‘LIFE WASN’T MEANT TO BE EASY’: SEPARATING WHEAT FROM CHAFF IN TECHNOLOGY AIDED LEARNING?

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ABSTRACT

The paper commences by reviewing some of the issues currently being raised with respect to the use of technology in undergraduate mathematics teaching and learning. Selected material from three research projects is used to address a series of questions. The questions relate to the use of symbolic manipulators in tertiary mathematics, to undergraduate student attitudes towards the use of computers in learning mathematics, and to outcomes of using technology in collaborative student activity in pre-university classrooms. Results suggest that teaching demands are increased rather than decreased by the use of technology, that attitudes to mathematics and to computers occupy different dimensions, and that students adopt different preferences in the way they utilise available resources. These outcomes are reflected back on the literature, and implications for teaching, learning, and research discussed.

KEYWORDS: undergraduate; mathematics; technology; Maple; graphical calculators; attitudes; collaborative learning.

1. Introduction

In this paper I want to reflect on outcomes from three research projects that span the interface from senior school to undergraduate programs. The common elements in the programs are mathematics, students, and technology. The purpose is to describe findings from the selected research foci, and relate them to matters raised in the wider literature, and to implications for theory and practice.

Papers addressing the use of technology in undergraduate mathematics make for interesting and varied reading. For example:

- The impact upon educational practice of powerful software like Mathematica has been less profound than optimists hoped or pessimists feared…tendency to begin by looking for electronic ways of doing the familiar jobs previously done by textbooks and lectures. (Ramsden, 1997).
- Of all the flaws in our mathematics training this seemed to us to be the most dangerous and insidious, for as we removed mathematics from our courses in response to 'student failings', the need for mathematics to do real science was in fact increasing…firstly there was the pious hope that a computer assisted approach would require less staff…problems arose from attempts to use Mathematica in two ways- which were incompatible. Was software an arena for exploration of mathematical ideas, or a channel for their transmission? (Templer et al, 1998)
- There is growing evidence (in the UK and elsewhere) of a general decline in the mathematics preparedness of science and engineering undergraduates…one response has been to simply reduce the mathematics content and to rely on computer-based tools to do much of the mathematical computation…difficult questions (emerge) at the intersection of cognitive and epistemological domains; to what extent must the structure of mathematics be understood in order for it to be used effectively as a tool? (Kent & Stevenson, 1999)

These excerpts canvass some of the challenging and problematic issues that are emerging in undergraduate mathematics education. The discussion that follows will raise issues associated with the use of symbolic manipulators as central agents for teaching and learning undergraduate mathematics; with affective characteristics of students using technology in undergraduate coursework; and with the use of technology in collaborative learning activity. The latter project has been implemented with pre-university school mathematics students as subjects. The qualities displayed by the students, and their approaches to learning have implications for the undergraduate programs in which they subsequently enrol.

2. Background

One fundamental component of any discussion of undergraduate learning is the composition and background of the student cohort. As noted above (Kent & Stevenson, 1999) the widening of secondary education, and curriculum decisions in relation to school mathematics, mean that the mathematical preparedness of entering undergraduates is perceived to be changing. Clearly this perception is impacting on course design and teaching approaches, in particular in the way that technology is utilised. However a nostalgic review of the past should not obscure the reality that there were really no "good old days". Studies addressing the (mis) understanding of basic concepts and procedures displayed by undergraduate mathematics students have been reported over a substantial period of time. Findings from these studies have a common theme viz. that the standard of performance of the 'current' student group is much lower than hoped for, given the investment of time and energy that has been directed towards the teaching and learning of mathematics over many years.

Characteristics of flawed performance have been historically consistent over a quarter of a century:

> ...After twelve years of schooling followed by two years of university, they had all but accepted the mindless mathematics that had been thrust upon them...Misconceptions, misguided and underdeveloped methods, unrefined intuition tend to remain assignments, corrections, solutions, tutorials, lectures and examinations notwithstanding. (Gray, 1975)
>
> It appears that students have developed special purpose translation algorithms, which work for many text book problems, but which do not involve anything that could reasonably be called a semantic understanding of algebra. (Clement et al., 1980)
>
> Weaker students suffered from the continued misinterpretation that algebra is a menagerie of disconnected rules to do with different contexts. (Tall & Razali, 1993)
>
> In attending module after module, students tended to 'memory dump' rather than to retain and build a coherent knowledge structure...Their presumed examination strategy resulted in such a fragile understanding that reconstructing forgotten knowledge seemed alien to many taking part. (Anderson et al, 1998)

A common thread running through these studies is the powerful negative influence of fragmented learning, and the apparent absence of cognitive strategies to co-ordinate conceptual and procedural knowledge. The successive comments can be read as evidence supporting the constructivist paradigm, for students continue to carry mathematical 'baggage' and habits that inhibit the goals of instructors hoping to provide a fresh beginning in tertiary mathematics. Into the wake of this historical legacy, curriculum reforms and innovative teaching methods (often incorporating electronic technologies), have been injected as fountains of hope, at times accompanied by extravagant claims.

3. Focus A: Computer-Based Undergraduate Programs

The form of computer-based instruction varies widely, indicative of a range of beliefs among program designers and instructors - both about mathematics, and the nature of mathematics learning. Olsen (1999) discusses one of the most extensive examples of technology used to provide automated instruction. She describes (page 31) how politicians visiting Virginia Tech's Mathematics Emporium, a 58 000 square foot (1.5-acre) computer classroom:

> see a model of institutional productivity; a vision of the future in which machines handle many kinds of undergraduate teaching duties-and universities pay fewer professors to lecture…On weekdays from 9 am to midnight dozens of tutors and helpers stroll along the hexagonal pods on which the computers are located. They are trying to spot the students who are stuck on a problem and need help.

This program appears to be openly driven by economic rationalism, and an assumption that mathematics is something primarily to be delivered and consumed. By contrast Shneiderman et al (1998) describe a model, in which electronic classroom infrastructure is extensive and expensive. Courses are scheduled into electronic classrooms, following a competitive proposal process, requiring full use of an interactive, collaborative, multi-media environment. Active engagement with a variety of learning tools is highly valued here.

In between the extremes occur a variety of models of instruction, concerned in varying degrees with factory production on the one hand, and student understanding and engagement on the other, and it is instructive to note comments from those describing the characteristics of such programs: here are some selections.

Templer et al (1998) noted problems accompanying efforts to provide meaningful learning that were perceived to arise as a direct result of a symbolic manipulator (*Mathematica*) environment. They noted that typically having mastered the rudiments, the majority of students:

> "began to hurtle through the work, hell bent on finishing everything in the shortest possible time."

The following comment, or a close relative, was noted as occurring frequently among the students:

"I just don't understand what I'm learning here. I mean all I have to do is ask the machine to solve the problem and it's done. What have I learned?"

Kent & Stevenson (1998) in elaborating on their concerns about student quality (see Introduction), question whether mathematical procedures can be learned effectively without an appreciation of their place in the structure of mathematics. They argue that unless some kind of breakdown in the functionality of some concept or procedure (say integration) is provoked, students do not focus on the essential aspects of that concept or procedure. On the other hand they observed that the demands for formal precision that a programming environment places on its user, serves both to expose any fragility in understanding, and to support the building and conjecturing required in the re (construction) of concepts by learners. This comment interfaces with a debate about whether computer technology should be employed following prior understanding of mathematical concepts and procedures (Harris, 2000), or as a means integral to the development of such understanding (Roddick, 2001).

Interesting comment has been made also about specific issues relating to the introduction of technology into mathematics learning settings. Templer et al (1998) noted that the screen dominated the attention of most (although not all) students, and that some balance needs to be struck between directing students from paper to screen, and vice-versa. A lack of symmetry was evident in that some students are reluctant to move from screen to text, whereas the move the other way is more flexibly undertaken. An interesting slant on the 'how and when' debate is provided by the observation that mathematical 'tools' are forged through use, in contrast to conventional tools that are first made and then used. This then calls into question a sequence that seeks first to master a tool and then apply it. Specifically whether training in a manipulator such as *Mathematica, Derive, or Maple* requires prior time and effort, or whether a careful design can enable mathematics to be learned and applied contiguously with increasingly sophisticated manipulator use? Clearly this matter is not yet resolved.

3.1 Research Program

The teaching programs that form the background for this section of research took place at the University of Queensland during the period 1997-2000. As mainstream courses located between the extremes described above they represent models that may be located comfortably within present university structures and resources. The programs involve the use of *Maple* in first year undergraduate teaching, and issues associated with implementation connect with those of other researchers as sampled above. In keeping with Kent & Stevenson (1998) there is interest in the range of questions raised by students as they work with the software, as well as in their performance. With Templer et al (1998) there is concern with the links between computer-controlled processes and their mathematical underpinnings, noting the similarities and differences between the respective symbolism. This project had several aims, including the following:

1. *To classify the range of student-generated questions that emerge when learning of mathematical content interacts with a symbolic manipulator environment.*

2. *To identify structural properties associated with the Maple environment that can be identified as linking task demand and student success.*

The research was conducted within first-year undergraduate mathematics courses taken by students studying mainly within Science and Engineering degree programs. As taught in 1999 and 2000 the courses comprised a lecture series complemented by weekly workshops, in which approximately 40 students were timetabled into a laboratory containing networked computers equipped with *Maple* software. The lecture room was fitted with computer display facilities so *Maple* processing was an integral and continuing part of the lecture presentation. To support their workshop activity students were provided with a teaching manual (Pemberton, 1997), continually updated to contain explanations of all *Maple* commands used in the course, together with many illustrative examples. During laboratory workshops two tutors and frequently the lecturer also, were available to assist the students working on tasks structured through the provision of weekly worksheets. The students could consult with the lecturer during limited additional office hours, and unscheduled additional access to the laboratory was available for approximately 5 hours per week. The course was also available on the Web. Solutions to the weekly worksheets were provided subsequently.

The formal course assessment was constrained by departmental protocol and the availability of facilities. The major component comprised pen and paper exams at mid-semester and at end of semester (combined 80%). The balance consisted of *Maple* based assignments and a mark assigned on the basis of tutorial work (20%). To succeed students needed to transfer their learning and expertise substantially from software supported environment to written format, which means that they must be able to develop understanding through the medium with which they work, while simultaneously achieving independence from it. This involves the ability to learn and maintain procedures that a *Maple* environment does not enforce, so that attention is focused on the relationship between the mathematical demands of tasks, and their representation in a *Maple* learningscape.

3.2 Data sources

The data for addressing these questions come from two sources. Tutors assigned to the *Maple* workshops were provided with diaries in which they entered, on a weekly basis, examples indicative of the range of questions raised by students in the course of their workshop activity. The second source of data was a test given 7 weeks after the program started. This test was a voluntary exercise, and comprised a series of questions to be addressed with the assistance of *Maple* in its laboratory context. It provided formative feedback to the students on their performance, and ranged from simple school level manipulations to new material introduced in the tertiary program. Sample questions are included in the appendix, together with their *Maple* solutions. The test was directly relevant to preparing for the formal assessment at the end of semester, for the procedures required were ones that the students need to be proficient with, irrespective of software support. The tests were analysed and marked by two of the course tutors using criteria designed by the researchers. For this purpose the quality or indeed presence of a final interpretation of graphical output was not taken into account, so that the correct/incorrect dichotomy was on the basis of *Maple* operations only. On the basis of a review of the 250 (approx.) scripts submitted, it appeared that the first 16 questions had been attempted seriously by the whole group. For technical reasons two of these were deemed unsuitable for inclusion, so that responses to 14 questions formed the final data set.

3.3 Regression Analysis

Performance was analysed in terms of the influence of two categories labelled SYNTAX and FUNCTION respectively.

SYNTAX: refers to the general *Maple* definitions necessary for the successful execution of commands. These include the correct use of brackets in general expressions, and common symbols representing a specific syntax different from that normally used in scripting mathematical statements (such as *, ^, Pi, g:=).

FUNCTION: refers to the selection and specification of particular functions appropriate to the task at hand. Specific internal syntax required in specifying a function is regarded as part of the FUNCTION component, including brackets when used for this purpose. Complexity is represented by a simple count of the individual components required in successful operation. The way these definitions work is illustrated by applying them to the examples given in the appendix.

Q2. SYNTAX: Incidence of ^ [2] *plus* * [2]; total=4.
FUNCTION: General structural form of factor (argument); factor [1] *plus* ( ) [1] *plus* argument entry [1]; total=3.
Q8. SYNTAX: Incidence of ^ [1] *plus* *[2] *plus* ( ) [2] *plus* x1[1] *plus* := [1]; total=7.
FUNCTION: General structural form of plot (function, domain); plot [1] *plus* ( ) [1] *plus* , [1] *plus* function entry [1] *plus* domain entry [1] *plus* domain specification [1]; sub-total=6.
General structural form of fsolve (function, domain); sub-total [5] *plus* domain specification[1]; total =12.
Q14. SYNTAX: Incidence of*[2] *plus* ( ) [3]; *plus* y [1] *plus*: = [1]; total=7.
FUNCTION: General structural form of plot(function, domain); sub-total [5] *plus* domain specification [1];
General structural form of int(y, integ interval); sub-total [5] *plus* (subtraction) [1] *plus* integration interval specifications [2]; total=14.

Similar pairs were assigned to each of the 14 questions in the sample. Our diagnostic approach involves scoring on a correct/incorrect basis, as we are not (in this analysis) concerned with apportioning partial credit as would be necessary if grading student performance. The success rate on the questions is given by the fraction of students (N~ 250) obtaining the correct answer. We can regard these as providing a measure of the probability of success of a student from this group on the respective questions. For the questions in the Appendix the respective values are 0.89, 0.26, and 0.14. A linear regression analysis was performed using these probabilities as measures of the dependent variable (success), and SYNTAX and FUNCTION as input variables (Tables 1 & 2).

Table 1: Regression statistics

Multiple R	0.8710
R Square	0.7586
Adjusted R Square	0.7148
Standard Error	0.1419
Observations	14

Table2: Regression Statistics cont

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.0947	0.0961	11.383	2E-07
SYNTAX	-0.0482	0.0168	-2.874	0.015*
FUNCTION	-0.0396	0.0122	-3.246	0.008**

According to this analysis both the SYNTAX ($p<.05$) and FUNCTION ($p<.01$) complexity measures contributed significantly to the task demand of the questions.

3.4 Student-generated questions (question 2)

A total of over 1300 questions indicative of the range of concerns displayed by students in the 2000 cohort when working mathematically in a *Maple* environment, was assembled from the tutor diaries. The categories were selected using a mix of empirical judgment, theoretical positioning, and the results of a pilot study in the previous year. The distribution is shown in Table 3.The number of questions per category varied from a maximum of 333 (24.6%) to a minimum of 29 (2.2%). The number of questions in which some aspect of *Maple* was unequivocally involved exceeded 80%.

Table 3: Student Question Types

Question Category	Percentage
1. Identify problem caused by a typo (TYPO)	8.4%
2. Resolve syntax error (SYN)	24.6%
3. Problem with function choice (FCHCE)	4.2%
4. Problem specifying function (FSPEC)	14.6 %
5. Stuck on mathematics (STMATH)	14.9 %
6. Procedurally stuck on Maple (STMAPLE)	19.5 %
7. Interpreting aspects of output (INTOUT)	11.6 %
8. General procedural (PROC)	2.2 %

The patterns evident in Table 3 confirm that when students interact with mathematics through technology, questions are generated rapidly and their scope is vastly increased. We can identify at least four types of inquiry from the responses. Those that are simply procedural (what to do next); those that are mathematical in the traditional sense; those that are software related (syntax and symbols); and those that are generated by the interaction of mathematics with software (function choice and specification). The intensity and scope of student questioning has escalated in comparison with traditional practice classes, with software the major contributor through properties of fast processing, scope for formatting and specification errors, just plain knowledge blocks in bringing the mathematics and software together, together with student initiative in exploring. In examining the analysis relevant to the first question, it can be observed that while achieving more rapid and efficient closure to algorithmic procedures the use of *Maple* has not reduced the need for the mathematical attributes of understanding and attention to detail. We note this in the significant impact of the variables SYNTAX and FUNCTION on success rate. SYNTAX errors penalise those who lack sufficient care in expressing their work symbolically, while the demands imposed by FUNCTION are proportional to the principles and sophistication of the associated mathematics. On the other hand, for those students who possess conceptual understanding and due regard for precision, the *Maple* environment has provided a means to progress rapidly and successfully at a greater rate than could otherwise be achieved. Our conclusion to this point is that there is no ‘free lunch’ (indeed laboratory tutors are lucky to get lunch at all). The propensity of students to alter their approach to reduce the learning potential available to them is apparent. Properties arising from the mutual interaction of students, mathematics, and technology can support approaches extending beyond the models that still seem to motivate some proponents of automated learning – models with goals of doing faster and more cheaply that which was done formerly with blackboard, chalk, and paper. These are limited goals indeed. The present research contributes to this broader endeavour, both in terms of identifying and classifying student responses to laboratory activities, and in linking mathematical demand to the complexity of manipulator operations and task success.

3. Focus B: Student attitudes to mathematics and technology

While there have been enthusiastic claims for the positive impact of technology on the teaching and learning of mathematics, systematic evaluations of impact have been harder to access. And while the study of *attitudes* in mathematics learning has a substantial history, the relationship between *attitude* and *performance* is not clear-cut although positive correlations have often been noted between these characteristics. Early claims that affective variables can predict achievement (e.g. Fennema & Sherman, 1978) have been balanced by later comments (e.g. Schoenfeld, 1989) indicating that research does not give a clear picture of the direction of causal relationships. Ma & Kishor (1997) set out to assess the directional relationship between attitude and achievement but their meta-study was essentially correlational, so that the Tartre & Fennema (1995) comment that described *confidence* as the affective variable most consistently related to mathematics achievement is probably a safe summary of the position.

More recent studies among tertiary students have continued to pose the direction of the relationship between *attitude* and *performance* as an open question. Thus while Tall & Razali (1993) argued that the best way to foster positive attitudes is to provide success, Hensel & Stephens (1997) concluded that “it is still not totally clear whether achievement influences attitude, or attitude influences achievement”. Shaw & Shaw (1997) noted that among engineering undergraduates the top performing students (at entry) had a much more positive attitude to mathematics, and lower performing students a commensurately negative one – again leaving the direction of causality open.

The study of attitudes towards information technology (most frequently computers) has a shorter but more intensive history, probably because information technology, while newer, is pervasive in its permeation of curriculum areas. In considering attitudes to information technology among tertiary students it is useful to note that the disciplinary focus of target groups has tended to be in areas like Education, Psychology and Social Work. Reports involving mathematics students appear harder to come by, although some studies have included affective variables almost incidentally when evaluating general project outcomes (see below). It is this very breadth of discipline background, which has served to keep the investigation of attitudes to technology at a general level, appropriate to the majority who will not be called upon to use computers in the same technical sense as mathematics students working intensively with specialised software.

The relevance of studying attitudes to technology in conjunction with those relating to mathematics is emphasised and reinforced by the increasing use of technological devices in mathematics instruction. Several studies refer incidentally to attitudinal impacts as well as proficiency measures and Mackie (1992) in an evaluation of computer-assisted learning in a tertiary mathematics course indicated six positive learning outcomes, three of which were related to attitudinal factors. Park (1993) in comparing a Calculus course (utilising *Mathematica*) with a conventionally taught program, found some improvement in disposition towards mathematics and the computer in the experimental group. However Melin-Conjeros (1992), in comparing the performance of a group of Calculus students (equipped with limited access to *Derive*) with a control group, noted that the attitude of both groups decreased slightly. It has not been generally clear in the mathematically focused studies just which ‘attitudes’ have been affected by technology, as the reporting tends to be non-specific. By inference it appears that it is ‘attitude’ to mathematics that is referred to, and we are led to consider the implications of technology in impacting upon component attributes. The relationship between mathematics confidence and performance noted in the literature (whatever the direction of causality), means that the implications of a nexus between technology and mathematics needs specific research attention. The broad reporting of studies on the use of technology in mathematics instruction makes it difficult to disentangle whether reported affective outcomes are associated with changed attitudes to mathematics, or are linked directly to the technology. So theoretically we are moved to ask about the interpretation of outcomes if students possess high mathematics confidence and motivation, but low computer confidence and motivation, and vice versa. And beyond this, whether structural changes in attitudes will occur as technology becomes more and more a part of the students’ life experience, past and present. The specific research purpose addressed here may be expressed as follows:

To investigate the stability of attitude scales for use in programs in which computer technology is directed towards assisting undergraduate mathematics learning.

4.1 The Attitude Scales

Given the purpose of developing scales for use in settings involving interaction between technology and mathematics learning, the positions articulated by Hart (1989), Mandler (1989), and McLeod (1989, 1994) have proved helpful in fashioning approaches to the definition of terms and hence instrumentation. The distinction between an *attitude* and a *belief* is tenuous to a degree – an *attitude* focus has been sought by wording items so that the respondent is personally involved:

e.g. I feel more confident of my answers with a computer to help me; rather than

Computers help people to be more confident in obtaining answers.

The students for whom the measures are designed are tertiary undergraduates in mathematics courses. They have made this a deliberate choice - whereby mathematics has been selected as both useful in pursuing career aspirations, and as a subject compatible with themselves as individuals. Hence while an overall monitoring interest in gender and usefulness has been maintained, these emphases, which have figured prominently in attitude studies among school students, (e.g. Fennema & Sherman, 1976), have not played a dominant role in the design. Two of the nine attributes (confidence and motivation) represented in the Fennema-Sherman formulation have been reflected in scale development, with appropriate items constructed for use by undergraduates. The choice of these attributes was influenced strongly by the total purpose of designing instruments for use when computer technology is used in the teaching/learning context. *Confidence* and *motivation* have been selected because of their extensive appearance in the literature for both mathematics and technology, and because of their potential for discriminating between attitudes when technology and mathematics interact. These four scales are designed to measure attitudes on both dimensions so that such differences can be identified and their implications noted. In particular the choice of *confidence* and *motivation* enables two circumstances of particular interest to be identified viz. situations where students hold strong positive feelings towards mathematics and negative feelings towards technology, and vice-versa.

A further scale measures the degree of interaction between mathematics and computers that students perceive they apply in learning situations. The interactive significance of the learning and instructional context has been emphasised in general (e.g. McLeod 1989). In a computer environment students may simply respond to the screen or be active in note making, summarising, and experimenting. Indeed they may choose not to utilise technology when it is available and relevant. The physical separation of the learning components; pen and paper, computer screen, and human brain adds a further dimension to the co-ordinating processes required for effective learning strategies. The computer-mathematics *interaction* scale assesses the extent to which students bring their mathematical thinking into active inter-play with the computer medium.

Within each scale the eight items were arranged randomly with half requiring the reversal of polarity at the coding stage. Students were asked for a measure of their agreement (or rejection) with respect to item wording on a Likert scale. The item groups were presented in such a way that the underlying constructs were unknown to the students. The scale items themselves were theoretically determined from the respective underlying constructs and from cognate literature. See (Galbraith & Haines, 1998,2000) and Galbraith, Haines & Pemberton (1999) for more details on developmental aspects of this work.

4.2 Administration & Outcomes

The instrument was given initially in October 1994 to 156 first year students on entry to courses in engineering, mathematics and actuarial science at City University, London, and subsequently to the corresponding cohorts in 1995 and 1996. At the University of Queensland, Australia the scales were administered to 170 entering engineering undergraduates in 1997, and to parallel groups in 1998 and 2000. For present purposes the 1994, 1997, and 2000 results have been selected to be representative across time and place.

The responses have in fact displayed similar patterns across both place and time. Polarities have been adjusted so that a higher score means more of the property described by the scale label. Included below for sample scales, are the positively worded item(s) attracting the strongest support, and the negatively worded item(s) invoking the strongest rejection (L=London, B=Brisbane). L942&B971&B001 means that the item was the second strongest choice of London ’94 students, and the strongest choice of Brisbane ’97 students and Brisbane ’00 students etc.

mathematics confidence:	I can get good results in mathematics (L941& B971&B002) *No matter how much I study, math is always difficult for me (L941& B971&B001)
computer confidence:	I am confident I can master any computer procedure that is needed for my course (L941& B971&B001) *As a male/female (*cross out that which does not apply*) I feel disadvantaged in having to use computers (L941& B971&B001)
	* items whose polarities are reversed in calculating scale scores.

4.3 Scale reliabilities

These were obtained for each scale as shown in Table 4. London data first followed by Brisbane data in brackets (1997), [2000].

Table 4: Scale Reliabilities (Cronbach α)

mathematics confidence	0.77 (0.85)[0.81]	computer confidence	0.82 (0.88)[0.85]
mathematics motivation	0.80 (0.84)[0.82]	computer motivation	0.85 (0.86)[0.81]
		comp/math interaction	0.70 (0.70)[0.71]

The scales are coherent with reliabilities from strong to moderate. Internal scale statistics verify that all items contribute usefully to the respective constructs.

4.4 Scale validity

This rests primarily upon the theoretical base behind the construction of the scales. Additional structural evidence may be inferred from the sample items given above. For example the two items attracting the strongest responses for *mathematics confidence* (expecting good results, and rejecting that mathematics is difficult irrespective of effort), are both centrally to do with confidence. The coherence of the scale as indicated in the α value then supports the argument for validity without examining each additional item. Similar arguments apply to the other scales.

4.5 Differences in Attitude to Mathematics and Computing

A main purpose in this research has been to investigate the extent to which attitudes to computer use and to mathematics represent different inputs into technology based teaching contexts involving mathematics learning. In this section the student responses are analysed to address this issue further. London and Brisbane data indicated as in the previous table.

Table 5:Inter-scale correlations

	mconf	Mmotiv	cconf	cmotiv	cmint
mconf		.47(.68)[.51]	.29(.21)[.22]	.14(.19)[-.04]	.13(.16)[.04]
mmotiv			.25(.23)[-.07]	.29(.29)[.00]	.35(.26)[.15]
cconf				.71(.75)[.62]	.61(.58)[.56]
cmotiv					.68(.66)[.65]

Table 5 displays correlations between the five scales. The entries indicate that for all three cohorts the confidence and motivation scales are strongly associated within mathematics, and within computing respectively. However they are less strongly associated across the areas, as shown by the weak correlation, for example, between mathematics confidence and computer confidence. The computer-mathematics interaction scale is more strongly associated with computer confidence and computer motivation scales than with the mathematical scales, suggesting that computer attitudes are more influential than mathematical attitudes in determining the level of active engagement with computer related activities in mathematical learning. A Factor Analysis using the five scales as input variables with a two-factor solution (using oblimin rotation (SPSS) following a principal components analysis) yielded the loadings shown in Table 6. The two-factor solution confirms that the computer and mathematics related scales define different dimensions with computer properties dominant in the interaction scale.

Table 6: Factor Pattern Matrix

	Factor 1	Factor 2
mconf	.11(-.06)[.02]	**.55(.87)[.88]**
mmotiv	.14(.03)[-.02]	**.85(.89)[.87]**
cconf	**.89(.89)[.84]**	-03(-.03)[.05]
cmotiv	**.92(.90)[.89]**	-.05(.02)[-.11]
cmint	**.80(.83)[.85]**	.13(.02)[.06]

Percentage of variance 67.2(69.7)[75.3]

With respect to the research question we note the properties independently confirmed among students from different cohorts at different times and in different locations. Two further potentially significant inferences emerge from this stability and robustness. Firstly the confirmation that attitudes to mathematics and computing occupy different dimensions (the respective factors are almost orthogonal), with interaction loading with the computer scales. Secondly, at least an interim conjecture regarding the following question. Given that students' prior access to and experience with computers is continually increasing, will structural differences identified between mathematics and computer based affective responses diminish with time, or do they represent distinctive sets of characteristics with a permanent presence in computer-assisted mathematics learning? The data discussed here suggest the latter.

A final point of interest is associated with the data plotted in Figure 1 which shows an item-by-item plot of the differences between the means registered by females (F) and males (M) at the University of Queensland, using 2000 data.

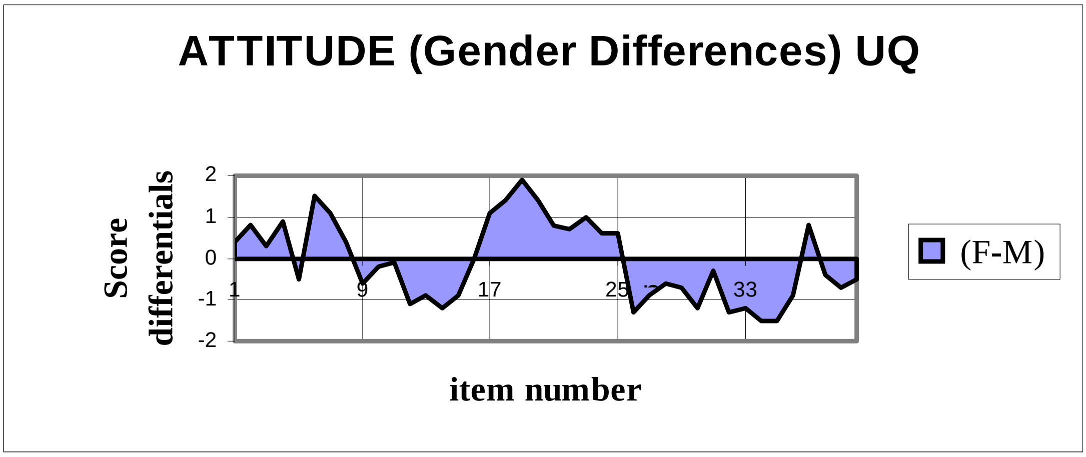

Figure 1. Gender differences on attitude scales (UQ 2000).

The vertical bars delineate the five 8 item scales, which, reading from left to right, are Mathematics confidence, Computer motivation, Mathematics motivation, Computer confidence, and Mathematics-Computer interaction. It is clear that females score more highly on the mathematics scales, and males more highly on the computer scales suggesting a systematic gender difference exists. A similar pattern occurs within other data. Both of these outcomes (robust scales and gender differences) suggest implications for the design and implementation of teaching programs that integrate computer-based activities into mathematics learning.

4. Focus C: Technology augmented Collaborative Learning

For this third focus the context is changed and the notion of technology broadened to include graphical calculators and also peripheral devices such as viewscreens. Different criteria apply when we allow the purpose of technology in mathematics teaching and learning to widen. If we are concerned purely with mathematical versatility and power, and features such as screen resolution then a symbolic manipulator may be a preferred choice. If we value portability, accessibility, and continuous access to a more restricted but still substantial range of mathematical functions then graphical calculators provide advantages. This is particularly so if the *learning environment* is a research interest. In a comprehensive review of research on graphical calculator use (in the decade to 1995), Penglase & Arnold (1996) noted a dearth of studies addressing learning environments and teaching approaches designed to maximise learning benefits. A subsequent review of research (Asp & McCrae, 2000) commented that this particular gap did not appear to have been seriously addressed, although substantial work on other aspects of graphical calculator use was noted. The teaching-learning environment remains an important context for examining alternative ways in which technologies, teachers, and students, combine in the pursuit of mathematical goals when these are not obscured by narrow definitions of desired outcomes.

Sociocultural perspectives on learning emphasise the socially and culturally situated nature of mathematical activity, and view learning as a collective process of enculturation into the practices of mathematical communities. The classroom as a community of mathematical practice supports a culture of sense making, where students learn by immersion in the practices of the discipline. Rather than relying on the teacher or textbook as an unquestioned external authority, students in such classrooms are expected to defend and critique ideas by proposing justifications, explanations and alternatives. Collaborative practices are called for, and in considering alternative models Brandon (1999) has usefully pointed out that the 'C' in Collaborative Learning has been used ambiguously to refer to both co-operative based learning (group members share the workload); and collaboration-based learning (group members develop shared meanings about their work). While interrelated there is a clear difference in the respective emphases. Collaborative activity in this latter sense, is characterised by equal partners working jointly towards an end (Anderson, Mayer, & Kibby, 1995), as a co-ordinated activity directed towards construction and maintenance of shared meaning and understanding (Rochelle & Teasley, 1995). A key element is *elaboration* (Webb & Palincsar, 1996), through which students: provide specific examples to illustrate concepts; use multiple representations (charts, diagrams etc) to explain concepts; create and evaluate analogies; translate terms; provide detailed descriptions of how to perform tasks or illustrate differences between concepts; provide detailed justifications for their problem solving; or use observations and evidence to support opinions or beliefs. These characteristics of collaborative learning, that emphasise the social construction of knowledge and shared conceptions of problem-based tasks, carry across as important elements in the design of computer based - supported collaborative learning (CSCL) as described by Brandon (1999). In generalising this property beyond computers to encompass technology in general we distance ourselves from models of 'Co-operative learning' wherein members of a group of peers are assigned individual roles (e.g. recorder, checker) prior to structured group activity. In this model role assignment may interfere with group processes by overemphasising organisational tasks at the expense of learning processes. Role assignment effectively restricts the opportunity of individuals to engage with problems freely, and to use their knowledge in the widest and most relevant way. This is in fundamental conflict with the goals that motivate a community of scholars.

A central tenet of sociocultural theory is that human action is mediated by cultural tools, and is fundamentally transformed in the process (Wertsch, 1985). The rapid development of computer and graphical calculator technology provides numerous examples of how such tools transform mathematical tasks and their cognitive requirements.

The approach then is predicated on three basic assumptions.

1. Human action is mediated by cultural tools, and is fundamentally transformed in the process.
2. The tools include technical and physical artefacts, but also concepts, reasoning, structures, symbol systems, modes of argumentation and representation.

3. Learning is achieved by appropriating and using effectively cultural tools that are themselves recognised and validated by the relevant community of practice.

The approach is informed by a Vygotskian framework, that has moved beyond the most widely known interpretation of the *Zone of Proximal Development* (ZPD) as the distance between what a learner can achieve alone and what can be achieved with the assistance of a more advanced partner or mentor. Two other representations are of particular relevance to our learning context. These are firstly the conceptualisation of the *ZPD in egalitarian partnerships*. This view of the ZPD, involving equal status relationships, argues that there is learning potential in peer groups, wherein students have incomplete but relatively equal expertise – each partner possessing some knowledge and skill but requiring the others' contribution in order to make progress. In the research context this feature becomes relevant through the collaborative activity of students in bringing technology to bear on mathematical tasks with varying levels of individual technological and mathematical expertise. One advantage of these groups is that, when the teacher withdraws, the students are provided with the opportunity to *own* the ideas they are constructing, and to experience themselves and their partners as active participants in creating and testing personal mathematical insights.

A second extension of the ZPD concept is created by the *challenge of participating in a classroom culture* constituted as a community of practice. Students as participants in a learning community are viewed as having partially overlapping ZPDs that provide a changing mix of levels of expertise that enables many different productive partnerships and activities to be orchestrated. (Brown et al., 1993; Brown & Campione, 1995) Through the establishment of a small number of repeated participation frameworks such as teacher-led lessons, peer tutoring, and individual and shared problem solving, students are challenged to move beyond their established competencies and adopt the language patterns, modes of inquiry, and values of the discipline. Such a classroom environment, representative of an active community of learners, is then augmented by the availability of technology as another agent in the search for powerful and meaningful mathematical learning and application.

To elaborate then, technology is viewed as one of several types of cultural tools - sign systems or material artefacts - that not only amplify, but also re-organise, cognitive processes through their integration into the social and discursive practices of a knowledge community (Resnick, Pontecorvo & Säljö, 1997). The amplification effect may be observed when technology simply supplements the range of tools already available in the mathematics classroom, for example, by speeding tedious calculations or verifying results obtained by hand. By contrast, cognitive re-organisation occurs when learners' interaction with technology as a new semiotic system qualitatively transforms their thinking; for example, use of spreadsheets and graphing software can alter the traditional privileging of algebraic over graphical or numerical reasoning. Accordingly, learning becomes a process of appropriating cultural tools that transform the relationships of individuals to tasks as well as to other members of their community (Doerr & Zangor, 2000).

This conceptualisation of technology usage in mathematics classrooms differs in its emphasis in that, in addition to its contribution in addressing mathematical concepts and processes, it encompasses also the sociocultural dimension: interactions between teachers and students, amongst students themselves, and between people and technology, in order to investigate how different participation patterns offer opportunities for students to engage constructively and critically with mathematical ideas. That is, while technology may be regarded as a mathematical tool (*amplifies capacity*), or as a transforming tool (*reorganises thinking*), it may also be regarded as a cultural tool (*changes relationships between people, and between people and tasks*).

5.1 Research procedures

A team of researchers, comprising a mix of academics and teachers, has been investigating the potential of collaborative learning in mathematics at pre-university level for a number of years. The student subjects are serious students of mathematics, many of whom enrol in undergraduate degrees in science and engineering in the year following their participation in the study. One particular study followed a group of students during their final two years of secondary education. On average a lesson was observed and videotaped every one to two weeks, with more frequent classroom visits scheduled if a technology intensive approach to a topic was planned. Each student had permanent access to a graphical calculator and spreadsheets were available as a normal classroom resource. Audiotaped interviews with individuals and groups of students were conducted at regular intervals to illuminate factors such as the extent to which technology was contributing to the students' understanding of mathematics, and how technology was changing the teacher's role in the classroom. This data triangulated information obtained from analysis of videotapes and questionnaires. At the beginning of the course and at the end of each year students completed a questionnaire on their attitudes towards technology, its role in learning mathematics, and its perceived impact on the life of the classroom.

The quality of mathematical exchanges is captured on the videotape record and is not reported in this paper. The interest here is in characteristics displayed as students work collaboratively, aided by technology, as a means towards collective and individual mathematical competence. While the most illuminating data are in the form of videotaped segments, featuring student and teacher discourse, (Goos et al., 2000) for present purposes we skip to a summary of some of the findings related to the learning characteristics identified. These have to do with the different ways in which students use technology, and see themselves in relation to it.

5.2 Metaphors for technology use

Observations have led to the development of a descriptive taxonomy of sophistication with which students work with graphical calculators. This is expressed in terms of metaphor.

Technology as Master. The student is subservient to the technology-a relationship induced by technological or mathematical dependence. If the complexity of usage is high, student activity will be confined to those limited operations over which they have competence. If mathematical understanding is absent, the student is reduced to blind consumption of whatever output is generated, irrespective of its accuracy or worth.

Technology as Servant. Here technology is used as a reliable timesaving replacement for mental, or pen and paper computations. The tasks of the mathematics classroom remain essentially the same—but now they are facilitated by a fast mechanical aid. The user 'instructs' the technology as an obedient but 'dumb' assistant in which s/he has confidence.

Technology as Partner. Here rapport has developed between the user and the technology, which is used creatively to increase the power that students have over their learning. Students often appear to interact directly with the technology (e.g. graphical calculator), treating it almost as a human partner that responds to their commands – for example, with error messages that demand investigation. The calculator acts as a surrogate partner as students verbalise their thinking in the process of locating and correcting such errors. Calculator or computer output also provides a stimulus for peer discussion as students cluster together to compare their screens, often holding up graphical calculators side by side or passing them back and forth to neighbours to emphasise a point or compare their working

Technology as an Extension of Self. The highest level of functioning, in which users incorporate technological expertise as an integral part of their mathematical repertoire. The partnership between student and technology merges to a single identity, so that rather than existing as a third party technology is used to support mathematical argumentation as naturally as intellectual resources.

Having constructed the taxonomy, through example and repeated observation the research team asked a group of students near the end of their course to reflect on its structure in relation to themselves as individuals. A selection of responses from the 2000 cohort is given below.

> *Master (M):* because I often don't understand how to use every specific function of the technology, thereby limiting my use of such technology. I often don't know if I've used it correctly and as a consequence I can't be sure if my answer is correct or not.
> I think I'm between *master* and *servant*. I tell the calculator what to do sometimes but only stick to what I know usually. I don't know exactly what it allows me to do, and if I haven't been taught, I won't look for it.
> *Servant (S):* because I do not have enough knowledge of technology to be able to investigate new concepts. However I do regularly use it for *familiar* tasks purely as a time saver and to verify and check my answers.
> *Partner (P):* Because my calculator has become my best friend. His name is Wilbur. Me and Wilbur go on fantastical adventures together through Maths land. I don't know what I'd do without him. I love you Wilbur.
> *Extension of Self (ES):* Because my calculator is practically a part of myself. It's like my 3^{rd} brain. I use it whenever it can help me do anything faster.

The student group had no problem reaching a personal decision and justifying it, and the 15 responses from the Year 12 students produced the following distribution. M (1), M-S (1), S (7), P (2), ES (4).

Following the earlier choice of metaphor to describe the taxonomy of sophistication with which students may work with technology, observation and discussion then suggested that a similar taxonomy may be useful in classifying instructional uses of technology.

Technology as Master

Here the teacher is subservient to the technology, and is able to employ only such features as are permitted either by limited individual knowledge, or force of circumstance. This seems clearly the case in large-scale transmissive programs where, as described by Olsen (1999), helpers are reduced to assistants responding to students on the basis of what the software has generated, and to marking computer generated quizzes. Here course organisation forces the relationship. However this circumstance may also apply in classrooms where teachers have individual autonomy. As described by Stuve (1997), pressure to be seen to implement technology following 'training', results in implementation dominated by whatever basic skill has been acquired, without consideration of impact beyond the present.

Technology as Servant

Here the user may be knowledgeable with respect to the technology, but uses it only in limited ways to support preferred teaching methods (Thorpe, 1997). That is the technology is not used in creative ways to change the nature of activities in which it is used. For example just as a calculator can be restricted to the purpose of producing fast reliable answers to routine exercises, a viewscreen may be limited to providing a medium for a teacher to demonstrate output to the class as an alternative to chalkboard, or a computer to crunching numbers faster.

Technology as Partner

Here the user has developed 'affinity' with both the class and the teaching resources available. Technology is used creatively in an endeavour to increase the power that students collectively exercise over their learning, rather than exercising it over them (Templer et al., 1998). This can occur both in the use of mathematically based technology (calculators and computers), for the purpose of enhancing individual prowess, and in the use of communications technology to enhance the quality of class learning through sharing, testing, and reworking mathematical understandings. For example, instead of functioning as a transmitter of teacher input, a viewscreen may be a vehicle for engendering otherwise non-existent student participation or act as a medium for the presentation and examination of alternative mathematical conjectures.

Technology as an Extension of Self

This is the highest level of functioning, in which powerful and creative use of both mathematical and communications technology forms as natural a part of a teacher's repertoire as fundamental pedagogical skills and mathematical knowledge. Writing courseware to support and enhance an integrated teaching program would be an example of operating at this level. Successful use of the rich electronic classroom (Shneiderman et al., 1998) would appear to demand this kind of expertise. However, ironically, too much sophisticated technology may exact a price! The sheer volume of technological choice can reduce opportunities to explore fully creative uses of individually productive items. It is noted that these levels of operating are neither necessarily tied to the level of mathematics taught, nor to the sophistication of technology available. Simple mathematics and basic technologies are sufficient to provide a context for highly creative teaching and learning. Conversely, powerful computers and expensive infrastructure can be associated with programs that are limited in what they are able to achieve, or indeed attempt.

6. Reflections

It seems almost fatuous to say that (without further qualification) the term 'technology assisted learning' is effectively meaningless. Much has been written that belongs to the genre of 'show and tell' rather than to information carefully collected and rigorously scrutinised. Almost anything can be argued to have enjoyed some success, in some form, with someone, at some time. Over a decade ago James Fey surveyed developments in the use of technology in mathematics education to that date. In noting that there was no lack of speculative writing on the promise of revolution that would follow from the application of various calculating and computing tools, he drew attention to the paucity of data available to back extravagant claims.

> It is very difficult to determine the real impact of those ideas and development projects in the daily life of mathematics classrooms, and there is very little solid research evidence validating the nearly boundless optimism of technophiles in our field. (Fey, 1989)

It is bemusing to reflect that this comment seems as relevant today as it was over a decade ago, even if the questions have become more refined. The literature confirms the existence of diverse factors that impact on the development and testing of theoretical frameworks, and on the conduct of practice. Such factors include not only inter-product competition (competing brands and genres) that extends also to users, but competing educational philosophies with respect to the teaching and learning of mathematics, and institutional politics.

It seems that one viewpoint of significance at all levels of debate, is whether technology is regarded primarily as a *learning tool* or a *power tool*. If we see calculators and computers as *power tools* then we use them as a high tech means of accomplishing mathematical tasks more quickly, or attacking problems that are intractable without the technology. Either way their use in these ways is enabled by the expert knowledge base of the user. Some of the most incisive discussion in the literature concerns the debate about whether students need to understand the mathematics independent of the technology, or whether it can be learned through technology. This raises the question of using technology as a *learning tool,* and what this means for educational practice. Those who treat mathematics as something to be transmitted and consumed, and see technology essentially as a means to this end, ignore both the message of history and the evidence accumulating from studies that pay attention to the learning context (e.g. Templer et al, 1998; Kent & Stevenson, 1999). Our work inhabits but a small corner of this domain: however consistent observations have indicated that access to technology impacts not only on task requirements, but on the culture of the learning approach, and on ways in which students reposition themselves with respect to the technology, the task, and each other. The fact that pages of output can be generated when operating with software packages gives a misleading measure of learning productivity, and creates even further need to subject such output to quality control and follow-up. Ironically this requires additional human resources at a time when institutional managers are looking to technology to reduce this very thing. The point has been underlined (Olsen, 1999) following her description of the 1.5 acre budget driven automated instruction initiative at Virginia Tech.

> Instructional software issues are unlikely to be resolved quickly... If we want the software to help at all… it's got to understand how students might misconceive what is presented to them--and to figure that out from the student's response. And right now, only people do that well. (p. 35)

The search for complexity measures for demands incurred in using *Maple* software, is an intended contribution to the 'replacement' debate - about the extent to which a student can adopt a black box mentality to software and focus on the purpose of a task. While results are preliminary they do not lend any support to the view that mathematics and technology are separable in the learning phase, and that technology essentially is a means to stronger mathematical capability among students. Put another way, it cannot be assumed that students use technology as experts use a *power tool* even when provided with sufficient enabling information. If learning is to be achieved then technology's role in initiating and consolidating understanding needs further intensive study and careful documentation. It is doubtful that enough of this is being done despite the plethora of projects using technology for instructional purposes. Studies such as Drijvers (2000) help to reinforce that obstacles arising when students work with computer algebra systems are generated by the interaction of mathematical and technological aspects. The idea then, of technology as simply a power tool to enable stronger mathematics, or as a replacement for transmissive models of teaching, is effectively rebutted by an increasing number of studies.

Work on attitudes has tended to be blurred by interactions between computers, calculators, and mathematics in programs involving technology-aided learning. Studies over many years have found that attitude and performance are related in school mathematics, although the direction of causality has been open to question. Several papers over the past five years have specifically made reference to attitude in relation to performance in undergraduate programs (e.g. Shaw & Shaw, 1997; Hensel & Stevens, 1997). Suspicion that in technology aided learning settings, confidence and motivation (in mathematics and technology respectively) may occupy different dimensions has been consistently confirmed in our research. Furthermore the results appear to be stable with no change apparent over a period of six years using students in different locations. An anticipated softening of the technology data due to increasing access and experience with calculators and computers has not eventuated. Gender differences in attitudes to mathematics and computers respectively, favouring females for mathematics and males for computers raise additional issues for course design, when technology and mathematics are brought together in undergraduate programs.

Studies on the impact of calculators and computers as cultural tools that change the nature of learning and relationships, as distinct from their agency as mathematical aids, promise to expand and challenge notions of what can be achieved in technology aided instruction. The emergence of different levels with which students see themselves using and interacting with calculators and computers also challenge approaches that see technology purely in terms of increasing mathematical power. Failure to recognise taxonomies of competence, preference, and confidence in using technologies increases the risk that inappropriate expectations and methods of instruction will drive course design and implementation. The risk that through unquestioned acceptance of a perceived authoritative source, a ‘tyranny of the text’ becomes replaced by a ‘tyranny of technology’ emphasises the role of the teacher as a custodian of mathematical values that must be continually articulated and embedded in instructional practices (Guin & Trouche, 1999). As the use of calculators and computers as cultural and mathematical tools in communities of practice approaches to learning become more prevalent in secondary education, there are implications for the design and implementation of undergraduate courses into which the students subsequently flow.

Finally, in order to make more systematic progress in evaluating quality and identifying problems we need to look at improving the relevance of research methods. It is probably fair to say that a substantial majority of us received research training within the scientific paradigm of the controlled experiment. Many have questioned its relevance in testing for outcomes of quality in educational settings-many more need to do so. What is valuable in knowing that approach A achieves statistically better results than approach B when both are terrible, and about 5% of variance is involved? Furthermore it is frequently not clear that the condition being ‘tested’ has been faithfully applied. Some unsuccessful attempts to replicate the success of Schoenfeld’s (1985) problem solving program with College students provide cases in point. Johnson & Fishbach (1992) and Lester et al., (1989) reported studies that foundered in their attempts to replicate the success of the teaching approach advanced by Schoenfeld. While these studies specifically implemented elements of that teaching program (in terms of strategies), they did not nurture and sustain the culture of “mathematics community” that was of equal or greater importance. In the former study, the College students, used to other methods of mathematics teaching, were uncomfortable with the learning approaches and setting. On the other hand, their teachers were uncomfortable with the teaching style required of them, which was substantially different from that developed over many years. No positive change was achieved over a ten-week period. In the latter study, two classes of primary year 7 students showed little ‘improvement’ in metacognitive control behaviours over the seven weeks of the trial. These students had limited domain specific knowledge on which to draw, were reluctant to reflect on strengths and weaknesses, and inexperienced in the small group settings which formed a key part of the instructional program. Failure to establish a community of practice culture renders invalid attempts to evaluate the effectiveness of teaching strategies that necessarily draw from such a culture. Yet parallels to this failure, often compounded by inadequate reporting, torment study after study. This is quite apart from an increasing concern with ethical considerations that would question the integrity of studies that allocate a group of subjects to a ‘treatment’ believed to be inferior! The social context of the classroom is an inextricable component in the development of a community of practice. It becomes central therefore to locate identifiers by means of which the operation of such a community can be recognised, monitored and developed, and within which the achievements of teaching approaches can be assessed. Such methods involve establishing criteria against which to measure the quality of outcomes, for which purpose the use of videotapes, transcript analysis, and other methods of triangulation augment written data. Qualitative research methods and Grounded Theory approaches need to complement appropriate applications of quantitative methods more than they have so far managed to do. The development and implementation of rigorous research within a rich environment of outcomes is perhaps our greatest challenge in seeking to test and improve the effectiveness of instructional strategies involving technology.

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Appendix

Sample Questions

(Questions in italics: *Maple* commands in bold: *Maple* output in ordinary type)

Q2. Factorize $x^3 - 6x^2 + 11x - 6$

Maple Solution

➢ **factor(x^3-6*x^2+11*x-6);**

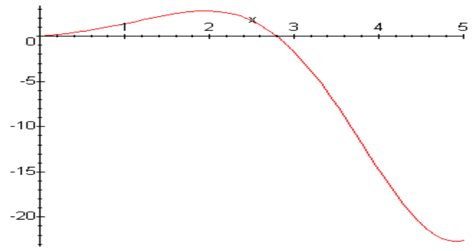
(x - 1) (x - 2) (x - 3)

Q8. Find where the graph of $x^2 \sin x + x \cos x$ *for* $0 \le x \le 5$ *is:*

(a) above the x – axis (b) below the x – axis (c) cuts the x – axis.

Maple Solution

> **plot(x^2*sin(x)+x*cos(x),x=0..5);**



> **x1:=fsolve(x^2*sin(x)+x*cos(x),x=2..3);**

x1 :=2.798386046

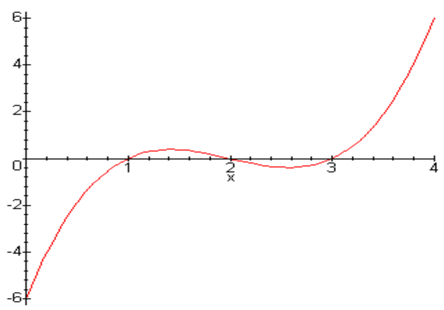
****

Q14. Plot the graph of f(x) = (x-1)(x-2)(x-3) and use this to find the physical area under the graph from x= 1 to x= 3.

Maple Solution

> **y:=(x-1)*(x-2)*(x-3);**

➢ **plot(y,x=0..4);**



➢ **int(y,x=1..2)-int(y,x=2..3);**

➢ 1/2