators of relevance, efficiency, effectiveness, efficacy, processes and resources. This study uses a qualitative-interpretative research methodology that includes observing the teacher working in the classroom at primary and secondary levels. Besides this, self-assessment, peer assessment, administrative assessment, and students' assessment instruments are devised. Carrying out this study allows us to know the professional competencies that the teacher teaching mathematics performs and, by this means, obtain qualitative indicators that regulate the changing processes.

Saving Math Jobs: Keeping Math Courses within Math Departments

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There has been a trend for post-secondary math courses to move to other departments (statistics being taught in the business school, for example) and for math requirements to be reduced. Since math jobs are at stake, how can we stem or reverse this trend? In this paper, we talk about a successful curriculum innovation project involving calculus for business students, and some lessons learned about working with other departments and colleges. The project involved collaboration with the business school and members of all of its departments from the beginning. We first listened carefully to the needs of our client disciplines, both in terms of overall philosophy as well as specific topics. Then we looked to see what course concepts and texts already existed that might meet our needs, but soon realized that nothing really fit well and that we would have to craft a new solution ourselves. Our concept was to make a two-semester course with integrity that was problem-driven, and relate it to students' other courses, careers, and personal lives as closely as possible. We applied for and received grants from FIPSE, NSF, Villanova University, and Prentice Hall, which helped give us the time needed to develop new materials and the foresight and discipline to organize evaluations of the new course sequence. We worked extensively and sometimes agonizingly with an Advisory Committee from the business school as well as their Curriculum Committee and the math department, but made sure everyone was on board. We were careful to provide gradual and plentiful training and development for our math colleagues. The bottom line is that the new course has been a great success at all levels (student learning and attitudes, business school enthusiasm, and math faculty satisfaction). In this paper we will discuss details about our process and lessons learned.

The Method of Teaching Mathematics based on the Pedagogy of Love

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Any person who is committed in the teaching and the learning of mathematics or any other school matter should have his own philosophy. We call it the pedagogical method. This one individualizes any teaching person, from teacher to professor. A good pedagogical method

- 1) can be learned from an older professor,
- 2) can be constructed by using his own experience,
- 3) can be learned from books or
- 4) can be obtained as a combination of the former methods.

But any method fails if it isn't based on Love. What kind of love or love of what? The answer is simply to give, but is difficult to apply. It is about

- 1) the love of science and
- 2) the love for students and
- 3) the love for work. In a religious interpretation we should add a very important point
- 4) the love for God.

The whole enumeration should be read with AND not with OR. Hence, the pedagogy of love means love or nothing. Without love we can't obtain anything. In the work we develop the above ideas and we add the following topics:

- I). The church-prayers done in Romania at the beginning of any school year.
- II). The relation student-professor necessary for a good work. The amphitheater atmosphere.
- III). How to prepare a good course and to deliver a good speech? Examples.
- IV). The rules to prepare and to keep a good seminar. Examples.
- V). Teaching the students in a cooperative and collaborative method.
- VI). On the integration of new computing technology into the fundamental courses of mathematics.
- VII). The professor's authority in the middle of the students. Remarkable examples in Romanian history of mathematics.

Old ideas: a new tool for teaching basic Mathematics

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Incorporating the History of Mathematics in everyday teaching can be much more than just giving anecdotic events a place in our classroom expositions. In fact, there are ideas and methods used in the past which may be no longer of practical usefulness today, but may nevertheless help our students to grasp the meaning of a theory, an algorithm or a simple formula. Just as, for Elementary School children, manipulating pebbles is a useful way of visualizing some properties of the basic operations, like commutativity, for example, also working at the High School level with Euclide's " Geometric Algebra" or the egipcians' " False position rule" may be of great help for understanding relations and concepts more clearly. In fact, the possibility of realizing that concepts and ideas in different topics of Mathematics are connected is one of the most important benefits for the student exposed to the historic development of some mathematical ideas. On the other hand, the teaching of Mathematics as a field of knowledge which is ever changing, instead of as a rigid set of formulas and algorithms, is, besides a tribute to truth, a way of encouraging our students to interact with the ideas developed in the classroom, since they will necessarily be exposed to several different ways of solving problems, and therefore their creativity will be stimulated. After a 4- year experience in Mérida, Venezuela working with High School teachers who introduced their students to Algebra and Geometry using the approach mentioned above, there have been positive results, not only visible in their Math grades, but also in their general attitude towards learning Science. Some examples of the use of certain elements of the History of Mathematics in the exposition of basic topics in Algebra and Geometry of the High School level will be shown.

Main Theme: Innovative Teaching

Secondary Theme: Curricula Innovation

Demonstrations, experiments and supplementary notes to motivate students in differential equations courses.

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A series of low-tech, low-cost experiments that drive and motivate a typical introductory course in differential equations will be described: emptying of a tank, mass/spring, pendulum, tuning forks, several instances of resonating systems, RLC circuits: filters and amplification. Also, a set of low-cost supplementary notes are being written to allow the student to see and listen to what goes on in class without the need to take notes.

PARADOX AND PROOF

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The definite integral is a major topic in Calculus with many student difficulties. In [6], we have traced the development of understanding as it progresses and found a curious phenomenon. When unable to proceed along a particular schema, students introduce a heuristic that helps them bridge the gaps in their understanding. In this particular situation such a gap is filled a change of a unit from that of a rectangle to that of a line segment of 0 width, the indivisible. They follow with the computation involving the sum of heights of infinitely many line segments to obtain the area under a curve - the definite integral. We suggest approaches to channel their thinking - a guided heuristic that confronts students with concrete physical scenarios where similar manner of reasoning leads to a contradiction. Using Zeno's paradoxes of the race between Achilles and the tortoise, we begin the process of introducing students to a directed heuristic. We follow with the mathematical context, using a construction by John Wallis [14], to provide a mathematical framework within which the intuition of indivisibles can connect with the notion of the area of two dimensional regions..

Using Computerised Tests as Support for Tutorial-Type Learning in Logic

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The course 'Introduction to Mathematical Logic' (32 h lectures + 32 h exercises + 56 h independent work) is mandatory at the Faculty of Mathematics and Informatics of the University of Tartu. In 1987-1991, we designed four programs for the exercises: Truth-Table Checker, Formula Manipulation Assistant, Proof Editor and Turing Interpreter. As result, we transferred 70% of the exercises to the computer class. The blackboard-based practical training was preserved for predicate logic. In the subsequent years we added one more program for truth-values of predicate formulas. The main benefit of the first round of computerization was the acquisition of real skills in two areas: Turing machine programming and proof construction. At the same time, rejection of blackboard exercises reduced the possibility of assigning to students small but important questions concerning new topics. In 1998, we decided to use a test administration system APSTest to introduce tutorial-type support for lectures. After each lesson, a test comprising 10-15 questions is available in the faculty network. The test has randomly selected questions and can be solved a number of times; before the beginning of the next lecture, however, the students shall have achieved at least a 75% success rate. The weekly duty to keep oneself up to date with theoretical material has reduced the dropout rate and improved the average grade results. The paper describes: intent of questions for different types of lecture topics, topics and questions proving to be easier and more difficult, eligibility of test-type questions for the different parts of the course, the use of different types of questions, some conclusions about the students' learning strategies drawn on the basis of the data saved in the database, changes made over time in test management.

A project-based approach to Numeracy practices at University focusing on HIV/AIDS.

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In this Information Age it is increasingly important that students at university are numerate at an appropriate level for their discipline. This paper reports on an attempt to achieve this goal through a project-based curriculum component in an Effective Numeracy course. The practice of numeracy within relevant contexts is emphasised, rather than the decontextualized acquisition of skills. We explore the way in which students engage with curriculum-embedded projects, and the manner in which the projects contribute to changes of attitude towards numeracy competencies. The choice of the HIV/AIDS epidemic as the project topic is motivated by the need to raise awareness of the magnitude of the threat and its social implications. Its obvious social relevance is also essential to motivate the students to engage fully with the project. The project develops an appreciation of the relevance of numeracy, by requiring the students to practice numeracy in a context where there is close linkage with other vital competencies, such as writing and information and computer literacies. The project design also includes the provision of scaffolding (especially for writing); and opportunity for co-operative learning. Students were required to present their research in a range of genres, which enabled different kinds of engagement with the material, and different affective reactions to the tasks. In all cases, the learning was not only through reception, but through synthesis and transformation of knowledge in the processes of production. One element of numeracy that was not sufficiently addressed by the HIV/AIDS project, and will receive more attention in future, was the critical assessment of the quantitative information that was contained in the source materials; and an exploration of the assumptions on which the mathematical models used to generate data were based.

A Methodological Tool to Analyze Verbal Arithmetical-Algebraic Problems

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We present a methodological tool to describe a structure of verbal arithmetical-algebraic problems. This tool allows us to analyze both the arithmetical or algebraic nature of this type of problems, and pupils' productions while solving them using the Cartesian Method. A solution of a problem by the Cartesian Method consists first of all on the analysis of the statement. This analytical reading transforms the statement into a new text that is ready to be translated into algebraic language. The need for this transformation comes from the fact that the translation from natural language to algebraic language does not try to reproduce all the meanings the original text have in the text written in algebraic language. Instead, a translation into algebraic language preserves only arithmetical meanings (and then algebraic ones), that is, quantities and arithmetical relations among them. The analytical reading consists then on the determination of the quantities known and unknown relevant to the problem and the relations among them. That's the crucial first step of the Cartesian Method. The next steps are the election of a quantity (or some quantities) to be named by a letter; the writing of algebraic expressions to designate other quantities, using the letter (or letters) introduced in the second step and the relations found in the analytical reading made in the first step; and the writing of an equation (or as much equations as letters introduced) by establishing that two (non-equivalent) algebraic expressions written in the third step designate the same quantity. The instrument we have developed describes the first step of the Cartesian Method, by representing the text it produces by means of a graph, in which each relation is represented by an arc and each quantity by a node. We will show how these graphs can be characterized as arithmetical or algebraic.

Geometrical Findings by Secondary Students

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The introduction in the mid-nineties of dynamic geometry software (DGS) such as The Geometer Sketchpad and Cabri has made it possible to drastically change the way we teach and learn geometry. The students using this software not only may develop geometric intuition by using traditional constructions of elementary figures, but they can also test or uncover their defining properties. Moreover, the students can deform or transform their constructions and observe which properties remain invariant. These capabilities allow them to explore, discover, test and conjecture new properties and relationships, anticipating the need for formal proofs. We remark that the impact of dynamic geometry is extending beyond the teaching and learning of geometry. Researchers [Vonder Embse et al, 1998] are beginning to use dynamic geometry to facilitate the understanding of fundamental concepts of algebra and calculus. Although the introduction of this DGS in the geometry classroom has just begun and has not reached many schools, we can already see some of its effects. Thus, during the last seven years there have been a noticeable number of mathematical findings by secondary students. In this conference we review a selection of problems solved by students, using mostly this type of software. Some of these solutions represent new mathematical results. The amount and depth of the problems solved in such a short time give an idea of the potential for growth that technology in general, and in particular this software brings to the mathematics classroom.

Calculus Machina: An intelligent tutor providing computer based support for teaching undergraduate calculus.

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Students arriving at University are far from homogeneous and there is a growing need to assess their active mathematical ability on entry to any course and provide suitable support materials when necessary. This paper explores how emerging technologies can provide an environment for diagnostic testing and follow up support material for such students. In particular, it discusses a new Computer Algebra System, called Calculus Machina. Although many Computer Algebra Systems are excellent at "Doing" mathematics they leave something to be desired when it comes to teaching and supporting learning in first year undergraduate mathematics, as many of the intermediate steps involved with basic calculus are not revealed. Calculus Machina is capable of solving many of the problems that arise in the standard Calc I and II sequence, but also disclosing the steps and processes by which these results are obtained. Calculus Machina can also function in tutorial mode where students are required to take an active role in learning, and where the program can "look over the shoulder" of a student as the steps in a calculation are performed, checking each step, and offer help when required. Finally, there is always a certain element of inertia when considering the adoption of any new teaching material so we conclude this paper with an evaluation of Calculus Machina in a teaching environment.

IS TRAINING NECESSARY FOR UNIVERSITY MATH LECTURERS?

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The training of teachers is associated, to a large extent, with those who conduct instruction at kindergarten to secondary schools. To a small extent, emphasis is laid on the training of teachers in the post secondary teaching. Apparently, at the university level the notion of having the subject knowledge is good enough qualification for good math teaching. This has left the math undergraduates' lecturer with a free for all approach. A good math lecturer, as the saying goes, "is that one that produces the highest passes". The graph of the number of candidates that register for math as academic ladder advances in the education line reduces tremendously compared to other subjects. Hardly any study conducted has explained this unusual trend that has continued for years. Most undergraduate math teachers acquire training through experience. It is this period of experience that the student is disadvantaged. Majority of students are able to solve math problems correctly. Unfortunately, very few are able to interpret and apply their results to practical situations, adding to the unpopularity often experienced in math. One observation that has been realized is that, the high school graduate joins the university with a high score that is rarely maintained throughout the course. Some scholars argue that the problem is caused by rote learning in high schools. So, how does the rote learning come about? Is it through the teaching approach? Are students motivated at this level? These are the issues that need to be focused on at the university level. The teacher needs to put across the math ideas in a progressive manner. Planning of the teacher's work is therefore, necessary, acting as a guideline to organizing problems that build on the computational, application and interpretational skills of the learner. Consequently, Plato's traditional approach where the learner is seen as an empty tin that is to be filled with knowledge has been practiced at some university levels. It is therefore, crucial that the training of the teacher needs to be given a chance at this high level. This is what this paper addresses.

COMPARING TWO METHODS OF TEACHING A COURSE IN DISCRETE MATHEMATICS

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In this paper, we aim to compare two methods of teaching proofs in discrete mathematics, i.e. "using the classical definitions" and "using the combinatorial explanations". By employing statistical methods, we shall reach the conclusion that, the students of mathematics prefer combinatorial proofs and it seems they understand better and grasp easier the combinatorial explanations of the theorems and their proofs.

The use of online interactive models and simulation in assisting students' development of mathematical concepts

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The authors of this paper teach mathematics to engineering and business students at the Higher Colleges of Technology (HCT) in the United Arab Emirates (UAE). This system of post-secondary colleges for UAE nationals was established to provide students with vocational and technical education. To further enhance student success, the HCT, in line with current trends, has moved to a more learning-centred education, with the creation of independent learning centres, custom designed labs for integrated learning and increasing use of technology and the Internet. Laptops are now becoming quite commonplace in classrooms around the system, and laptop technology is being integrated into learning goals. The development of on-line modes of learning is being encouraged in order to provide students with more flexibility in their learning. Students are familiar with the Internet and, in general, quite comfortable with computers. Students' laptops are fitted with PCMCIA cards, enabling a wireless intranet connection. The majority of classrooms are equipped with smart-boards, and teaching with technology is being encouraged. In teaching Arabic-speaking students, learning through the medium of English, particular attention is needed in providing meaningful understanding and sound concept development. With students having immediate access to the college intranet via their laptops, on-line materials can be used effectively as supportive tools in the classroom to enhance learning. The authors are involved in the development of such on-line materials, which offer students opportunities to interact meaningfully with mathematical content. The emphasis is on concrete and visual approaches with a high level of interactivity. This paper outlines some of the methods the authors have found useful in creating interactive and simulation models for the learning of engineering and business mathematics, and presents examples of such models.

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Main Theme: Education Research

Secondary Theme: No secondary theme

Three Experiments in Teaching Undergraduate Students in Mathematics

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The aim of this presentation is to describe the pleasures and the problems in teaching undergraduate students. We look at three experiments. One experiment looks at the methodology of teaching a large class of 250 students using the overhead projection method. The advantages/disadvantages of this method from the viewpoints of students, teachers and the administration are discussed. The other two experiments look at teaching a class size of about 40 students in an examination free set up, in a more interactive way. The feed back of the students about the several aspects of these methods are discussed.

**'Understanding solutions to first order differential equations:
A map of the conceptual terrain**

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Advances in technology and mathematicians' evolving interests in dynamical systems are prompting changes to the learning and teaching of differential equations. For the purposes of informing and guiding these reform efforts, theoretical elaborations that are classroom-based and address student's conceptual advances are needed. This report addresses this need by delineating key aspects of the conceptual terrain through which students progress. The results, which extend previous research on students' understandings and difficulties in differential equations (Artigue, 1992; Rasmussen, 2001), are based on data collected during two classroom teaching experiments (Cobb, 2000) and data analysis (videorecordings of each class session, copies of student's written work, and videorecordings of individual student interviews) followed the approach described by Cobb & Whitenack (1996). The details of this paper will address specific aspects of students' reasoning with rate of change. My use of the term reasoning includes a focus on quantities, operational-structural ways of thinking, and students' images, and goals. I view understanding as an act that comes forth in a series of meaningful events constituted through interaction (Yackel, Rasmussen & King, 2000) rather than reflective of fixed cognitive structures that an individual may be thought to possess. This paper will articulate ways in which students' mathematical realities evolve and unfold in the context of ongoing learning experiences. The development of students' mathematical growth will be framed in terms of five interrelated mathematical practices: construing approximate solutions, construing exact solutions, organizing the space of solutions, reasoning about individual solutions and collections of solutions, and reasoning about changes to collections of solutions. The full paper will elaborate and illustrate each of these practices. The significance of this work is that it offers instructors and curriculum designers a framework for anticipating and responding to the mathematical growth of students in differential equations.

Mathematics That Changes Lives

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We are developing a general education mathematics course that will introduce students to mathematical reasoning and applications. The one-semester course will cover the history of, motivation for, and an introduction to cryptography, fuzzy logic, graph theory, and non-Euclidean geometry. Three weeks (nine discussion-lecture hours) will be devoted to each topic and the remaining weeks will be used for intensive group work and project presentations and refinements. Classroom activities will build from the students' current knowledge to an understanding of the value and importance of each topic. Through discussion and exploration students will experience some of the insights, frustrations, and excitement experienced during the development of the four basic concepts. Group projects will enable students to work together in diverse groups applying mathematics with detailed exactness to particular problem situations, planning written descriptions of their projects, and then presenting them to the class. The cryptography unit will emphasize the classic cryptosystems and will focus on the mathematics that explains how and why the schemes actually work. The fuzzy logic unit will emphasize the basic notion of fuzziness and real-world applications. Topics in graph theory will highlight applications in management science, including Euler and Hamiltonian circuits, the traveling salesman problem, minimum spanning trees, and ideas in scheduling and planning. The non-Euclidean geometry unit will teach the basics of spherical geometry and focus on developing an understanding of an axiomatic system. Pre-service teachers will be encouraged to take this course in order to expand their understanding of the nature of mathematics. The hope is that they will begin to experience mathematics as a process of finding patterns and making and verifying conjectures, so that they will be comfortable enough to encourage their own students' explorations. We will encourage them to work on projects that can be useful in their own classrooms.

What is probability? What events and random variables?

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That is a proposal for an undergraduate course on probability. In fact, most of the courses and books on probability are affected by an initial fault, which makes hard the initial learning. The fault relies on two points: the introduction of the probability concept and the starting point of mathematical modelling, leaning upon the definition of events and random variables. Usually, the concept of probability is introduced by means of the different approaches to the theory (namely classic, frequentist, subjectivist, etc.), as if they were alternative answers to the same question "what is probability?". The consequent muddle can be avoided, by realizing and underlining that the various approaches are rather answers to different aspects of the same question, and as such they can be used, so to complete each other, rather than compete with each other. In the first part of my talk I point out these different levels of the question and I show how they should be used in different ways. In the second part, I consider the definition of events as subsets of a set and of random variables as measurable functions. In my opinion, such definitions are wrong; then, I would like to show simple and common examples, which prove that such definitions are inconsistent and that we can understand the students panic when asked to practice with them. Almost all of a first course can be well developed, without incurring on such definitions, but introducing the two random objects by following the naive logic-subjectivist approach. On the other hand it is undeniable that the set-theoretical view of events and variables is necessary to deepen the level of the study. Here it is shown how to recover such point of view, by means of a representation theorem. Clarifying the two points in the approach of a course increases the clearness and the rigor, without losing the treatment depth.

Main Theme: Preparation of Teachers

Secondary Theme: Curricula Innovation

One Interpretation of "Profound Understanding of Fundamental Mathematics"

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Liping Ma has received widespread acclaim for her book, *Knowing and Teaching Elementary Mathematics*, where she advocates teaching teachers "profound understanding of fundamental mathematics." Consequently, the idea of helping teachers gain this knowledge is the focus of programs with widely different philosophies of mathematics education. That such different schools agree indicates that "profound understanding of fundamental mathematics" has different meanings. It would be helpful to look closely at different interpretations of that concept, and eventually to try to come to some consensus as to what is needed in teacher education. This talk will present one way that profound understanding is understood and is being implemented in a series of mathematics courses for middle school teachers. Lessons learned while developing and teaching the courses will be discussed. One lesson is that profound understanding is different for people who operate at different levels of mathematical abstraction. What a mathematician would expect as profound understanding of a concept among undergraduate majors need not be the same kind of understanding that is appropriate for a middle school teacher. The understandings of teachers serve their teaching needs so well that it seems they should be labeled as a "profound understandings," although they are different from the understandings of mathematics majors. One way of fostering profound understanding of concepts is by helping students make different kinds of connections. These connections can be made through the senses, by connecting various representations of a concept, or by using a metaphor. Examples will be given from the courses of these connections. Finally, active engagement of students working with concepts is an essential element of attaining profound understanding. Different notions of active engagement and different ways of encouraging it will also be discussed.

Main Theme: Innovative Teaching

Secondary Theme: Curricula Innovation

Rich Resources for the Mathematics Classroom

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Close work with faculty of other disciplines creates an enlightened need for approaches and materials that used to be confined to the art, physics, or music class. The Center for Mathematics Education at Dartmouth College maintains a collection of instructional aids drawn from a variety of disciplinary sources. In this paper we will describe the collection we maintain and how faculty in various subjects use these materials. The use of visual and manipulative aids has a positive impact on the educational experience of the college mathematics student. We tend to assume that the role of the lower grades is to wean students from those needs, as if that kind of learning no longer added anything after middle school. Learning in different "modes" is still helpful, even in college, allowing students to solidify and deepen their mathematical understanding. A good choice of laboratory demonstration or other activity can also provide a welcome connection of mathematics to another discipline.

An Approach of Linear Algebra Through Examples and Applications

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For a class consisting of students in mathematics, computer science, physics, engineering, microtechnics, chemistry, we use an pluridisciplinary approach of this field by example and application. We start with three examples of linear systems. First: we show that it is impossible to cover a sphere with (curvilinear) hexagons only: In a repartition using hexagons and pentagons, a fixed number of twelve pentagons is needed. This is shown by row operations on a system of 4 equations in 5 variables. Here, the surprise is that although the system is underdetermined, one variable has a fixed value. Several natural examples may illustrate this necessity: Football ball, architecture, protozoa,... The dodecahedron illustrates a special solution with no hexagon: Second: we consider a chemical reaction, in which the coefficients have to be determined. The superposition principle for homogeneous systems appears quite naturally in this context. Third: digitalization of a potential on a grid. There is one solution for each data on the boundary having a mean property. In our opinion, these are accessible to undergraduate students. Linear equations may be amplified and added: Linear combinations appear. They can be dependent,... here is the rank.

Graphing Calculator as a Tool for Enhancing the Efficacy of Mathematics Teaching

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We have observed new trends in mathematics teaching during the last years. These trends are connected with current approach to school mathematics as a component of general basics of education - it means mathematics is a tool for practical life. The rapid progress of technology is one of aspects that have affected the mathematics teaching at all levels including the preparation of prospective teachers. In the traditional mathematics teaching the teacher passes complete information or knowledge to students and the students are passive recipients mostly. On the other hand the integration of technology (computers, calculators, www resources) allows to introduce new procedures in mathematics teaching and learning - e.g. to investigate problems, to discover the connections between phenomena. Furthermore, technology can help to develop understanding of abstract concepts in mathematics via its visualization or graphic representation, we can also show the relationships between terms and their properties using this tool. These procedures can help students in their learning and understanding of mathematics and consequently to their better memory of mathematical knowledge. The contribution deals with utilization of graphing calculators in pre-service education of prospective teachers of mathematics at Faculty of Mathematics and Physics of Charles University in Prague. We use this type of technology in some subjects of their study program, specifically in "Didactics of Mathematics" and "Problem Solving Methods". According to our experiences it is appropriate to use graphing calculators in the situation mentioned above - it means when we introduce new concepts in mathematics lessons. It is very easy to draw the graphs of function of one variable on the display of calculator and therefore it seems to be convenient to use graphing calculator for explanation of new concepts which are connected with term of function. Several examples will be presented.

Hot and Abstract: emotion and learning undergraduate mathematics

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Undergraduate mathematics students' affective responses to their studies have been collected from interviews, questionnaires and observations as part of a three-year longitudinal study of a cohort of mathematics students at two UK universities and from other opportunities from working with undergraduates and post-graduates. The central point of this report is that emotion has a significant part to play in learning mathematics at this level. Far from mathematics being cold and abstract it is hot and abstract! Affect has been classified into the three subdomains of belief, attitude and emotion (McLeod 1992). Attention here is on emotion, the least researched of these subdomains in undergraduate mathematics education. Reasons for the lack of attention in this area are attributed to the elusive task of tracking others' emotions as well as the abstract nature of mathematics with its concomitant 'cold' image. This image belies the strong feelings expressed by or observed among mathematics students or recent graduates, and anger is more prevalent than joy. Students mostly attribute their original choice of mathematics as a specialist subject to enjoyment. Enjoyment is highly correlated with skill. When these students become unable to understand the mathematics presented, frustration, fear or anger often arise. What role does the mathematics lecturer have in harnessing their emotion to pull them through to success? For emotional engagement, rather than just a good attitude or compatible beliefs, is the real key to desire to learn something which is abstract. The report will be in three parts: firstly, a selection of data will be presented which will be interpreted to indicate the presence of emotion; secondly, the importance of emotion in learning undergraduate mathematics will be discussed; and thirdly, attention will be drawn to the role of mathematics lecturers. The longitudinal project is funded by the Economic & Social Research Council (R000238564) UK.

Cooperation - The Basic Principle of Cooperation Learning

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It was established that in every group there are three different possibilities of action: individuality, competition or cooperation. Cooperation is most complex and most natural. This is one of the bases for developing the cooperative learning. The second base is the fact, that higher thinking structures are developing through positive social contacts. Different teaching methods give the possibility for appropriate conditions in the classroom through which pupils develop their knowledge and cooperation skills. It is very important to understand the basic principles of cooperative learning, that we could develop them successfully in the classroom. There is very important to understand the importance of this principles for healthy and harmonious growth of children, for their academic, social and personal development. In the presentation there will be discussed about this principles. We will look at it from different points of view. Further there will be discussed about implementation of mentioned principles into mathematics lessons and about connections between mathematics, cooperation skills and personal development of pupils.

Using computer-based projects with cooperative learning in first year mathematics

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As part of a continuing 10-year, classroom research project to the effective use of cooperative learning in first-year mathematics, the use of computer-based projects has been investigated. The Maple Computer Algebra System (U of Waterloo) was adopted and the impact of various strategies has been investigated. The points discussed are: elaboration vs discovery, degree of complexity (difficulty and length), frequency of assignment, and the assessment of the teamwork. This report discusses the impact of the various choices and suggests possible alternatives to be considered in the context of one's own institution and students. Student reactions to various choices are also discussed.

Diversity in Mathematics Education: Building Infrastructure for Learning and Teaching Mathematics with Understanding

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Over the next decade, primary and secondary schools in the United States will need to replace over two thirds of the mathematics teachers currently teaching. The situation is equally critical at the university level, where the new teachers and leaders in mathematics education will be prepared. At the same time, the K-12 student population of the United States is growing increasingly ethnically and linguistically diverse. These data suggest the scope of the formidable challenge facing the United States to maintain and rebuild-at all levels-its infrastructure for providing mathematics instruction so that it supports high-quality instruction for all students. To meet this challenge, the University of Wisconsin-Madison, the Madison Metropolitan School District, the University of California at Berkeley, the Berkeley Unified School District, the University of California at Los Angeles, and the California Subject Matter Project are collaborating in a Center for learning and teaching mathematics with understanding that is focused on Diversity in Mathematics Education (funded through the National Science Foundation's Centers on Learning and Teaching program). The Center brings together university faculty, post-doctoral fellows, doctoral students, and pre- and in-service teachers to study issues of diversity in mathematics education and to create and analyze videocases on the learning and teaching of mathematics of diverse student populations as a catalyst for professional development and teacher preparation. In this presentation, we will report on the organization, activities, and progress of the Center, focusing on three major components:

- (1) the model of professional development and teacher preparation
- (2) the Web-based database of annotated videocases and the related software tools that support cross-disciplinary, cross-institutional analyses of the videocases
- (3) the Language Study, an innovative qualitative evaluation of the Center's ongoing activities which focuses on language and discourse as the analytical site of institutional change.

Usage and Usability of the Mathematical Web Pages - An Example

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The main goal of this paper is to consider of usage and usability of the virtual mathematical web pages which are mostly collected from materials what others have done I will also consider some moral and effectiveness aspects concerning this kind of procedure. As an example Laboratory of Applied Mathematics in Lappeenranta new web pages will be introduced. This project was at first intended only to build material which would help mathematically not very skillful students to get a grip about mathematics studies and for which they could use by themselves without any supervision. But since money and time was short we did not wanted to do our material from the beginning cause this would have for one thing been expensive, secondly time consuming and finally there would have been no guarantee that we would at the end have in our hands any better material than what already has been done and which is available for use also. I started to look over what kinds of materials have been done by others. Soon I had more than enough good materials what could be used for previously mentioned needs. Background is that for some reason the mathematical knowledge and ability of the new students at a universities - in Finland, but probably also in any other country - is not so firm as one should be able to expect. Some revision of the school level mathematics is needed before the actual university studies can start. Here, computer-based study materials can give one solution. It has been claimed that development of software and network technology would give enormous possibilities to teaching and learning. However in practice, it does not seem so clear how these possibilities should be used or even what these so called new possibilities actually are.

DESIGN OF THE SYSTEM OF GENETIC TEACHING OF ALGEBRA AT UNIVERSITIES

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First, one should accomplish the analysis consisting of two stages:

- 1) genetic elaborating of a subject matter and
- 2) analyses of arrangement of a material and possibilities of using various ways of representation and effect on students.

The genetic elaborating of a subject matter consists of the analysis of the subject from four points of view:

- a) historical;
- b) logical;
- c) psychological;
- d) socio-cultural.

In designing of the system of genetic teaching very important is to develop problem situations on the basis of historical and epistemological analysis of a theme. It is important to organise an educational material so that to reveal the necessity of the construction and development of concepts and ideas. It is necessary to arrange problem situations or tasks, for which the important concepts or ideas, which should be studied, would serve as the best solutions. It is necessary to analyse those problems of knowledge, for which the considered concepts and ideas serve as the necessary solutions. For this purpose, both historical analysis and epistemological considerations, and special search for appropriate problem situations and tasks can help. When studying university algebra courses, the students usually are encountered with a ladder of abstractions. According to Leontyev, actions on learning concepts, as well as any actions, consist of (unconscious) operations. These operations are essentially contracted actions with the concepts of the previous level of abstraction. In our view, for reaching a contraction of an action with algebraic objects into operation it is necessary, after sufficient training with this action, to include it in another action on the next step of abstraction. After two stages of analysis, it is necessary to implement the project of the process of study of an educational material.

Main Theme: Preparation of Teachers

Secondary Theme: No secondary theme

MODERN STATE AND CHANGES IN MATHEMATICS TEACHER TRAINING IN RUSSIAN FEDERATION

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Serious changes in social and economical life in Russian Federation during the last decade had remarkable impact on mathematics teacher education. Generally, approaches to the higher (university) education have changed. The amount of lectures and seminars has decreased, and the amount of hours for the independent studying by students has increased. Thus, the Government obtained the possibility to decrease the number of university teachers. Instead of strict and uniform (all over Soviet Union) curricula for maths teacher preparing, new standards have been elaborated by the Ministry of Education and are being adopted by pedagogical universities. On the basis of these standards, universities construct their curricula for themselves. In some of pedagogical universities, two-stage curricula have been elaborated: after first 4 years, students become Bachelors and may teach at lower secondary schools. After 2 years of additional studies, they become Masters and have the right to teach at upper secondary schools. As earlier in Communist times, maths teacher education programs cover all contents, issues, methods and resources one only can imagine. Although, generally, higher education is now more popular among young people, teacher's profession is not popular because of bad employment possibilities, low salaries which are, moreover, often delayed for many months. Most popular in our poor country are now professions of finance managers, bankers, bookkeepers and lawyers. Many new private universities preparing for these professions appeared. However, teachers are being prepared only at state universities, and thus rather high standards of teacher education are saved.

Main Theme: Mathematics And Other Disciplines

Secondary Theme: No secondary theme

Efficiency of teaching of mathematics to the students of economic speciality.

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The University of World Economy and Diplomacy prepares the specialists on three directions - international economic relations, law and international relations. The students of economics have 400 academic hours in mathematics, including - high mathematics (first year students - 108 hours), quantitative methods in economy (second year students - 144 hours), theory of probabilities (second year students - 72 hours) and econometrics (third year students - 72 hours). Quantitative methods in economy and econometrics have an applied nature, while high mathematics and theory of probabilities have auxiliary nature and are used to study other subjects as quantitative methods in economy, econometrics and special courses. We conducted the researches to estimate level of adoption of high mathematics by the students. High mathematics consists of the following sections: differential calculus, integral calculus, functions of several variables, linear algebra, elementary differential equations, theory of series. The researches were conducted among the second, third and fourth years students and for two groups of students at the same time. One group was tested without the preliminary preparation and the second group was tested after the preparation and introductory lectures. The given tasks were identical for everybody. The gap between the marks of these groups of the second year students was insignificant while the gap between the third year groups was more sensible. The fourth year students showed the very big difference. Moreover, we conducted the researches to determine the level of students's adoption of high mathematics on the sections. There are high results on the sections differential and integral calculus, less results on linear algebra and functions of several variables. The results on differential equations were very bad and the worst exponents were get on the theory of series. We explained these facts that exactly the mentioned sections are poorly used in the special disciplines. The results of this research helped us improve our teaching program, by enlarging some sections of the course and reducing others.

MATHEMATICS PROBLEM-BASED LEARNING THROUGH SPREADSHEET-LIKE DOCUMENTS

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Problem-based learning is particularly suitable in fields like Mathematics. It is quite usual to find abstract descriptions of solving methods to express mathematical knowledge. However, these descriptions happen to be somewhat hard for students. For instance, when a learner faces the integration-by-parts method, one possibility is to try to understand the general formula (by using the abstract description), while on the other hand another possible scenario is to learn by watching a particular example. Both approaches are complementary and, in fact, teachers usually move back and forth in order to manage their students to understand the underlying concepts. In this paper we introduce ConsMath (CONStraint-based MATH teaching), that is being developed as a part of the ENCITEC project, a system based on a problem-based learning approach. ConsMath is an authoring tool for the creation by the teacher of interactive math documents. The teacher can establish spreadsheet-like relations between parts of the document and require certain conditions to be hold when used by the student. The documents generated are dynamic and change depending on the math formulae filled in by the student. Furthermore, ConsMath uses the Mathematica software when delivering formulae to be evaluated. The system has been implemented in Java, and it runs both as a standalone application or in an applet within a web page. An important feature of the system is the possibility for the teacher to create interactive documents starting from static ones. Besides, the student can also ask ConsMath to generate math data appropriate for a given problem pattern. ConsMath has a math data generation engine that provides the student with such data. Consequently, students can choose between filling input data themselves, or ask the system to generate different problems corresponding to the same problem pattern.

PRIMARY STUDENT TEACHERS' CONCEPTUAL SYSTEMS

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In order to be able to understand teachers and the types of decisions they make, it is important to investigate both their conceptual systems, that is, their beliefs about teaching, the subject matter and how children learn, as well as their subject knowledge. The relevant literature shows that primary teachers' knowledge and understanding of mathematics are frequently very limited, whereas their conceptual systems are dynamic, permeable mental structures, susceptible to change in the light of experience (and that their relationship with) practice is a dialectic, not a simple cause-and-effect relationship (Thompson, 1992). A teacher's subject knowledge and his/her beliefs are determined by experience acquired during his/her school years as a student, the trainee teacher period and his/her in-service time. The present work focuses on Greek student teachers and, in particular, on their conceptual systems and subject knowledge in mathematics during their four-year course. Three types of data were collected for the purposes of the study reported here: (a) the responses of 450 first year student teachers to a questionnaire concerning teaching, learning and the subject matter taught in primary school, (b) the performance of 94 first year student teachers on a number of simple arithmetic and geometrical tasks and (c) the responses of 75 third year student teachers to a questionnaire on attitudes towards and beliefs. The analysis of the data showed that student teachers' conceptions of mathematics and its teaching and learning are deeply rooted, strong and very narrow. This, combined with their limited subject knowledge, provides a rather poor basis for development. This suggests that teacher training courses, instead of focusing solely on providing good instructional skills or competencies, should also pay attention to the student teachers' conceptual systems.

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Algebra in the transition from High School to University

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Algebra in the transition from High School to University Fabiana Saldivia(1) and Carmen Sessa(2)

This report covers the design and implementation of a proposal for teaching algebra to local undergraduates who require advanced Mathematics (Universidad Nacional de la Patagonia Austral - Province of Santa Cruz, Argentina). In devising this proposal, we have considered various dimensions of analysis: - A theoretical framework of the teaching and learning of mathematics (we have taken into account the theories of G. Brousseau, G. Vergnaud and Y. Chevallard, among others) from which arises, among other things, a methodological option for the work in the classroom, which involves a great change of the teacher's role. - A reflection about the meaning of algebraic symbols in their use as a tool to solve problems (for this we consider the theoretical issues furnished by A. Arcavi and J. Drouhard, among others). In particular, we consider the dimension of algebra as a validation tool as relevant to this course. - A critical analysis of the selection of problems that previously formed part of the practical work, keeping those which could allow a work centered in the construction of the sense of mathematical objects and the particular methods of algebra. - The knowledge of the characteristics of the population of students to whom this would be directed. As a result of this work, new exercise booklet was produced for a part of the course. Then, subject teachers attended a workshop to discuss the problems proposed in the booklet and some questions relative to its implementation. Finally, changes have been made to the course. These three stages of the course's development are analyzed and evaluated in the full text of this report. This work will show the difficulties that appear when a change is introduced, a change that requires the reformulation of the personal relationship of each student with the study of mathematics, as well as repositioning students and teachers in their roles in the classroom.

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Geometry as Epistemology

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This study is based upon the experience accumulated while lecturing a in course in "Foundations of Geometry", both to Students in Pure Mathematics and to future High-School Teachers. It represents an essay in presenting not only new facets of Geometry (mainly for the second group), but in introducing Geometry as an archetype of scientific method (for the first group). As such we investigate the differentiated shock produced by the paradigm shift upon the two groups. In particular we accentuate not only the importance of the contact with the Axiomatic Method and the beauty of unconventional (i.e. not "common sense") way of thinking; but in principal the discovery of Geometry in particular and Science in general as the Gromovian "art of posing questions". [A living science - as opposed to the norm of formulating problems and solving them (this being the best case: the normative one consisting in seemingly infinite concatenations of unsavory exercises.)] We ascertain the strength of the introducing Geometry as Physics, i.e. as "Natural" Science, with its methodology, rigors and philosophy. We also show how both groups profited by the new light "Foundations of Geometry" reflected upon some othr fields, such us Analysis [and not only in yhe "classical" aspects, i.e. 'length' and 'area', but also on the very concept of continuity (with applications, of course, to a better undrestanding of Topology.)]

Symbolic Calculators and CAS in Math Education: More Evidence from Germany

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Following the survey on the use of technology in math education in all secondary schools in the (former East) German state of Thuringia in spring 2001, a similar survey will be carried out in early 2002 in the neighboring state of Hesse. This state has always been part of the Federal Republic of Germany, implying that its school system is rather different from the one in Thuringia. First results from this new survey will be presented. The actual usage of pocket calculators, as approved by the school authorities, is compared to what it would be if the decision were to the discretion of the teachers alone. It is then analyzed how well teachers know, and how often they use, certain symbolic calculators, as well as the three major CAS. In the final question the teachers were asked to mark three of six statements regarding the use of symbolic calculators and CAS in the classroom. Since the statements described an equal number of advantages and disadvantages associated with the use of technology in math education, the teachers' choices indicate their attitudes towards the use of technology in math education. The results are, in part, differentiated for the different types of secondary schools represented in the survey.

Self-guided and cooperative learning - scenarios and material

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Educational research shows that there might be interdependencies and interactions between mathematics, learning in general and the use of new media (not yet been completely analysed that can improve the quality of learning. What can scenarios of self-guided and co-operative learning look like? How can knowledge be consolidated by means of intelligent practice? How can new media make the teaching and learning of mathematics more exciting? These are three main questions that the German pilot study 'SelMa - self-guided learning in mathematics in senior high schools' tries to answer. Teachers of the pilot study have created different scenarios of self-guided learning. They describe the role of new media for mathematical exploration (e.g. CAS) as well as for presenting the topic to be learned (e.g. hypermedia), they focus on co-operative working and point out the teacher's role in each learning arrangement. Up to now suitable classroom material for some scenarios have been developed and systematically tested by other schools (evaluators) whether it works in everyday usage. The talk will present learning arrangements and materials of a learning carousel, an electronic learning environment for constructive learning and a jigsaw-puzzle. How learning diaries, mindmaps and communication via email and www can support the individual learning processes, will also be demonstrated. Results of the first evaluations are included. The current state of affairs can be viewed online (<http://www.selma-mathe.de>). This site (in German) offers a wide range of material that can be tried out and adapted to the teacher's individual needs. It is to be a platform for communication and co-operation between teachers working in the field of self-guided learning of mathematics as well.

Logic Used in Students' Transition Course Proofs

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This paper reports on the relationship between the logic taught and the logic used in students' proofs in one transition-to-proof course. In the U.S.A, such courses are often taken by second year mathematics majors and preservice secondary teachers. The focus is on teaching students to construct proofs of theorems. Typically logic is presented early in a somewhat abstract, decontextualized way and the students' experiences with theorem proving come later. In the course under study, which was taught at a large public university, only a little of the logic taught appears to have been used in students' proofs (e.g., modus ponens, DeMorgan's laws). The main data consisted of all (62) student-generated proofs constructed collaboratively during class sessions. The textbook and the professor's teaching notes were also analyzed and additional information was obtained from student information sheets and tests. The analysis was done on a "line-by-line" basis, recording how each inference or step of a proof might have been made, whether by using a logical rule (e.g., modus ponens, modus tollens), by some other "logic-like" move or schema, as a result of a calculation, or by invoking some algebraic transformation or a theorem. In addition to a detailed comparison of the logic taught to that used, we will discuss the way the students used it. Much of the logic appears to have been used as "logic-in-action" -- an idea similar to Vergnaud's "theorems-in-action." For example, children (and others) often use properties like commutativity of multiplication automatically without knowing (or resorting to) some theorem or principle. We will discuss implications for the amount of logic that might profitably be explicitly taught in transition courses (from the point of view of developing students' abilities to prove theorems) and how it might be taught. We will also raise questions for future research.

The Frontal Competitive Approach to Teaching Computational Mathematics

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Mathematics as a subject has that specific feature that it can not simply be put in student's memory. A student has to pass all the information through his mind by doing large number of exercises independently, as we say, to "adjust his head and hand". But even this is not enough. Whether in so doing he shows independence, critical approach and creativeness is what matters. Although mathematics is often said to be a "chamber" science, many universities traditionally practise teaching it to large student audiences. It is almost evident that a large "stream" of students has certain obstacles to develop independent thinking, as many of students get used to "flow over the stream" and prefer to be as ordinary as their classmates. How can teacher transform this obstacle into advantage? How can he encourage students' independence? How can he help them to feel their outstanding abilities? And finally, how can he prove that mathematics is a live, beautiful science but not a collection of non-refutable recipes? A possible approach how to achieve these goals is presented in this paper. Distinctive features of the approach may be determined as follows. "Frontal" means general, involving all audience to meet one goal. "Competitive" means opportunity for a success due to individual's creative and non-standard solutions or actions. To implement FCA we need to do the following: (1) Organize creative environment. (2) Encourage students' creative potential. (3) Give start to students' instinct of competition. (4) Ensure transparency of assessment. In this paper we examine these components in more detail as applied to teaching Computational Mathematics giving as an example such courses as Numerical Linear Algebra, Computational Techniques of (Simultaneous and Sequential) Least Squares, and Linear Programming.

Cryptography and statistics: a didactical project.

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Cryptography is a stimulating way to introduce and consolidate ideas in statistics, computational linguistics, combinatorics and algebra (modular arithmetics). Two of the authors have been carrying out didactical experiences in this direction at primary school level, starting in 1989, without any special technology (paper-and-pencil cryptography). The most obvious type of ciphers, i.e. substitution ciphers, has been used (each letter in the cleartext is replaced by a corresponding symbol in the ciphertext). One builds up histograms by 'parallel work', so as to achieve what is being felt as 'statistical significance'. The game involves both cryptographers (encryptors) and cryptanalysts (decryptors). Pupils quickly discover Markov models and the slight non-stationarity of the linguistic stochastic; polyalphabetic and homophonic substitutions are pointed out as clever tricks (actually historically used) to 'cheat' statistical cryptanalysis. The new experiments now in progress are at different level of age, 14-16, and of technology. By use of computer software a much deeper analysis of cipher systems and Markov models can be carried out. We are tackling also (pseudo)perfect ciphers by use of the friendly technology of graphic calculators, inside a wider project concerning simulations and use of (pseudo)random digits. We consider (pseudo)perfect ciphers, in which a sequence of (pseudo)random bits is added mod 2 bit by bit to a binary cleartext, by so producing a ciphertext which is itself a (pseudo)random sequence. Algebraic techniques are implemented on graphic calculators and are used to break pseudoperfect ciphers (genuine perfect ciphers are provably unbreakable). The team includes a person active in cryptographic research (A. Sgarro), a researcher in Mathematics education (L. Zucheri) and two teachers in charge of the class project (M. Borelli, A. Fioretto).

INNOVATIONS IN TEACHING UNDERGRADUATE MATHEMATICS IN AFRICAN UNIVERSITIES

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During the last three decades, the economic and social development at the international level was largely facilitated by the rapid expansion of knowledge, science and technology (S-T) and their applications to the production and management. Mathematics played a major role in this development. In order to promote the use of S-T in the process of human development, several African countries have recently undertaken reforms of higher education curricula and teaching and learning methods in science and mathematics. Indeed, for various reasons, during the last 15 years, African universities attracted very few students in mathematics. In several countries, the students arrive at the university with a low background and lack motivation to enrol in mathematics, which they consider to be abstract, complicated and leading almost exclusively to teaching careers in rural areas where the working conditions are quite poor. Moreover, the university teachers do not usually have adequate pedagogic skills in order to introduce teaching methods likely to improve the motivation of students and to ensure effective learning. In order to address these issues, I was asked to supervise from 1998 to 2000, in my capacity of Senior Specialist in higher education at the UNESCO Regional Office for Education in Africa, the publication of a guide to teaching and learning in higher education and to organize training workshops for more than 1.000 higher education teachers. For mathematics teachers, the emphasis was put on the teaching methods used for small classes, in particular seminars, small groups discussions, case studies and simulations. For the my presentation at the ICTM 2, I will select 5 to 6 universities which introduced some innovations in teaching undergraduate mathematics following their participation in the UNESCO training workshops. I will carry out a critical analysis of these innovations, including the achievements, the problems encountered and the strategies used to overcome them and from this analysis, I will propose the way forward.

The Use of Cases in Mathematics Teacher Education

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Many programs for teacher education look for ways to help undergraduates build new knowledge of mathematics which recognizes and connects with their practical experiences in the elementary classroom, in their field experience or practicum. The intent is to strengthen their fundamental knowledge of elementary school mathematics, starting with the students' own understandings and misunderstandings of the mathematics they are encountering in real classrooms with a particular classroom of children. One approach we have used successfully with pre-service elementary teachers is to integrate their own "cases," examples that they write about mathematical thinking in their current classroom teaching assignments which highlight children's thinking strategies, misconceptions, and special representations of mathematical ideas. These cases they have written are shared and analyzed with their peers in our undergraduate courses, and examined in light of the depth of the mathematics and the connections to mathematics education research. This "case-based" approach uses the advantages of learning the mathematics within the context of particular situations which the students identified as problematic, challenging, or fascinating. It also is a model for their life-long learning as mathematics teachers, to see within the children's thinking those very opportunities for their own learning and professional development.

Kepler's Wine Barrel Problem in a Dynamic Geometry Environment

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Attending a workshop for mathematic teachers at the Haifa Technion, I was exposed to Kepler's wine barrel problem and its history background [1]. In his book, *New solid geometry of wine barrels* Kepler described how the wine salesman measured the distance from the barrel's entry to the bottom's heel and could tell its volume just according to this measurement. He was impressed and started mathematical investigation. This investigation results in Theorem V [2] (Part Two): Of all cylinders with the same diagonal, the largest and most capacious is that in which the ratio of the base diameter to the height is $\sqrt{2}$. I asked myself what else can I, as a teacher, do with this interesting problem? How can I work it out in my class? I came up with the idea it would be a good problem to model in a Dynamic Geometry environment. It would enable achieve some teaching goals stated by the N.C.T.M Standards:

§ Visualize the problem.

§ Represent it in another way and find connections between calculus and geometry.

§ Use technology in order to solve the problem without calculus by appropriate means for students who did not have calculus yet.

I thought about the Geometry Inventor as an appropriate tool for this purpose: It can shed light on the problem and help the students grasp it better. In the proposed article, Kepler's problem and its generalizations, will be represented in a dynamic environment (with the help of the Geometry Inventor software) in two cases: of a cylinder barrel and of a barrel, which is composed of two truncated cones having a common base. The focus of the investigation is: the influence of parameters defining the barrel's shape on its volume in general and on the maximal volume in particular. The solution of the maximum problems is based on using the graphical features of the Geometry Inventor and verified by calculus means. The advantages of exposing students to such problems by using computing technology in the spirit of connected mathematics are discussed at the end of the article.

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Analyzing Functions' Behavior in a Computational Environment

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This proposal describes a research carried out with first year undergraduates in Calculus in a computational environment. In the last five years, the authors have been researching whether the exploitation of functions in a computational environment builds up an improvement in the students' performance. This is a report on the study about the 'Analysis of Functions' Behavior'. In a first instance, a theoretical discussion about the subject was carried out in class and, soon afterwards, activities were developed in a computer lab and the students insisted upon getting exact results. Therefore, a change in perspective was made. The authors believe that a good learning atmosphere involves 'bouncing', that is, students go from computer to theory and back to computer, thus forwarding the construction of knowledge. This new proposal, which excelled in promoting an active, critical and investigative students' behavior as well as a more independent attitude towards the teacher, was centered on a breakthrough from the Didactic Contract. The main tool in this methodology is the computer, whose use must be criteriously made; the professor should have control not only over the subject but also over the software being used. This finds support in the works of Luc Trouche. In steaming up the subject by means of a theoretical exposition, the teacher may aim at 'passing on knowledge'. Yet, we do believe that there is no passing but rather a construction of knowledge, and the teacher contributes in the process by propitiating learning situations that boost in the students the aforementioned behavior. The computer lab proved to be the right atmosphere to bring up this sort of student's behavior. And the activities, which were prepared for the analysis of functions' behavior, proposed a break in the Didactic Contract (open questions, conjectures, ...) which, added to a constant renegotiation, resulted in a shift in attitude.

FACTORS AFFECTING THE STUDENTS' PERCEPTIONS ON THE USE OF GRAPHING CALCULATORS

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In this paper, outcomes of the perception analysis of students on graphing calculators will be presented and discussed. The subject of the study was students in advanced algebra, pre-calculus and calculus of three different private schools in Istanbul, Turkey. Although the schools follow the same curriculum topics there are main differences in teaching methods/materials and students' background. Our aim was to see the affects of various factors on students' perceptions and to find out the dominant factors in the implementation of graphing calculators into the mathematics curriculum and school setting. In the design of the study, six independent and one dependent variables are considered. Independent variables are gender, grade level, previous year math grade, schools, teachers/teaching methods, and duration of the instructions using graphing calculators. The dependent variable is students' perception on the use of graphing calculators, mainly on the functions and various representations of functions. Considering the students' visual thinking and symbolic representations we have designed instructional materials that integrates various forms of functions like tabular, graphical and algebraic forms. These materials were mainly used at Uskudar American Academy, where the researcher was working as a mathematics teacher. Also an instrument, namely questionnaire on the students' views, was designed to score students' perceptions on the use of graphing calculators, mainly in the function concepts. The scale has 28 items covering several subgroups like how students' attitudes are toward the graphing calculators and how they do perceive graphing calculator as a cognitive tool. More specifically, we found out that there are various factors affecting the efficiency and students' attitudes on the use of graphing calculators. The results of the analysis show that instructional materials and teaching methods are the dominant factors among the others. In the presentation, we will give the details of the analysis, samples of the teaching materials, which shows the characteristics of the teaching methods of each school, and reflect our personal views and experiences, on the subject matter.

Connecting Undergraduate Number Theory to High School Algebra: A Study of a Course for Prospective Teachers

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Most universities in the US require prospective high school mathematics teachers to major in mathematics. In most cases, these students will encounter a course in abstract algebra and number theory, usually in the third year. In my dissertation study, I followed six above-average performing students enrolled in a third-year number theory course at a large state university in the southwestern US. The study focused on the students' understanding of congruence of integers developed during a unit on modular arithmetic. Though the topics studied in these types of courses are closely related to those of high school mathematics, students generally do not see these connections and regard these courses as completely unrelated to the mathematics they will be teaching in the future. For example, the students in my study did not appear to view congruences as being analogous to equations. When solving a congruence such as $5x \equiv 3 \pmod{7}$, they did not tend to think of "dividing" both sides of the congruence by 5, or of using a "guess and check" strategy. A solution method introduced in the course was viewed by the students almost exclusively as an algorithm to be memorized, and they generally did not recognize the connection between this method of solving equations and the solving of equations in elementary algebra. This study has several implications for teacher education. I believe that we should make explicit for future teachers the connections between the abstract algebra and number theory that they study as undergraduates and the high school algebra that they will teach. Placing emphasis on the connections between the mathematics they are learning at the undergraduate level, the mathematics they already know, and the mathematics they will be teaching will emphasize the importance of understanding why algorithms and processes work. We expect them to emphasize this understanding with their own students; thus expecting it of them is important.

Correlation between student performance in linear algebra and categories of a taxonomy.

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This paper concerns a study of the performance of students in a recent linear algebra examination. We investigated differences in performance in tasks requiring understanding of the concepts with those that required only the use of routine procedures and factual recall. Central to this study was the use of a taxonomy, based on Bloom's Taxonomy, for characterising assessment tasks, which we have described in previous publications. The full taxonomy has 8 categories, which fall into 3 broad groups. The first group (A) encompasses tasks which could be successfully done using a surface learning approach, while the other two (B&C) require a deeper learning approach for their successful completion. Tasks on the examination paper were grouped into the three groups and comparisons were made concerning the performance of individual students in each of these areas. There are several interesting areas, which the data allow us to investigate. The first is in identifying those students whose performance in category A was markedly different to their performance on categories B and C. Students whose performance in categories B and C is markedly inferior to their performance on category A are probably persisting with a surface learning approach. We would need to investigate ways of encouraging such students to move to a deeper approach. We have also investigated whether or not the data supports any systematic effect of differences in sex or language background in the performance on the 3 categories. The sample contained a large cohort of students with who had a home language other than English. A reasonable hypothesis to test would be that such students would have difficulty with the conceptual aspects of the course, since these normally require greater language facility. This paper will discuss these matters in detail and provide quantitative results to support the conclusions reached.

IFORS tutORial Project

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Everyone knows that the Internet in general, and the World Wide Web in particular, provide new exciting tools for the development and usage of teaching/learning resources. Unfortunately, it is also a fact of life that the development of educationally-rich web-based resources is not easy and can be expensive. One way to alleviate this difficulty is through cooperation and professional societies can play a major role in initiating and coordinating such joint projects. In this presentation we describe a project called tutORial that was initiated by the International Federation of Operational Research Societies (IFORS) in 1999. The goal of this project is to provide a framework for an international collaboration in the development of educationally rich tutorial models for standard Operations Research (OR) and Management Science (MS) subjects. The project will be officially launched at the IFORS 2002 conference (July 8-12, 2002, Edinburgh, UK) but its web site is already open for preview (www.ifors.org/tutorial/). The site currently features more than twenty highly interactive modules covering topics from areas such as linear algebra, discrete mathematics, linear programming, integer programming and dynamic programming. The goal is to expand this collection over time with contributions from OR/MS professionals and organizations world-wide. Details concerning preparation of contributions to the project can be found at the project's web site. All you need in order to use these modules is access to the Internet and a web browser. These modules are accessible free of charge. In this presentation we shall give a very broad overview of the project, explain how its modules can be incorporated in undergraduate applied mathematics courses and illustrate some of these modules.

The teaching of creative mathematical modeling via an educational toolkit for design optimization (TDO)

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The attempt at solving a real-life problem via mathematical modeling often leads to the posing of a mathematical optimization problem. In many cases, even if the modeling exercise is relatively simple, the solution of the finally formulated optimization problem represents a non-trivial and time-consuming process. In the teaching of mathematical modeling, this fact often inhibits the student from carrying out the repetitive but essential evaluation of various alternative models in order to arrive at an acceptable solution. To overcome this difficulty in the teaching of modeling to undergraduate students, the Toolkit for Design Optimization (TDO) was recently developed [1]. This system allows the student to easily solve his or her formulated constrained or unconstrained optimization problem on a computer, through the interactive use of a graphical user interface (GUI) without doing any formal programming. This paper describes the experiences of the authors with TDO in teaching a course in creative modeling to a group of senior undergraduate engineering students. The system, which employs gradient-based optimization algorithms developed at the University of Pretoria, will briefly be described and demonstrated. This will be followed by the presentation of evidence indicating that, using the TDO system the students were capable of performing the modeling cycle, i.e. analyzing the real-world problem, constructing the mathematical model, solving the mathematical problem and evaluating the results, etc., with relative ease. Of particular importance is the finding that the system not only enables the students to be creative in solving non-trivial design problems, but also allows them to have fun in doing so.

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AN INTERDISCIPLINARY APPROACH TO TEACHING NUMERICAL LINEAR ALGEBRA

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The theory of linear algebra, in particular, the methodologies focusing on solving systems of linear equations, can be applied to many real- world problems. For example, inversion techniques are used in finding current flows in electrical circuits, computer graphics, cryptography, and tomographic imaging. By bringing applied problems into the classroom, students are able to make connections between the theoretical constructs of linear algebra and the physical quantities modeled in these applications. This allows them to better appreciate the theory and enhances their interpretive capacity. In this paper, we discuss an interdisciplinary approach to teaching numerical linear algebra, by focusing on a fundamental problem in emission computed tomography (ECT). In ECT, the goal is to obtain an image of the bio-distribution of radio-pharmaceuticals that have been administered to the patient. The image is reconstructed from projection data measurements, that have been acquired at various angular positions around the patient by radiation detectors. The image is used as a diagnostic tool by a physician, to make a decision regarding the patient's health state without using invasive techniques. Data acquisition can be modeled as a matrix equation $Ax = b$ and so the reconstruction process is tantamount to solving a linear system of equations. Using various detector configurations, which affects the determinancy of $Ax = b$, students geometrically derive the elements of A . Then, given a measurement vector b , students apply inversion techniques, including LU decomposition, direct inversion, SVD and the iterative Landweber algorithm. By examining the eigen/singular values of A , students discover the link between truncating the spectrum and its effect on the solution. Additionally, students are able to quantify the convergence of iterative algorithms. General methods of matrix inversion are investigated, including least squares and minimum length. Several examples of classroom materials will be presented and applications will be demonstrated using the MAPLE program.

A graphical exploration to the concepts of Eigenvalue and Eigenvector using Cabri

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The topic of eigenvalues and eigenvectors is included in the first course of Linear Algebra which is offered at the Universidad de Sonora, Mexico. Despite professors acknowledge the importance of these concepts, the teaching practice usually reduces to the presentation of algebraic procedures to calculate them. As a consequence, students can acquire these procedures, but the comprehension of concepts turns out to be limited. With the purpose of offering a base of graphical significance, which enriches the comprehension of these concepts, an interactive computational environment has been designed, based in graphic-dynamic representations that can be directly manipulated by the student. This environment, which is created with Cabri Géomètre, allows the exploration of the concepts of eigenvalue and eigenvector for 2×2 and 3×3 matrices, and works with three simultaneous on-screen representations, which are:

1. The graphical representation of vectors v and $T(v)=Av$, where v can be directly manipulated and $T(v)$ changes after modifying v ;
2. Matrix A , which entries can be varied, which in turns modifies vector $T(v)$ and the characteristic polynomial $P(x)$ of the matrix A , associated with the transformation T ;
3. The graphical representation of the characteristic polynomial $P(x)$ of A , which cannot be directly modified, but only through modification of the entries of matrix A .

These representations allow the student to perform exploration at two levels:

1. Vector v is an eigenvector of A if it is collinear with $T(v)$. The search consists, then, in "dragging" vector v until this occurs. Once an eigenvector v has been found, the student can calculate the corresponding eigenvalue dividing $\|T(v)\|$ by $\|v\|$. In addition the student can move vector v without changing its direction to verify that this motion does not alter the computed eigenvalue. At this level, the graph of the characteristic polynomial remains fixed and its roots shall coincide with the found eigenvalues.
2. By varying the entries of matrix A , the student can search for eigenvectors and eigenvalues of any matrix. This allows the exploration of the behavior of eigenvalues and eigenvectors for some interesting cases, such as diagonal, symmetric, triangular and singular matrices. In the present work we describe this environment and some experiences we have had in using it with students of science and engineering which take their first Linear Algebra course at the Universidad de Sonora, Mexico.

The evolution of professional development activities designed to meet the changing needs of graduate student teaching assistants

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The preparation to teach that graduate student teaching assistants (TAs) receive is critical for a number of reasons. TAs are responsible for a considerable portion of undergraduate instruction in the U.S. Furthermore, future mathematics faculty come from the current population of graduate students, who are likely to carry habits they develop as TAs into their careers. In addition, recently undergraduate mathematics instruction in the U.S. has experienced some changes. As a result, now TAs may be asked to teach in ways that they did not themselves experience as students (for example, using collaborative group learning). The preparation and support TAs receive, especially early on, has the potential to shape instructional experiences for a substantial number of undergraduates now and in the future, and is especially important during this time of change. The topic of this paper lies at the intersection of two conference themes: preparation of teachers and innovative teaching methods. I describe how a learn-to-teach course evolved in response to TAs' needs. These TAs taught classes where students spend significant time working challenging mathematical tasks in small collaborative groups. In contrast to "traditional" teaching assignments (where TAs may be expected to answer homework questions and present solutions), these TAs assisted students as they worked in groups, provided problem solving support, and led whole-class discussions. As more was learned about challenges TAs faced and difficulties they encountered, activities were designed and revised. The activities were designed both to model active learning and to promote reflection on issues of teaching and learning. Through the cyclic process of observation and development, activities evolved such as in-class discussions about learning and collaborative group work. In addition, TAs watched videotapes of their own teaching and met with consultants to discuss strengths and strategies for further improvement.

Prospective primary teachers' experiences as learners, designers and users of open mathematical tasks

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This paper explores prospective primary teachers' views of open tasks. The study has been realized in the framework of developing student teachers' awareness of mathematics teaching and learning through a number of activities that aimed to relate theoretical perspectives to the mathematics teaching practice. Students experienced mathematics teaching by being exposed to research in mathematics education, and by being involved with lesson planning, the actual implementation in the classroom and discussion and reflection of their experiences. Data was collected from students' portfolios, and those parts that refer to open tasks have been analyzed. In particular, students' ways of analyzing two different kinds of open problems, their approaches in designing and their experience from using open tasks in the classroom have been explored and aspects of their views have been identified. Our preliminary analysis shows that students talk about open problems in terms of their mathematical and practical relevance, the information provided, the number of possible solutions, and their potentiality for extension. The students seem to link the importance of open problems to the mathematics learning and teaching mainly to a better understanding of concepts and or their use and application and to the process of problem solving. The tasks developed by the students cannot all be characterized as open and they can be broadly classified in terms of their wording and structure, context and the type of solutions. These dimensions concerning in the analysis of students' reflections on using those tasks in the classroom. Furthermore, the role of open tasks on pupils' mathematical development seemed to be central in students' evaluation of the classroom experience. Overall, the study contributes to our understanding of the development of students' awareness concerning open tasks as developed through their involvement in different kinds of experiences.

Optimam partem elegit

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The computational methods for solving optimization problems are generally known as hill climbing techniques. That is because they mimic the strategy that a climber may use in trying to reach the summit of a mountain. Different strategies are opened to the climber to reach the summit and we illustrate the rationale behind them by showing, graphically, the path to the optimum for alternative methods. Optimisation is frequently used to fit models to data with the intention of summarizing, interpolating or extrapolating the observations. Extrapolation carries the implication that the estimated parameters are physically meaningful. However, it is very possible that parameters, which produce a very good fit to the data, lead to disastrously unsuitable extrapolations. The question arises: When it is not safe to extrapolate? The paper shows, through an example, when not to extrapolate. This is called the problem of ill-conditioning. The paper reflects on the famous case where a good fit of the data led to a great discovery, that of Neptune, but the problem being ill-conditioned, meant that the model gave erroneous predictions for the location of the future positions of the newly discovered planet.

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Main Theme: Education Research

Secondary Theme: No secondary theme

Use of Informal Cognition in Teaching Mathematics

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The new curricula are based on the point that the including of mathematics of every day life in teaching mathematics is very important in order to make school mathematics meaningful. In Greece also, there is the same spirit in the new curriculum, without being noticed that mathematics of every day life is not the common for all students. Through a comparative study, we have already conducted, in two different-culturally groups of students we found out that they carry in their school different informal cognition. In particular, we have posed activities, based on conditions of every day life, in a group of gypsy students and in a non-gypsy one. The way these groups negotiated the activities made obvious that, since they come from different cultural context, acquire different informal cognition. The above, leads to the idea that, the curricula should be adaptable and thus to become suitable for all cultural groups.

Geometrical transformations – constructivist analytical approach

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The contribution illustrates a constructivist approach to the teaching of geometrical transformations to future mathematics teachers at the Faculty of Education, Charles University. Traditionally, this subject was presented as a series of logically connected definitions and theorems and students were asked to apply them in problems. A lot of material was covered like this, however, students' understanding was often formal and superficial. Several years ago, the course was completely re-designed in such a way as to let students deduce most knowledge themselves through a series of carefully prepared problems. A textbook adopting the Klein approach to geometry was written for the course (in Czech). Only isometries and affine transformations in line and plane are covered, however, our experiences show that the investigative approach leads to a better understanding of the subject matter and improves students' ability to study transformations independently of the teacher. A year ago, the author taught geometrical transformations in English to a group of practising teachers and the course was refined again. When it was possible, no mathematical result was presented as a ready made product, students had to discover it themselves. As the analytical approach to transformations lends itself to using software (e.g. Maple), the emphasis was placed on using it to help with tedious calculations. The problem of presenting computer calculations to the class was solved by using a data-projector. Later, files with calculations were sent to students by e-mail so that they had an opportunity to revise and see the process of deriving new knowledge in detail. During the presentation, an illustration of a series of tasks leading to the discovery of the matrix 3×3 of glide reflection and to its geometrical interpretation as well as examples of some interesting problems given to students to solve in a seminar work will be shown.

Problem solving for future teachers – an individual learning course

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At the Faculty of Education, Charles University, Prague, problem solving represents one of the key subjects in the future teachers preparation and is divided into four terms. Four years ago, the first course of problem solving started to be organised as an individual learning course as the only course during the study. By that we mean that students have no scheduled classes, they work individually and meet their teacher at consultations. The main aims of this form of study (besides the obvious goal to teach different strategies of problem solving) were to acquaint students with the range of mathematical books and textbooks and to develop their ability to (a) work independently, (b) take responsibility for their learning, (c) critically evaluate mathematical texts, (d) write mathematically. The course comprises three topics: Equations and their systems, Number theory and Plane geometry. Students have to submit one seminar work for each topic which include solutions to (a) problems given by the teacher (different for each student), (b) problems chosen by students from the assigned literature, (c) an “extra” problem chosen by students from any book but with a short justification of their choice. The fourth and last seminar work has a different character – it is an essay in which at least two books or textbooks used during the course are evaluated according to a student’s criteria. Finally, students sit for a short test. After the term, students are asked to write a short anonymous evaluation of the course (students mention advantages and disadvantages of an individual form of work and give suggestions for improvement). These written evaluations and their analyses contribute to the running modification and improvement of the course. During the presentation, our four year experiences with a distance form will be given, including students’ responses.

An Application of a Partial Differential Equation to Finances for Tertiary Education

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Revolutionary developments that have taken place on Wall Street and other stock exchanges around the world over the last twenty years have led to a financial world that is rapidly changing. One of the most interesting developments has been the growing popularity of derivatives in the economy. A derivative is a financial instrument whose value depends on the values of other variables and an example is a stock option, whose value is dependent on the price of a stock. The option price and the stock price depend on the same underlying source of uncertainty, namely stock price movements. The measure of how uncertain man is about future stock price movements is called the volatility of a stock price and options are therefore dependent on the volatility. Since 1973 the original Black and Scholes Option Pricing Model has been the subject of much attention. The latter has brought to the finance industry the modern methodology of sophisticated mathematics like partial differential equations and stochastic calculus. Myron Scholes together with Robert Merton were the recipients of the Nobel Prize for Economics in 1997. Options are traded on exchanges and are valuable to any person interested in making money on the stock market. In deriving their formula for the value of an option in terms of the price of the stock, Black and Scholes assumed ideal conditions in the market for the stock. In this paper exact pricing formulas for options are deduced from the Black and Scholes partial differential equation by making use of probability distributions and stochastic calculus. The above-mentioned equation resembles the well-known partial differential heat transfer equation that is taught in undergraduate Mathematics and is an excellent example of such partial differential equations applied to finances. Any student that has access to a computer can use the spreadsheets of Excel to calculate with alarming accuracy the exact values for the prices of call and put stock options, by making use of the Black-Scholes Pricing Formulas. The latter can be used to value options on stocks, stock indices, currencies and futures. In the banking industry there is currently a need for people with a thorough background in Mathematics with analytical, quantitative and numerical skills and in this regard the above-mentioned content can play an important role in outcome based research projects at tertiary institutions.

A profile of first year students' learning preferences and study orientation in mathematics

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This paper reports on action research activities during 2000-2001 with first year engineering students on an extended study programme in the School of Engineering at the University of Pretoria. Students in the 'experimental' group were enrolled for a support course aimed at facilitating the fundamental concepts underpinning a study in calculus as well as complementing the development of academic, communication and information skills. The thinking style preferences of students in both groups as well as the study orientation in mathematics of the 'experimental' group were determined. The possible effects of thinking preferences and study orientation on performance in a first course in calculus were assessed. Analysis of the thinking style preferences of the students indicates a diversity representing an array of preferences distributed across all four quadrants as measured by the Herrmann Brain Dominance Instrument. The same trend of diversity was found in distribution of learning styles according to the Soloman Felder Index of Learning Styles. Analysis of data obtained from the Study Orientation Questionnaire in Mathematics shows that students of the 'experimental' group enter tertiary education with a high level of mathematics anxiety and a history of unfavourable study environments. Analysis of student performance in a mainstream first semester calculus course indicates that the mathematics performance of the 'experimental' group was better than that of the other students on the five year programme who were not enrolled for the support course and also compared favourably with that of the engineering students on the four-year programme. In this paper it is envisaged that freshman mathematics students seemingly can benefit from a learning facilitation strategy for mathematics that endorses a student centred and a brain based approach. Such a strategy is aimed at developing the mathematics potential of the learners and improving their study orientation in mathematics.

PREPARATION OF TEACHING MATERIALS ON SELECTED MATHEMATICAL TOPICS OF DISTANCE COURSES

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The activities of the future teachers of mathematics in the preparation of a database for distance courses on mathematics are considered. The postgraduate students are realized during their training and writing the diploma works. The database, created by the students, consists of small units of an educational material on the selected themes of mathematics. Two substantial themes - symmetry and polyhedrons are considered. The educational material, submitted in a database, has the following structure: 1) The educational material on the certain theme is actually based upon knowledge base containing theoretical knowledge, practical skills and educational tasks. 2) The elements of structure of an educational material correspond to the purposes of training on the given educational material and find obvious reflection in their contents. 3) The elements of an educational material include elements of previous educational materials. Theoretical knowledge should act as a basis for formation of the appropriate knowledge in the distance courses. Practical skills are the set of algorithms, samples of activities. The educational tasks look like exercises, control questions, which are usually resulted at the end of the unit of the textbook. The structure of a particular course is determined as a choice of a subset from the model of a subject domain. The theoretical elements of an educational material and connection between them give its theoretical substructure. The practical substructure consists of the samples of activity and the connections between them. The training of the students were realized during their learning of computer science. The special course "Development of the training programs" gave them the entire framework. The work is carried out on the technology accepted for Web-courses creation and is realized with the help of HTML language.

Guidelines of a geometric-transformational section in High School and Undergraduate Mathematics Curricula.

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Since F. Klein introduced groups as structural particles, Mathematics is enriched as regards methodology, the emergence and utilization of the productive interaction between different structures defined upon the same set, fruitful new applications in Theo-retical Physics, and the emergence of “latent” elements of interrelation between Art and Science. These, among others, constitute an implication of a special productive strength of the “globally viewing” that is inborn in transformations (considered primarily as notions), and that, as we believe, should be trans-planted in the educational “corpus”:

In High School: By intuitive introduction and basic study of Geometric Transformations via natural-physical notions (in accordance with their first introduction by Helmholtz) aiming to provide the framework for:

- (a) Training the pupils to a kind of “globally thinking”; a special attitude due to the character of the transformations, that is considering the space as a whole and transforming “shapes” as rigid. There are ways to, intuitively, introduce the “basic knowledge” needed with minimum time- and procedure-weight for the pupils.
- (b) Enriching mathematics education with interdisciplinary and cultural elements that are inborn in Mathematics and may, for example, refer to “symmetries” in Physics, or to primitive or newer tribal decorative arts.

In Universities: By analyzing Klein’s view of Geometry (“Kleinian Geometries”) with applications referring to diverse branches of Mathematics, pointing-out Transformation Groups as a modern founding and research framework of structures and exhibiting the inner consistency of Mathematics: For example, by considering the “classical” Geometries as “subGeometries” (in the Kleinian sense) of the complex plane, since their groups of “symmetries” are subgroups of the group of Möbius transformations. In the same framework, we may exhibit the interrelation between human aesthetics and “mathematical creativity”, for example by analyzing details enabling the recognition of the highly interesting fact that the seven forms of “symmetries” present in diverse tribal decorative arts are all the possible Euclidean symmetries of the (infinite) zone.

We shall give brief details and indicative examples enriching and explaining the above.

Sociocultural factors in undergraduate mathematics: The role of explanation and justification

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Polya argued that mathematics is an act that is socially constructed and socially transmitted. In recent years, we have witnessed a growing tendency in studying social interaction patterns as a means to characterize mathematics learning. Yet, little work has been done at advanced levels; the bulk of the research in this area has been conducted in elementary and secondary school classrooms. This paper proposes to extend this work to the learning and teaching of mathematics at the undergraduate level. To achieve our goal we designed a teaching experiment in the context of a discrete mathematics course at the university level. In this course, the instructor strove to sustain an inquiry-based environment. The analysis of the teaching episodes is used to document the social and sociomathematical norms regarding explanation and justification and how these norms were constituted. First, we focus on the social norm that students justify and explain their thinking while they try to challenge and make sense of other students' thinking. We show explicitly how the instructor of the course established an expectation for explanation and justification, and how students' interactions developed in accordance to this normative understanding through the semester. That is, we trace students' development from the passive acceptance of the instructor's authority to the expectation that students become contributors to the class and that they all share common understandings. We then shift our focus to the sociomathematical norms' normative interactions specific to mathematics. We discuss the development of students' explanations from the procedural level to ones that are grounded in deeper conceptual understandings. The analysis presented in this paper makes explicit the process in which the aforementioned social and sociomathematical norms are constituted in ways that enhance the learning of advanced mathematics. We finally discuss the social interactions with respect to reformed instruction in advanced mathematics classrooms.

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Main Theme: Innovative Teaching

Secondary Theme: No secondary theme

A quasi-qualitative approach to limits

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The classical definition of limit of a function involving ‘epsilon and delta’ is not readily understood by students studying calculus for first time. Though teaching/learning calculus from Non-standard models of number system and infinitesimals is relatively easier , it is not widely practised. Under these circumstances increased use of Landau symbols is suggested. This will promote a greater qualitative understanding of limits and the rate of growth of functions.

How to prepare the prospective teachers to teach mathematics - some remarks

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Started in 1998 the reform of the Polish education system is a new challenge for teachers of mathematics, especially at the early teaching level and at the primary school level. According to the reform concept, at the early teaching level mathematics is a part of integrated educational block. Since that time mathematical content is bound up with other items of education. It causes the real danger of losing mathematical content in a space of many other information and topics. In addition to that the teachers of early level are not strong enough in mathematics - they are not sure that their mathematical knowledge is sufficient, they are afraid to look for their own didactical proposals. There was a necessity to prepare a different kind of teachers training - to prepare teachers to go through the new content of the subject -math-. This necessity creates also a big chance to extend an offer for the students - prospective teachers - in the framework of their professional preparation. Among other, there is a chance to:

Create a new approach to teaching early geometry.

Use a new tendency (based on recent didactical researches) in teaching arithmetic.

Create new connections between arithmetic and geometry with keeping the essence of arithmetical and geometrical cognition.

This change made also a change in approach to teaching math. (mainly geometry) in the next level of education (children 10-12 years old). Ideas created on the lower level should be to continue. Some problems, which have to be taught in the new way at the lower secondary school level are as follows:

Intuitions of the basic geometrical figures, measurements and geometrical transformations.

Fusion of two dimension and three dimension geometry.

Vector intuition.

During conference we will show the examples of our proposals.

Main Theme: Preparation of Teachers

Secondary Theme: No secondary theme

Algebra for Adults

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In this paper I will describe a new mathematics course I have recently developed which prepares secondary teachers for the specific task of teaching algebra in grades 8 through 11. The objectives of the course are to take future teachers who already know the procedural aspects of algebra and revisit the fundamental ideas of the subject from a deeper perspective. The goal is to develop a conceptual understanding of the overall structure and the unifying themes of algebra. While pedagogical issues are often addressed, the focus of the course is the mathematics. Students who cannot pass a pretest demonstrating a good foundation in basic algebraic skills are not allowed in the course, since experience has shown me that these students have little chance of succeeding in this course. In my presentation I will explain the philosophy of the course, give a justification for it, describe the course outline and organization, and give examples of how some standard algebra topics are addressed using non-standard methods. I will also give examples of some of the assignments, projects and exams in the course, including a new community service component that I plan to implement in the Spring semester, 2002.

Teacher Training: A Necessity for Successful Mathematics Education Reform

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As a result of insufficient teacher training, reformed mathematics courses often fail to be implemented as intended. A wonderful curricular construction can quickly disintegrate into a pedagogical nightmare for both the teacher and the students, especially when the new course represents mathematics education reform at many levels. Professor Christopher G. Lamoureux and Professor Richard B. Thompson from the University of Arizona co-created a reformed business mathematics sequence that presents challenges even to veteran instructors of the traditional finite mathematics/business calculus sequence offered at most U.S. universities and colleges. These relatively new courses involve thorough integration of technology, inter-disciplinary project-driven mathematics, as well as significant collaborative work in the form of student teams, team written reports and oral presentations. In addition, the text for this course sequence is entirely electronic. Pima County Community College District in Tucson, Arizona recently adopted this reformed course sequence. In an effort to provide a smooth the transition while ensuring that the courses are delivered as intended, Pima County Community College District offered a weeklong, hands-on instructor training workshop during the summer of 2001. This workshop was specifically targeted to mathematics instructors interested in teaching these courses. The majority of instructors in attendance were from the four Pima County Community College District campuses. Others in attendance were instructors from the University of Arizona and Arizona State University. Although a few of the instructors were initially skeptical, the workshop was very well received. Entrance/exit survey results and highlights of this workshop will be presented.

Language as a communicative and interpretive tool in mathematical problem solving

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Language is a major component in the thinking process. Lately the importance of language in mathematics has been addressed in classroom practices. Different aspects of language and its use have been the focus of analyses. On the one hand, there are analyses using linguistic methods focusing: on language itself and on some discursive patterns that occur during problem solving. On the other hand there are analyses treating language as a medium to gain some insight into the cognitive, cultural or social factors that underlie the problem solving process. Our study intends to combine these two approaches by creating a dynamical analysis of mathematical texts. For the analysis of transcribed protocols we have used symbolic interactionism as our basic theoretical framework, and aspects of linguistic analysis techniques. The subjects were undergraduate students who worked in pairs to solve a geometrical problem. Our main aim was to observe the role of everyday and quasi-mathematical language in problem solving situations. Particularly we analyzed incidents in which: (i) a new meaning was introduced by one participant or both participants(ii) a meaning or a strategy already used was transformed or connected to another meaning by a participant, after the interpretation of the other participant-s actions or intentions(iii) a strategy was changed, abandoned, retained or revised by a participant, due to the other participant-s actions or intentions. More specifically, our interest was centered not only to the study of the role of language in a particular problem solving setting, but we also aimed to expand the potentials of pure linguistic analysis methods and combine them with interactional analysis methods.

Technology and Graphical Interpretation of linear systems

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Technology and Graphical Interpretation of linear systems The purpose of this work is to present and discuss data and findings on graphical interpretation for linear equations and systems of 2 and 3 incognita using a computer software called .Winplot. Undergraduate students have great difficult in understanding linear systems solutions in R^2 and R^3 , regarding geometrical interpretation those difficulties increase. The investigation took place in a teacher education (licenciatura) class in Brazil. The work was divided in phases: ·

The students solve systems of equations with 2 incognita and represent the lines in a Cartesian plane (using pencil and paper)·

They compare the results, classifying the systems regarding the number of solutions. ·

They identify the relative position of the lines and look each case regarding the classification they had created. ·

They go to the computer lab and besides doing a comparison to the pencil-and-paper solutions, the students use the software to represent geometrically linear equations with 3 incognita. ·

From the graphical representation they classify the systems regarding the number of solutions·

The students, then solve the systems in order to verify their classifications.

To mention one of the main findings, we observe that before the computer activity most of the students believed that it would be impossible to represent geometrically a system with 3 equations and 3 incognita. We will show different ways the students carried on to solve the problems in each phase. We will also discuss what are the implications for mathematics classrooms.

Main Theme: Preparation of Teachers

Secondary Theme: No secondary theme

Preparing Teachers for a New Challenge: Teaching Calculus Concepts in Middle Grades

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An interdisciplinary team of professors from the Colleges of Science (Mathematics), Education (Mathematics and Science Education), Engineering at the University of Texas at El Paso (El Paso, TX, USA) in partnership with local public schools is working on issues of teaching and developing students' conceptual understanding of advanced Calculus principles in middle grades. They are conducting research and exploring pedagogical conditions (what actually works) for the successful teaching of a 'Visual Calculus' course (integrated 3-D Geometry and multi-variable Calculus concepts of differentiation, integration and optimization) in a middle school with a culturally diverse student population. Research is based on the assumption that conceptual learning leads procedural development (L. Vygotsky, V. Davydov, R. Skemp, etc.). The main distinction of the 'Visual Calculus' course is its orientation toward method of ascending from general to specific, multiple connections with science and technology, as well as multiple representations with focus on the power of cognitive visualization in the development of students' conceptual understanding of advanced Calculus ideas. Final research destination of the project is the measurement of an impact that early conceptual development of students' advanced mathematics principles has on students' progress in Calculus at the high school and college level. This research is taking place in conjunction with ongoing NSF funded PETE (Partnership for Excellence in Teacher Education) program at UTEP, with its emphasis on field-based intervention for improvement of pre-service mathematics and science teachers preparation. We will discuss preliminary research results focused on the relationship between pre-service teachers' content and method knowledge in mathematics, and students' achievement in 'Visual Calculus' and in regular middle school mathematics classes. The influence of the 'Visual Calculus' course on students' eventment on standardized tests will be also discussed during the presentation session.

Why students (mis-)understand mathematics and science: Intuitive rules and infinity

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Infinity is a major concept in mathematics. Many researchers describe common difficulties that students encounter when dealing with various facets of this concept. In this paper we suggest that students tend to react in similar ways to a wide variety of tasks that share specific, external features but differ with regard to their content area. So far four types of responses were identified, two relate to repeated division (Everything comes to an end, and Everything can be divided endlessly), and two to comparison tasks (More A - more B and Same A-same B). Based on such observations, the Intuitive Rules Theory has been proposed to explain and predict students' responses to mathematics and scientific tasks. Students in various grades (from grade 1 to undergraduate students) were presented with tasks related to repeated division of mathematical, physical and biological objects. As expected, most young subjects (up to grade 8) gave finite responses to all tasks, regardless of the nature of the object. Unexpectedly, the percentages of the older students (grade 9 and up) who gave responses assuming infinity to all tasks were relatively high. These high rates could be interpreted as instances of the intuitive rule: Everything can be divided endlessly. Subjects were also asked to compare the number of points in two geometrical objects and the number of particles in two material objects. Surprisingly, most subjects in all grade levels argued, in line with the intuitive rule "More A - more B" that the large object is composed of a larger number of points/particles. These results suggest that students' responses to given tasks related to the concept of infinity are mainly based on external features of the tasks. More generally, our findings suggest the importance of using a great variety of tasks, embedded not only in mathematics but also in other related domains, to assess students' ways of thinking about mathematical concepts.

Algebraic Calculator Technology in First Year Engineering Mathematics

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Algebraic calculators have made minimal inroads to most Engineering mathematics courses in Australia. Indeed, many still forbid normal graphics calculators in assessment despite their wide usage in the school systems which feed the undergraduate courses. This is curious as even the algebraic calculator technology is no longer very new and reminds us of the resistance to change in undergraduate mathematics teaching. Currently we are developing an engineering course in product design, which combines traditional course objectives with handheld CAS. For several years now, our Engineering students have used Mathematica from second year of course (although not in tests) and normal graphics calculators are used in all work in first year. The emphasis on facts and skills in the extant course means that over 60% of examination questions previously given in the first year course could be solved much more simply using an algebraic calculator. The transition period requires that the traditional course be essentially maintained, partly to ensure student mobility between engineering courses, but some topics are modified for the new course and assessment is independent. Current engineering textbooks usually restrict themselves to traditional algebraic and calculus approaches, although graphics calculators are now more commonly used. Indeed many of these books explicitly state opposition to the extension of CAS within the framework of the traditional course. This forces the provision of resources in-house to service the CAS approach to engineering algebra and calculus. In this paper we discuss the introductory course and its implementation problems, illustrating how algebraic calculators can solve basic questions in a normal course, and how the calculators may be used in the future.

An Interdisciplinary Approach to teach ODE - Development and Implementation of the EV & C UBB platform.

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An active learning approach has been developed and implemented to teach Ordinary Differential Equations (ODE) for Food Engineering undergraduate students using an Internet-based package (EV & C UBB). A variety of learning strategies have been introduced to support and extend the traditional lectures making it easy for instructors to design and deliver online learning. To achieve those goals we have implemented an Internet-based package that includes several sections for learning and teaching, some of them interactive. The internet-based package works like a distance educational platform, so the student can use it from anyplace. EV&C includes class calendar, interactive tests, class bulletin, secure access for students and instructors, forum for homeworks and projects, peer review, and resources area. The ODE course was originally designed so the student could make his or her own projects. The student was evaluated at the end of each project. Now with the introduction of the Internet-based package the student can receive help whenever he or she wants, and it is possible to know at every step of his or her work all advice that has been given by the instructor or peers. We believe that EV & C UBB is an extraordinary teaching aid strategy to learn from research projects in ODE. The student can interact with instructors, peers to improve his or her project; therefore we have contributed to develop a teaching and self- Learning system that reinforce learning with understanding, active learning, and constructive learning.

THE NEW MATHEMATICS MATRICULATION/ENTRANCE EXAM SYSTEM IN HUNGARY

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We are witnessing a long-term educational reform after the political changes in Hungary. The main elements of this reform - beside the question of educational management and finance - are the changes of the curriculum and the maturity examination. Maturity examination will have double function in the future that is, on the one hand, a final exam for secondary education and, on the other hand, an entrance examination for the tertiary level of education. Within the frame of this examination reform we analysed the advantages and disadvantages of the present examination in the mirror of the expected social, educational policy and curriculum changes. During our research and developmental work we considered the international trends and the applicable Hungarian traditions. We concentrated on the development of a new examination model and new types of tasks and items. We had the opportunity to field test the new tasks and items and also to collect teacher's opinions and suggestions. After a careful analysis the experiences has been built in the new examination model. In our presentation we would like to demonstrate the new crystallized examination model and some of the new examination tasks. The main characteristics of this model:

- Two levels, the upper level has the selective function for the tertiary education.

- The examination has a centrally developed written and oral part.

- Among the tasks there are short answer questions and some complex mathematical problems with multiple questions.

- The evaluation of the written part is based on a detailed evaluation guide.

The model will be illustrated with concrete examination tasks and their solutions.

Expression Equivalence Checking in Computer Algebra Systems - Application Possibilities in Mathematics Education

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In connection with the spread of computer algebra systems (and algebraic calculators), the natural question arises: how to change the requirements and emphases of mathematics syllabuses? One possible domain that might be given more consideration in the future is expression equivalence checking. It plays an important role in solving equations, manipulating expressions, and other domains (be it performed on a computer or by hand). In this paper, we examine some possibilities of integrating expression equivalence checking more fully into the educational process. We present different schemes that describe the teacher's and student's activities in different situations, considering the particular goal, problem setup, student's preparedness, the specifics of the computer algebra system, access to computer algebra systems, etc. The schemes are based on sample problems selected from various areas of college algebra. They include step-by-step (line-by-line) solutions as well as solutions in which computers are used for solving larger blocks in one step. The schemes are primarily designed for the current computer algebra systems; however, apart from the available features, mention is made of those that do not (yet?!) exist directly. The schemes also lend themselves to investigating the possibilities of using computer algebra systems as the expert modules of intelligent tutoring systems. The domain of expression equivalence is closely related to those of equation equivalence and equation systems equivalence yet there are some differences between them, which we also address. However, to what extent are the computer algebra systems of today capable of determining the equivalence of expressions altogether? This article addresses the widely used systems Maple, Mathematica, Derive and MuPAD. An overview is given of how different computer algebra systems cope with different expressions from school and college algebra. In many respects, the computer algebra systems can cope well with equivalence checking. Nevertheless, they may encounter some challenges as well.

The University Goes to High School

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In Brazil the number of youth in high school interested in mathematics has been decreasing in the past ten years. In this paper, we address the way we endeavoured to challenge this problem at the Institute of Mathematics of Federal Fluminense University, Niterói, Rio de Janeiro. In 1998 we had 3.8 applicants for each vacancy. In 1999, we formed a team of three teachers in order to develop an educational research called "The University Goes to High School". The objective is to attract better students in mathematics. The main activity is to supply itinerant conferences in high schools to students that had not yet made a decision about their career. We address issues as varied as the presentation of problems in Topology, the possibilities of obtaining support of financial agencies during the course, the job opportunities, the University's Distance Learning of Mathematics etc. An unexpected favorable by product of this action is the return of the high schools teachers to the University. Indeed they begin to pay more attention to continuing education, in order to update or broaden up their knowledge in our University's Specialization Course on the Teaching of Mathematics. The number of regular students has steadily increased since our project was set up. Another goal we pursue is to detect gifted students with an outstanding talent for Mathematics, and to put them in contact with teachers of the University in order to develop a study program as earlier as possible. In 2001, we had the greatest number of graduate students with major in mathematics, 6.74 applicants for each vacancy, and we strongly expect to achieve a better selection in the coming years.

STUDENTS' ALGORITHMIC, FORMAL AND INTUITIVE KNOWLEDGE: THE CASE OF INEQUALITIES

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In his analysis of students' mathematical performance Fischbein (1993) related to three types of knowledge, i.e., the algorithmic, the formal and the intuitive knowledge. Fischbein described the algorithmic knowledge as the ability to activate procedures in solving given tasks, and understand why these procedures "work". The formal knowledge refers to a wider perspective of the mathematical realm, what is accepted as valid and how to validate statements in the mathematical context. The intuitive knowledge is described as an immediate self-evident cognition of which students are sure, feeling no need of validation. For each of these three components students may have either correct or incorrect ideas (Fischbein, 1987). We use this framework in analyzing Italian and Israeli students' solutions to algebraic inequalities. A population of 170 Italian and 164 Israeli high school student who had studied the topic of inequalities in a traditional "showing the algorithm" approach, were given a number of standard and non-standard tasks. The standard task were similar to the ones solved in class (e.g., solve $9x+4 > 0$) and the non-standard ones were not dealt with before (e.g., discuss the statement $ax < 5 \Rightarrow x < 5/a$ for any a in \mathbb{R}). In their solutions students exhibited a number of intuitive ideas, such as "the solution of an inequality should be an inequality". They also exhibited a number of algorithmic ideas, which were occasionally consistent with the intuitive grasp of the solution. Among the students who had formal knowledge, we identified phenomena of correct algorithmic performance, but also signs of algorithmic confusion, such as "I know that I have to find the values that make the expression a correct statement, but I have no idea how to go about it." (see also, Bazzini & Tsamir, 2001; Tsamir & Bazzini, 2001).

In the oral presentation, examples of correct and incorrect expressions of algorithmic, intuitive and formal knowledge will be presented and discussed in view of possible educational implications and suggestions for the assessment of students' learning.

**What can we know about pre service teachers' mathematics
content knowledge through their e-mail discussions with 6th grade students?**

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In this study my aim was to understand whether pre-service teachers (from a nontraditional mathematics classroom) have developed good understanding of fraction concepts and have improved their ability to do algebraic thinking. Furthermore, I sought to determine whether they could use their knowledge of fractions and algebra clearly and effectively in their communications with 6th grade students (also from a nontraditional mathematics classroom.)The approach in this college mathematics class, ·Learning Mathematics via Problem Solving·(Masingila& Lester& Raymond, 2002), is very different from a traditional approach. In this class, students construct their knowledge through active involvement with challenging mathematics problems while the instructor facilitates, guides, and helps students share their own knowledge. During 2000 Spring semester, pre-service teachers at our university studied mathematics in groups of four, shared their mathematical ideas and thinking with the entire class, kept daily math journals, and participated in a math communication project in which they discussed mathematics problems about fractions and algebra with sixth grade middle school students via e-mail. Simply stated, the instructor and I wanted to determine whether a ·learning via problem solving· approach enabled our students to better understand fraction concepts and engage in algebraic thinking. In this research, I analyzed the e-mail messages/discussions written to the sixth graders by the pre- service teachers in order to understand the pre-service teachers' content knowledge. This analysis enabled me to relate the pre service teachers' content knowledge to their implicit hypotheses about middle school students' knowledge (Simon, 1995) of fractions and algebra. In the presentation, the analysis and the findings of the study will be discussed.

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Changes of Names, Contents and Attitudes to Mathematical Units

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Will this material be on the exam? Why do I need to know this stuff? These are the sorts of questions that have been regularly asked from our mathematics students. Pre-service mathematics teachers often suggest that they do not need to learn anything that they do not have to teach. Generally, these students appear to have very little aesthetic appreciation for mathematics and its applications. Currently, we teach five traditional mathematical content units that are provided mainly for pre-service mathematics teachers. These units have been adapted and modified over the years from units that were designed primarily for science students. They contained a heavy focus on calculus with a limited breadth of mathematical experience. After consulting widely on the best mathematical practices throughout Australia and internationally, it was decided to reform all of the mathematics units to make them more attractive to a wider audience. The units that are currently being developed are: Profit, Loss and Gambling; Upon the Shoulders of Giants; Logic and Imagination; Modelling and Change; Algorithms, Bits and Bytes; Space, Shape, and Design; and Modelling Reality. The overall goal of this redevelopment is to improve student attitudes and motivation by exposing them to a wide range of topics in mathematics that are usable and relevant. All of these units will incorporate current technology, contain realistic problems, and include visiting speakers. Student assessment in these units will consist of portfolios, projects and examinations. The introduction of these new units will result in students having a greater choice of the units they wish to study. In order to overcome potential logistical problems of a small mathematics department, innovative changes to the structure of the units will also be examined. This paper will provide the details of the establishment of these units and some preliminary teacher and student responses from the first semester.

ON THE RELATION BETWEEN MATHEMATICS AND PHYSICS IN UNDERGRADUATE TEACHING

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History suggests that:

(a) Mathematics and Physics have always been interwoven (a two-way process):

- Mathematics is not just the "language" of Physics. It often determines the content and meaning of physical concepts and theories.

- Physics is not just a domain of application of Mathematics, providing problems ready-to-be-solved by existing mathematical tools. It also provides, ideas, methods and concepts, crucial for the creation and development of new Mathematics.

(b) Any distinction between Mathematics and Physics, seen as general attitudes towards the description and understanding of an (empirical, or mental) object, is related more to the point of view adopted while studying this object, than to the object itself.

(a) , (b) imply that:

- The histories of Mathematics and of Physics, seen independently, are incomplete.

- Accepting the importance of the historical dimension in education, the relation between Mathematics and Physics cannot be ignored in their teaching.

A historical approach summarized in the following scheme illustrates these points:

- The teacher though not a historian, has a basic knowledge of the subject's historical development. -Hence, the crucial steps of this development are appreciated by identifying key ideas, questions and problems, which opened new research perspectives.

- To become didactically appropriate, these steps are reconstructed in a modern context, often using modern terminology, notation and conceptual framework.

- Many details are presented as sequences of activities of increasing difficulty, each one building on its predecessors.

This approach is beneficial for both undergraduate students of Mathematics and of Physics. Examples (also supporting (a), (b)) range from the introduction of a concept to the development of a theory:

- Lorentz transformations, as an example of a transformation group with far reaching mathematical and physical consequences.

- The emergence of the abstract Hilbert space concept, from quantum mechanics.

Development of Calculus Concepts Through a Computer Based Learning Environment

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This paper investigates the students learning of calculus in undergraduate calculus course in a computer based learning environment in which Interactive Set Language (ISETL) and Derive were used. ISETL was used to help students to construct mathematical concepts on a computer, followed by the discussion held in the classroom. Derive was used to do the manipulations and to draw graphs. The study was carried out with first year undergraduate mathematics and mathematics education students. An essay type test measuring students' understanding of limit and derivative was developed and administered as a pre-test and post-test. Follow-up interviews were conducted with some randomly selected students. The analysis of written and verbal responses to the tasks given in the test or in the interview revealed significant information regarding the nature and the characteristics of students' concept images for key calculus concepts. The paper will address the following questions: ·How are the ISETL and Derive incorporated in the calculus course? · What are the students' errors and misconceptions on the tasks related to the derivative? ·Which of the errors or misconceptions seem to endure over time? ·Is there any improvement in learning derivative?

Avoiding Mathematics Trauma: Alternative Teaching Methods

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Children in primary education first encounter mathematics having picked up a general fear of mathematics from the society around them; this results in lack of confidence, avoidance of non-standard thought processes, weakness in problem solving strategies and other negative consequences. To overcome these, Mathematics Society, a student club at Izmir Institute of Technology (IZTECH), has developed a Mathematics Drama programme to present mathematics as dynamic entertainment. How successful have this amateur group been in addressing the needs of the students attending their shows? How have the undergraduate students in this group developed their knowledge and confidence? This presentation will first consider possible causes of the fear of mathematics, then look at the work of the Mathematics society and discuss its validity as a possible educational model. Finally we will present data from a survey of 500 pupils randomly selected from the 15 thousand who have attended the shows.

Main Theme: Distance Learning

Secondary Theme: Distance Learning

A New Kind of Precalculus Course

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This is a new computer-based course designed to reinvigorate students who have grown bored with standard issue mathematics. Unlike existing precalculus courses, it is also designed to confront actual calculus issues such as growth and areas without employing calculus tools of derivatives and integrals. The style is visual, experimental and fully interactive. By the time of the conference, this course will have been tested in a Chicago vocational high school.

Course Outline:

1. Functions and growth
2. Iteration. Car loans, battery discharge, logistic growth, predator-prey and Lancaster war model
3. Linear and exponential functions. The linear functions are those that post a constant average growth and the exponential functions are those that post a constant percent average growth. Exponential and linear data fit and analysis.
4. Oscillating functions, period and frequency. Approximating periodic functions with sums of sine and cosine waves.
5. Power functions. Exponential growth eventually beats power growth. Geometric sums. Approximating other functions with polynomials. Factoring.
6. Area measurement estimates. Monte Carlo and Riemann sums. Normal and exponential probability distributions.
7. Trig-Radian. Radians as lengths of trips on the unit circle. Unit circle and $\{\cos[t], \sin[t]\}$. Parametric plotting.
8. Rotate-Reflect and dot product. Rotations. Addition formulas for sine and cosine established through rotations. Reflecting across a line through $\{0,0\}$. Two successive reflections result in one rotation.
9. Equation solving
10. Arithmetic-Geometric mean inequality.

INVESTIGATION TASKS ON PREPARATION OF TEACHERS

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In last decades both problem solving and investigation tasks became an important approach to school mathematics (e.g. NCTM, 1980, 2000). Thus in the national portuguese mathematics instructional programs we can read that with an investigation, students can explore an open-task, recognize and look for patterns, make and test mathematical conjectures, draw conclusions and communicate with others in an oral or written way (DEB, 2000). We defend that any mathematical content can provide and create many opportunities to develop investigation tasks. These tasks should enable all students to apply a variety of mathematical knowledge that can be used purposefully and appropriately in contexts inside and outside mathematics; develop abilities of reasoning, communication and investigation; become more active in their own learning; and reinforce their elementary mathematics meanings. However, it becomes clear that successful implementation depend upon the ability of teachers to incorporate such approach into their classes. But, those recommendations for a change in mathematics education enable to a reform into the teachers preparation programs. So, research efforts must be directed to provide teachers with classroom ideas for translating those recommendations into classroom practice (Vale, 2000). We believe that future teachers must know and recognize the essential and power of investigation tasks, then we have to propose them the same kind of activities that we want they will propose to their students. The underlying idea of this exploratory study deal with the need to get a better understanding about preservice teachers when involved in investigation tasks. So we work with the students of the 3rd and the 4th years of the Math and Science course in a School of Education, in two disciplines (Number Theory and Didactics of Mathematics, respectively). Our fundamental question is the following: what level of performance and knowledge reveal the future teachers in the written investigation tasks? This poster presentation will try to answer the question above and will consider drawing some conclusions about mathematical investigations, mathematical learning and teacher preparation.

FRAMEWORK FOR INSTRUCTION AND ASSESSMENT ON ELEMENTARY INFERENTIAL STATISTICS THINKING

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The main objective in this paper is to describe a framework to characterize and assess the learning of statistical inference included in the Spanish secondary level curriculum. We first analyze briefly the innovations in the mathematics curriculum for the compulsory teaching levels in Spain and give a proposal for statistical inference teaching in classroom. Then we describe an framework to characterize statistical thinking in elementary statistics inference. The key constructs of the framework are: populations and samples and their relationships; inferential process; sample sizes; sampling types and biases. To refine and validate this scheme we have taken data from a 49 secondary students sample using a questionnaire having 12 items in tree different contexts: concrete, narrative and numeric. The analysis of the obtained results in this first phase has permit us to establish the key constructs described below and determine levels in them. Besides this has allowed us to determine the students' conceptions about the inference process and their perceptions about sampling possible biases and their sources.

An approach for the effective integration of computer algebra in an undergraduate calculus and linear algebra course

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In this paper we describe an approach for an effective integration of computer algebra systems in an elementary calculus and linear algebra course. In our mathematics courses at Wageningen University, an education and research centre for the biological, environmental, agrotechnical and social sciences, we have noted that students often show a lack of conceptual understanding while using computer algebra systems. A reason for this seems to be that the students do not establish a right link between the computer algebra techniques and their mental approach of mathematics. We have composed a framework that aims at establishing such a link. Because the students have developed their mathematical way of thinking in close relation with paper and pencil methods, this framework is based on an integration of computer algebra and paper and pencil techniques. We have used this framework for the set-up of an elementary calculus and linear algebra course for first year students in social sciences. In our paper we first describe this framework, which is made up of several steps. In these steps the use of paper and pencil and computer algebra alternate and reinforce each other. Next we show how we worked out this approach for an example from calculus: the determination of the stationary points and extremes of functions of two variables. In this example also the graphic facilities of the computer algebra system can be successfully exploited. The last part of the example is an application on maximising the total revenue of a production process, both without and with constraints.

Fostering student engagement in undergraduate Mathematics learning using a text-based online tool

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One of the big challenges academic teachers face today is the decline in student involvement with the university and in their academic performance. An Australian study on trends in the first year undergraduate experience (McInnis, 1995 and McInnis, James and Hartley, 2000) found that students are spending less time on campus and more time in paid employment. The studies indicate that compared to students who do not work, younger first year students who work part time are more likely to not work with other students on areas of their course, and to study inconsistently throughout the semester. Similar trends have also been reported in the US (Astin, 1998 and Kuh, 1998). Academics are being urged to put forward creative ideas to address this apparent lack of commitment, to think of new ways of engaging students that would fit with their lives. On-line environments and communication tools offer unparalleled opportunities to enrich the learning experience, to provide students with more flexible programs, and to foster student-student and staff-students interaction. However, given that the communication technologies available today present serious challenges for the communication of mathematics, the use of on-line environments in undergraduate mathematics teaching and learning is very limited. This paper will present an example of the introduction of the on-line text-based tool Interlearn to partially emulate the lively exchange of a face-to-face campus environment. The purpose of the on-line activities was twofold: firstly, they served as an organising device to help students work consistently throughout the semester, and secondly, they provided an opportunity to students to learn from each other. The trial involved a first year core unit that focuses primarily on the development of numeracy and science communication skills. We will describe our approach of integrating on-line activities to on-campus activities, and discuss the results of the trial, including student and staff evaluation. Finally, we will look at possibilities that on-line text-based tools such as Interlearn may open in the context of undergraduate mathematics teaching and learning.

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Piece-wise Affine Functions and Structures

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Piece-wise defined real functions on finite point-interval partition of the real line are reminded, esp. piece-wise affine (linear) ones. They can be understood as a union of affine functions $y = sx + t$, $s, t \in \mathbb{R}$, restricted to components of pi-partition of \mathbb{R} . All the information can be concentrated into a two row pi-table with changing columns of two kinds: intervals $(u_i; u_{i+1})$, with parameters s_i, t_i , of the corresponding affine part, $i=0,1,\dots,n$, and points u_i with functional values $f(u_i)$, $i = 1,\dots,n$. Such a function can be expressed uniquely as a linear combination of $x, 1, -x-u, \operatorname{sgn}(x-u), \operatorname{sgn}^2(x-u)$, where $u \in \{u_1, \dots, u_n\}$ are shifts. The formal and geometric approaches are combined. The disjoint union and restriction of functions are discussed, (dis)continuities classified. Step functions with special cases called mean, hiccup, "continuous", and esp. sign function of a function are studied. Presentation uses aggregation, level (fork, storey) schema, and pi-table structures. Attention is paid to methodical elaboration. The topic serves also as an example of structuring a text, where its internal relationships are used to reorganise it along the string, across its lines, and between its blocks with the aim to get together differences/similarities. Column close differences and alternatives are easy to recognise, remember, and discuss. The piece-wise defined functions in general do appear in many applications. Step functions are used e.g. as round-off, post-office fees, and pixels (another argument for full emancipation of these functions). Sgn reduces substantially the information about the internal function but its pi-table is sufficient for solving (in)equalities.

Statistical data analysis course via the Matlab Web Server

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Internet based courses are at present a quite common tool of learning. While still the most of them consist just of the static texts located on web pages, the objective of many new instruction systems is to make the learning process more dynamic and interactive in comparison with a mere reading the textbook or listening the lessons. Moreover, it is a feature of mathematics that it can hardly be studied by a mere memorizing the texts. That is why we searched, when preparing a series of internet courses on mathematical statistics, data analysis and quality control, for an environment enabling such an interaction and supporting the preparation and use of numerical and graphical procedures. Finally we decided to utilize the Matlab Web Server. In this environment, the author can combine the text with Matlab computational algorithms and graphical tools, the user can work with them without having its own Matlab installation. The system thus consists from text files (in html format), the Matlab programs and the procedures controlling the interface, input and output connection between the web pages accessed by the user and the Matlab algorithms (prepared by the author or used directly from Matlab toolboxes). In such a way, a student is provided simultaneously with relevant information, the examples, and graphs, he can enter his own data, too. The system we developed has also certain features of controlled and adaptable system. The student is offered the tests checking his knowledge, the sequence of tasks can be adapted along the feedback information on student's progress.

Teaching Mathematics

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1. The basic requirements of Mathematics teachers are learning and understanding; these are different but coexist. Learning is to gain knowledge, where understanding is for reproduction. To learn and teach mathematics is to do mathematics. Long term retention power with cognitive approach is desirable.
2. A teacher is a lifelong student. Teacher should possess qualities like abstract thinking, visualization, practical application with firm theoretical foundations, critical, experimental learning, dedication and perspiration for the cause of work.
3. The mission of teacher is to develop an urge and thirst of learning mathematics; to motivate the student and create an insight of the subject. It is his responsibility to make mathematics popular; Teacher has to take feedback and diversified evaluation. Teaching is a two way process. Teaching = lecturing + evaluation.
4. The foundation of Mathematics teaching is three dimensional! These are
 - (i) didactic (teaching plan in reference to philosophical (humanistic approach),
 - (ii) didactic (teaching plan in reference to students caliber) and
 - (iii) evaluation with efficacy (effective teaching).
5. Teacher's role is: institutional, instructional, expertise and self-identification. Teacher's strategies are
 - (1). A mediator in learning,
 - (2). Developer in pedagogic skills,
 - (3.) Integrator of multidimensional faculties and
 - (4). Evaluator of listener's ability.
6. Three aspects of mathematics teaching are (1) pedagogy (science of teaching), (2) androgogy (adult learning) and (3) mathetics (science of learner's behavior). Mathematics is a creative art and science of inductive -deductive combination. Mathetics of a student depends on the environment, caliber.
7. Various methods of Mathematics teaching are: inductive, deductive, analytic, synthetic, heuristic and genetic, cooperative and collaborative in addition to non-frontal and concentric and dogmatic one.
8. Good teaching of mathematics is subtle and intuitive.
9. Teaching aspect of mathematics can be put under the four heads: methods, devices, group communications; oral communications; technological approaches and mathematical modelling.

Using the web to enhance student learning

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When mathematics interacts with technology, the possibilities are complex. Technology can affect the way mathematics is done. It can also, though, have a profound effect on the way that we as academics manage our teaching and our interactions with students. The web provides opportunities which can significantly enhance the quality of the support which our students experience, while still maintaining the personal contact so necessary to a good education. We present here some of our recent experience in implementing a range of initiatives concerning the use of the web with mathematics undergraduates, as part of curriculum innovation involving the integrated use of technology both for doing and for learning mathematics. The particular module here involves the explicit critical study of mathematical technology, at first year undergraduate level. Matters arising have included:

- Full material support provided on the web.
- Communication networks (lists, discussion groups, etc).
- Automatic monitoring of student activity on the material of the module.
- The need for a new approach to assessment. This final point warrants further discussion. Traditional approaches to assessment of mathematical activities most frequently involve an examination, with a pass mark of say 40%, but little other feedback available to the student. This would be entirely inappropriate in this context. In this new approach the students may score up to 1000 points! Some of those marks are available for particular activities such as evaluating a piece of software, but for example small regular amounts are automatically accounted as they work from week to week and fill in a continuously monitored online learning diary. Thus as part of the approach, students acquire a marked profile of their range of skills and experience, and automatically receive a high degree of feedback on their progress.

Main Theme: Mathematics And Other Disciplines

Secondary Theme: Curricula Innovation

"Soon Unaccountable"

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The Mathematics Across the Curriculum Project at Dartmouth College produced a number of new courses integrating mathematics with a humanistic discipline such as literature, art, or philosophy. These courses were all free of any prerequisite and attracted a wide variety of students. The mathematical topics were chosen for their relative modernity and sophistication, e.g. group theory, infinity, or the fourth dimension. How does one come up with math that can be offered in these interdisciplinary courses? How do you present it in a way that isn't trivial? What sort of understanding is it reasonable expect students to carry away as a result of such a class? Why is it worth the trouble to educate this body of students in this particular way? In this talk we will consider these questions and get a glimpse into some unusual courses. This paper complements but does not duplicate the one by Jane Korey.

Factors Impacting Success in Developmental and Subsequent Mathematics Courses

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This presentation will report on a major developmental mathematics collaborative among five community colleges in Virginia carried out during a spring, 2000 sabbatical. Demographic and descriptive data-including gender, age, credit hours, enrollment, teaching method, attendance, class size, classroom participation, student success and retention rates-will be collected and analyzed to determine best scenarios for success in developmental mathematics classes. A recent report by the Institute for Higher Education Policy debunks (p.v) the misconceptions that remediation is too expensive and is an inappropriate function for colleges. Instead the report argues that remediation is a core function of higher education and a good investment for society as well as for colleges and universities. According to Jamie P. Merisotis, the institute's president, one of our concerns with the debate about college remediation is that there really hasn't been a whole lot of factual discussion about what remediation is, how it works, and the impact proposed policy changes might have. (Remedial). According to David W. Breneman, Dean of the College of Education at the University of Virginia, the report's findings mirrored those of a remedial-education study that he and another researcher published this summer. (Woodham). The Institute argues that as higher education strives to educate the populace, remediation will continue to be a core function of college and universities (p.6) and proposes a set of strategies designed to reduce the need for remediation in higher education while also enhancing its effectiveness (p.v). The executive summary of the report presents information, which should be considered in any debate regarding developmental education. First the report argues (p. vi) that the "financial costs of remediation are modest and generally comparable to or lower than the costs of other academic programs" (p.vi). Remediation absorbs less than 1 percent—\$1 billion of the \$115 billion annual higher education budget (p.12)—of expenditures, a relatively small proportion. The report goes on to posit that even if "remedial education were terminated at every college and university in the country, it is unlikely that the money would be put to better use" (p.vi). A 1995 survey by the National Center for Education Statistics (NCES) found that 78 percent of higher educational institutions that enroll freshmen and 100 percent of public two-year institutions offered remedial courses (Institute, pp. v-vi). Twenty-nine percent, as compared to 30 percent in 1989, of first-time freshmen enrolled in at least one of these remedial courses, and freshmen were more likely to enroll in a remedial mathematics courses than in a remedial reading or writing course. In fact, a recent study of remediation by the Maryland Higher Education Commission found that for students who completed college-preparatory courses in high school and immediately attended a community college, 40 percent needed math remediation (Institute, p.8). Consistent with the commission's finding, a recent local community college study (July, 1998) showed that students taking developmental work account for over 40 percent of the graduates. Such statistics mandate needed research on developmental courses to implement changes to ensure quality programs which will adequately prepare students for college level courses. A final concern of the Institute was that evaluation of remedial programs was minimal. Findings from their study of 116 two- and four-year colleges and universities found "that only a small percentage conducted any systematic evaluation of their remedial education programs" (p.10). Furthermore, the Southern Regional Education Board has raised the issue

about the effectiveness of remedial programs by observing that “few states have exit standards for remedial courses” (Institute, p.11). The Institute’s report concludes by proposing strategies for the future—two mutually reinforcing goals (p.ix):

- (1) Reducing the need for remediation in higher education, and
- (2) Improving the effectiveness of remedial education in higher education.

This study concentrated on the latter of these two charges—to improve the effectiveness of the developmental mathematics programs in higher education. The Institute’s report lists three strategies to improve the effectiveness of remedial education. The first of these strategies—(c)reating interinstitutional collaboration among colleges and universities in a state or system, allowing best practices and ideas to be shared and replicated—is consistent with the charges to the 1998-99 state Developmental Studies Implementation Task Force. These charges require systemwide collaboration for standardized test interpretation, common objectives, exit criteria, and assessment methods for developmental courses. This research is a response, first, to the Institute’s recommendation to collaborate with other college faculty for best practices and second, to the state Task Force’s efforts toward standardization in the field of developmental mathematics programs in the VCCS. The specific courses under study are Arithmetic, Basic Algebra I, and Basic Algebra II. Descriptive data, including credit hours, enrollment, attendance, class size, classroom participation, and success and retention rates will be collected. In addition placement procedures for developmental courses, methods of instruction, and other anecdotal data provide further insight. Findings from this study assist in maximizing the effectiveness of teaching strategies for developmental mathematics. Recommended changes will ensure an optimal environment for student learning to take place.

Students Understanding of Exponential and Logarithmic Functions

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Investigations have shown that pre-calculus and calculus students have difficulty understanding the concepts of exponential and logarithmic functions. In this paper, we analyze students' difficulties with these concepts within Dubinsky's APOS theoretical framework. In Dubinsky's theory, concepts are first understood as actions. Actions are transformations of objects which are usually performed by the student due to an external cue. If a student reflects upon the steps needed to perform the action, the student may interiorize the action as a process. Students with this understanding can imagine the output of applying an action without actually applying it, describe how the action is implemented, and even reverse the action. This theory also describes how a process can be encapsulated into an object and assimilated into a schema, but these two constructs will not be used extensively in this paper's analysis. In this paper, we use the APOS theoretical framework to propose stages that students progress through when they learn the concepts of exponential and logarithmic functions, including what tasks students at each stage should and should not be able to complete. We also report on in-depth interviews conducted with 15 college undergraduates in a pre-calculus course. These interviews illustrate that most students' knowledge of exponents, exponential functions, and logarithms consists of a combination of an action-level understanding of exponents and a large set of symbolic rules that are unstructured to the students. Students frequently misapply these rules, often cannot use these rules to solve relatively simple problems, and seldom can explain why these rules are valid. Further, these interviews suggest that most students do not have a process understanding of exponents, and this lack of understanding can account for many of their difficulties.

The roles of instrumental and relational understanding in constructing proofs about group homomorphisms

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The ability to construct proofs in advanced mathematical domains is crucial to any mathematician, yet proof construction is notoriously difficult for students. The purpose of this study is to investigate what deficiencies in students' knowledge lead to students' failures to construct proofs about group homomorphisms. Our findings suggest that a key deficiency is that students often possess an instrumental rather than a relational understanding of the theorems from group theory and thus cannot use these theorems effectively. In this paper, we report on a study of undergraduates and doctoral students. This study incorporated the following measures:

- (1) The participants were asked to "think aloud" while attempting to prove a small set of propositions about homomorphisms.
- (2) After completing a proof, the participants were asked why they used the proof technique that they did.
- (3) Afterwards, tests were performed to see if the participants had the instrumental understanding necessary to prove the theorems in this study.

Specifically, we checked if participants were aware of key facts and theorems and if they could apply them in specific instances. We observed many instances where undergraduates failed to construct a proof despite appearing to possess the instrumental knowledge required to do so. All participants appeared to have an accurate conception of what constitutes a valid proof; hence, possessing an accurate conception of proof and an instrumental understanding of relevant facts does not appear to be sufficient for one to construct proofs. Our observations of and conversations with the participants indicate that doctoral students possessed a relational understanding of many group theory facts while undergraduates generally did not. (e.g. doctoral students thought of isomorphisms as demonstrating "sameness" while undergraduates viewed them strictly as bijective mappings). This difference in relational understanding can explain why doctoral students chose to use effective proof strategies while undergraduates did not.

Main Theme: Preparation of Teachers

Secondary Theme: Curricula Innovation

COLLABORATIONS: Rethinking Excellence in Teacher Education (C.R.E.T.E.)

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Mathematics educators in urban areas are faced with multiple crises, including severe shortages in local schools, demands for higher standards at all levels (resulting in greater emphasis on testing), and the lack of qualified individuals in the pipeline. During the past decade the preparation of teachers has shifted dramatically from traditional programs for undergraduate mathematics majors to graduate level programs for career-changers whose mathematics backgrounds vary as widely as their professions. These programs must be designed or modified to meet demands of new technologies, linguistically diverse student and teaching populations, and the strategies used by local school districts to reform mathematics teaching. Brooklyn College has met these challenges through a series of creative collaborations - interdepartmental, with local schools, and with museums, etc. to design recruitment and stepping stone programs to meet the needs of the non-traditional prospective teacher. These programs have begun to fill the secondary school pipeline with qualified middle school teachers of mathematics. These collaborations have led to: * new pedagogical models used implicitly in mathematics courses,* the integration of writing in paired mathematics and pedagogy courses, * the use of reflective journals to promote problem solving,* the implementation of portfolios and exhibits as assessment tools,* the use of new standards-based curricula,* museum-based projects,* online foundations courses which focus on mathematics/science teaching, and* co-teaching among mathematicians, mathematics educators, and master teachers (K-12). Samples of the above and student work will be shared during the poster session and on our website.

Mathematics for Computer Games Technology.

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Despite their entertainment focus, Computer Games are at the forefront of computer science. Successful computer games (programs) make strong calls on a number of other disciplines, psychology, mathematics and statistics to name but a few. Enrolment numbers in mathematics courses across Australian Universities are in decline, however we have found that students who have enrolled in our new degree, Bachelor of Computer Science (Games Technology), are not just tolerant, but are enthusiastic about the mathematics component of the degree. In this paper we describe the mathematical demands of games technology along with the sub development of the mathematical component of the degree. Our program, being a full strength computer science degree demands some standard mathematical components, discrete mathematics, linear algebra, numerical analysis, statistics and ordinary differential equations. The strong emphasis on computer graphics programming needs a firm foundation in aspects of linear algebra; the virtual world of the game scene development is underpinned by the physics of movement. Acceleration, cornering, collisions explosions and disintegration all require ODE's. A subtle shift in the shape of a probability distribution can help to maintain game balance and player interest by giving the underdog an unseen helping hand. Logic, computational complexity and numerical analysis speak for themselves. Mathematics educators have always known that relevance is a strong motivator of mathematics. This has again been ably demonstrated by our first cohort of games technology students who want more maths.

Perceptions of difficulty

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Seventy students from a first semester calculus course ranked 8 mathematics tasks as to perceived difficulty before the task and actual difficulty after completing the task. The tasks were designed to fit into a taxonomy of mathematical skills. This was our MATH taxonomy, based on Bloom's taxonomy. The full taxonomy has 8 categories, falling into 3 main groups. The first group (A) encompasses tasks which can be successfully done using a surface learning approach, while the other two (B&C) require a deeper learning approach for their successful completion. Students also completed two examinations, one based on facts and procedures (category A) and the other based on applications and concepts (categories B and C). There were significant changes in the rankings after the students had completed the tasks and the students reflected this in open-ended comments. Some of the biggest changes were in the tasks that required routine procedures. Typical of the responses was the following comment "We totally underestimated the hardness of the questions at first glance. Closer inspection of the question revealed the exact nature of the question." These findings suggest that the style of the examination paper may have a significant effect on student performance, with students adopting ineffective strategies or avoiding certain questions, based on misconceptions of difficulty. This paper provides empirical evidence to show that students approach learning tasks in very different ways and that providing questions that support a deep approach encourages students to take that approach.

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Main Theme: Technology

Secondary Theme: No secondary theme

NetMath Paving the Way on the Web

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NetMath at University of Illinois has been offering Internet math classes on the web for the past 10 years. These courses are based on lessons written for use with technology providing an interactive environment in which students can learn hands-on. Part of the success of the program is reflected by a 72% retention/pass rate as opposed to a less than 50% retention rate reported by other on-line programs. Undergraduate mentors and interactive chats support the NetMath students with live interactive guidance. The web-based interface was custom written by a former NetMath student. The talk will concentrate on how we run the courses.

Using PC and TI-92 in teaching Statistics in Austrian Secondary Schools

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After the recent reforms of the curricula in mathematics, statistics and the use of the computer were fixed in mathematical instruction for ten to fourteen year-old students (1993, 2000). In grades 5 and 6 the concepts absolute and relative frequency, mode, arithmetic mean, median and different possibilities to plot graphs (pictogram, pie graph, bar graph, line graph, polygon) were integrated. The students are allowed to use hand calculators and often also to use spreadsheets when they work on the computer. As spreadsheet the teachers generally use EXCEL if the secondary school has determined to make the introduction in computer science a new subject in grade 5 or 6. Since the Austrian CAS II project in 1997/98, the use of the TI-92 has been tested in many classes. With the TI-92 it is possible to get a boxplot with the different quartiles of a set of data very quickly. But it also offers the teacher a good chance to acquaint the students of grade 8 with such difficult concepts as linear and geometrical regression and correlation. In the lecture I will show the way I have worked with students of grade 8 and with teacher students at university. It is very important not to take sets of data out of the school books only. I allow the students of grade 8 to work with their own dates (length and mass) or I let them find real data with the help of CBL (calculator-based laboratory) in an experimental way. Thus they get a better understanding of the concepts of regression and correlation.

USING MATHEMATICA IN TEACHING ROMBERG INTEGRATION

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High order approximations of an integral can be obtained by taking the linear combination of lower degree approximations in a systematic way. One of these approaches for approximating 1d integrals (I) is known as Romberg Integration and is based upon the composite trapezoidal rule approximations (T_{i1}) and the well-known Euler-Maclaurin expansion of the error formula. Composite approximations of polynomial order 1 are obtained by bisecting the domain of integration, and applying the original trapezoidal rule to each of the subintervals repeatedly. Linear combinations of T_{i1} form the so-called triangular Romberg table entries, T_{ij}'s and each column (with index j) increases the polynomial order by 2. Because of the complexity of the underlying theory, students in an introductory Numerical Analysis class usually find the theoretical aspects difficult to comprehend. In order to overcome the difficulties, a symbolic manipulation software, Mathematica, is utilized to illustrate the different aspects of the method and the underlying theory. For this purpose, a Mathematica program is written and a set of experiments are designed to explain the intermediate steps in deriving the method following a stepwise approach. Variations of the method are demonstrated by means of symbolic and numerical examples.

Main Theme: Distance Learning

Secondary Theme: Mathematics And Other Disciplines

Quality Connection: going the distance

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Virginia Beach City Public Schools launched a new initiative in February 1999 - distance learning videoconferencing. The program was initially designed to offer additional curricular choices to students and expanded training opportunities for staff in real time, but the program has grown exponentially. Not only has DL created expanded opportunities, the technology has liberated students and staff from the confines of budget and schedules. In fact, the DL program · Quality Connection: Going the Distance · has revolutionized the way the division does business. Though modest in its beginnings with the installation of DL labs at only three of the district's high schools and with only one course offering in discrete mathematics, the DL program expanded rapidly. DL capabilities have continued to expand where all 11 Beach high schools originate and receive over 20 courses this fall. Most recently, five middle schools have come on line, with more classes scheduled to begin in January. In addition to the discrete mathematics offering, other DL courses available to our students include AP Statistics and Pre-IB Algebra II/Trig. Videoconferencing technology supports the various pedagogical strategies promoted by standards-based mathematics educators. The document camera is the heart of most instruction. Technology such as the graphing calculator, algebra tiles, and PC applications are effectively employed. The current emphasis on student learning through communication of mathematics is complemented utilizing site-to-site communications enabled by DL. A demonstration can be arranged provided there is comparable videoconferencing technology at the conference. The division's motto is ·Ahead of the Curve·. As far as we are concerned, that is where we are collectively, all 86 schools, and 10,200 employees. And, that is where we intend to stay.

Algebra, Computer Algebra, and Mathematical Thinking

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Mathematical symbolism generally---and symbolic algebra in particular---is among mathematics' most powerful intellectual and practical tools. Knowing mathematics well enough to use it effectively requires a degree of comfort and ease with basic symbolics. Helping students acquire symbolic fluency and intuition has traditionally been an important, and sometimes daunting, goal of mathematics education. Cheap, convenient, and widely available technologies can now handle a good share of the standard symbolic operations of undergraduate mathematics: differentiation, integration, solution of certain DEs, factoring and expansion in many forms, and so on. Does it follow that teaching these topics, and even some of the techniques, is now a waste of time? The short answer is "no." On the contrary, as machines do more and more lower-level symbolic operations, higher-level thinking and deeper understanding of what is really happening become more, not less, important. Numerical computing has not made numerical viewpoints obsolete; neither will computer algebra render symbolic mathematics obsolete. The key question is how to help students develop that bred-in-the-bone "symbol sense" that all mathematicians seem to have. What really matters is that students use mathematical symbolism effectively to pose worthwhile problems in tractable forms. Once properly posed, such problems are well on the way to solution, very often with the help of technology. The longer answer, which I'll explore in the paper, concerns choosing mathematical content and pedagogical strategies wisely in light of today's technology.

Implementing a "European" approach to mathematics education in Indonesia through teacher education

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This paper reports on the results of the final phase of a four-year study called CASCADE-IMEI that is a learning environment (LE) in the form of a face-to-face course and a web site (www.clix.to/zulkardi) which aims to introduce Realistic Mathematics Education (RME), Dutch approach to mathematics education, as an innovative teaching methods in Indonesia through prospective mathematics teachers in initial teacher education. It also presents the background of mathematics reform in Indonesia by adapting RME as a promising approach. Then, the paper describes the process of a development research methodology in which three prototypes of the LE have been developed and evaluated both by prospective mathematics teachers in Indonesian Educational University in Bandung and several experts in the Netherlands. Finally, it will discuss the changes on the prospective mathematics teachers after they followed the LE program with a more detailed on their teaching performance in junior secondary mathematics classroom.

Key words: mathematics learning environment(www), RME teaching method, development research.

