LOOKING FOR LEVERAGE: ISSUES OF CLASSROOM RESEARCH ON "ALGEBRA FOR ALL"

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ABSTRACT

In the United States, African Americans, Latinos, and Native Americans have lower success rates and higher drop-out rates in mathematics than other racial or ethnic groups. Given that quantitative competency serves increasingly as a vehicle for economic enfranchisement, these differential success rates make mathematics achievement a civil rights issue. Failure and dropouts start early. Moreover, "algebra" is becoming a major stumbling block: many states require students to pass algebra tests in order to graduate high school. This social/mathematical problem is becoming increasingly urgent.

This paper describes the American context and suggests its relevance world-wide. It then explores the following issue. Suppose one wants to do classroom-based research on Algebra for All: one will observe what takes place in middle school mathematics classrooms where there are diverse populations of students. What kinds of data should one gather in order to determine which practices support the learning of mathematics by diverse groups of students and how they work? What theoretical frame will provide the best purchase on these issues?

Issues addressed include: whether mathematics is "culture-free" and what the implications for instruction might be, even if it is; the institutional support necessary for high quality instruction; the differential treatment of student groups; pedagogical practices that enfranchise a wide range of students; the roles of language and discourse in learning and classroom communities; individual agency; and what it means to engage meaningfully with mathematics. The challenge is to conduct classroom research that helps to explain, at a level of mechanism, how classroom interactions can be structured to help students who vary widely in terms of cultural backgrounds and prior mathematical success to *all* learn some very solid mathematics.

First.
You have to *understand* the problem.
– George Pólya, *How to Solve It*

Introduction

This paper differs from those I am accustomed to writing in one fundamental way. Typically, researchers spend a fair amount of time working on a problem. Then, after significant progress has been made, they write up the results. The purpose of writing such a paper is to share understandings with others. I will do some of that here. But my goal is also to problematize a research arena – to grapple with the question of how one can productively study classroom attempts to help middle school students with widely divergent cultural and socio-economic backgrounds learn the mathematics that leads to and includes the study of algebra.

Here is why the topic matters. Issues of "algebra for all" are absolutely central in the current America context. In the United States, poor children and under-represented minorities (African Americans, Native Americans, and Latinos) tend to earn lower grades and to stop taking mathematics courses much earlier than others; access to and treatment in mathematics classes also differs by gender. Broadly speaking, a lack of mathematical competence and credentials constitutes a barrier to full participation in the economic mainstream. Hence differential participation and success rates in mathematics become an issue of social justice. Moreover, the stakes are about to be raised. California and other states have instituted standardized examinations as a prerequisite for high school graduation. The mathematical content focus of the examinations is on algebra. Students who do not succeed at learning algebra will be denied a high school diploma – and thus seriously marginalized.

A team of researchers from three universities (The University of Wisconsin at Madison, the University of California at Berkeley, and the University of California at Los Angeles) has received funding from the U. S. National Science Foundation to address these issues. Our project, "Diversity in Mathematics Education" (DiME), covers a lot of territory. Project goals include preparing a new generation of researchers to work on issues of diversity and mathematics education, working in partnership with local school districts to create enhanced models of teacher preparation and professional development, and creating a set of resources that can be used by teachers and school districts to address these issues. Central to such resources is developing a deep sense of what happens in classrooms as students grapple with the ideas of algebra.

There is always uncertainty in research; that is the nature of the process. As an established researcher, I have of course developed my own *modus operandi* and a substantial level of comfort for dealing with uncertainty. Typically I approach a problem with some sense of what is likely to be important, in both theoretical and pragmatic terms. I identify phenomena of interest, gather relevant data (which might include videotapes and various artifacts), labor over the data until they begin to make sense, draw some tentative conclusions, and look for more data or perspectives that will yield triangulation. The results of that work may be some or all of the following: theory refinement, methods development, or a deeper understanding of a particular problem. (For me, problems tend to be of the type "how does something work"; answers are usually at a detailed level, describing the way things fit together.) I am accustomed to starting with rough ideas of problem, theory, and method – with some notions of what things are important and what will help me make sense of them – and then living with the phenomena until a reasonably clear picture emerges. Indeed, much of the pleasure of being a researcher is in figuring out how to turn one's intuitions into new methods, perspectives, and findings. When my intuitions feel solid, they often pay off – not necessarily in the ways I expect, but often in ways that are close.

As I begin this project, I do not feel comfortably equipped to address classroom issues at the heart of DiME's "diversity and algebra[1]" agenda. Despite having spent many years of thinking about issues of mathematical thinking, teaching and learning; despite having spent one morning every week in local public school mathematics classrooms for the past decade; and despite having read widely and thought hard about issues of "mathematics for all," I am not at all confident that I have an appropriate framing of the issues or that the methods I know are appropriate for grappling with them.[2] This paper represents an attempt to think through some of those issues – to lay out some of what is known and seems to be relevant, and to see if I can elaborate some of the conceptual and methodological problems that need to be confronted.

The paper proceeds as follows. In the next section I start with a bit of international context, to show the relevance of the issues discussed here to non-American readers. Then I focus on the American context, providing a bit of historical background – how high school mathematics moved from a subject to be studied only by the elite to a subject to be studied by all. I proceed to discuss plausible goals for mathematics instruction, and the reason that learning a solid core of mathematics is an important and plausible goal for all students. This is followed by a brief discussion of demographic data. These data on the mathematical performance of diverse groups indicate clearly that in the United States, mathematics education is an issue of social justice.

Having established context for DiME's agenda, I move on to review some of what is known about making mathematics accessible to a wide range of students. That section of the paper is where I try to untangle the issue of classroom research on algebra for all. As I work through various dimensions of what is known, I point to issues that still strike me as problematic.

Before moving to my announced agenda in the next section, I want to conclude this introductory section by posing and reflecting on some questions about the nature of mathematics and mathematics instruction. These questions have provoked me, through the years, to think about issues of diversity and mathematics. I begin with a question that haunted me for a long time as a mathematician, then move to ones concerned with pedagogy and research.

- Isn't mathematics "culture-free" or "culture-independent?"

At international mathematics conferences, for example, it's astounding how people who have never met each other and may share only a few words in a common language can communicate

[1] In what follows I shall say a fair amount about diversity and rather little about algebra. That is because issues concerning algebra are somewhat more straightforward, and do not cry for elaboration here: see, e.g., NCTM's (2000) *Principles and Standards* and the U.C. ACCORD Mathematics working group's (2000) report *Pathways to algebra for all of California's children*.

[2] This sense of discomfort is, of course, intimately tied up with my sense of what counts as understanding or explanation. My goal as a researcher is to understand how and why things work, so I'm not satisfied personally until I have a sense of how things fit together.

meaningfully about deep mathematical ideas. While it may or may not be the case that "a rose is a rose is a rose," there is no doubt that from the typical mathematician's point of view a Banach space is a Banach space is a Banach space: once the definition is made the properties are established, and anyone who plays by the rules can determine those properties. At a more elementary level, a square is a square is a square: once one says that a quadrilateral in the Euclidean plane is a square, then (for example) its diagonals must be perpendicular and must bisect each other. The point from the mathematician's perspective is that the properties follow from the definition, no matter who does the proving. At an even more elementary level, it doesn't matter who counts a finite set of objects, or what culture that person comes from– the answer will always be the same.

An affirmative answer to the first question leads to a corollary question:

- If mathematics is culture-free, then how does it make sense to speak of "teaching mathematics to students of different cultures"? That is, if mathematics is culture-free, shouldn't mathematical pedagogy be culture-free?

How one answers this question depends, of course, on how one conceptualizes teaching and learning.

One view, which predominated when I began to teach mathematics and is still, I suspect, rather common at the university and perhaps secondary levels, is that the responsibility of the mathematics teacher is to present lucid explanations of the mathematical ideas at hand. In this view, the truly competent teacher is the one who has three of four (maybe more) different ways of explaining a topic or concept, so that students who don't "get" the first may find the second more accessible, or perhaps the third, or fourth.

It is important to recognize possible concomitants of this view. When the teacher has presented mathematically clear explanations at the right level, he or she has met his or her pedagogical obligations. Thus this approach creates a clear division of responsibilities. The faculty's job is to make the material accessible to students; the students' job is to learn that which has been clearly presented. In consequence, this perspective allows the faculty to abdicate responsibility for some student learning: if the presentation has been clear, then it's the student's fault if he or she didn't learn the material. It also supports "deficit" models of instruction, with the assumption that students from particular backgrounds have particular deficits. (Students for whom English was a second language might, for example, be taken out of mathematics classes until their English was deemed adequate for full participation in the mathematics classes. The net result was that those students got further behind in mathematics.)

When it is presented in such stark terms, the "lucid explanation" perspective described in the previous paragraphs might well be rejected by a fair percentage of today's teachers. It harks back to the "old days," when teachers lectured and students took notes. In the United States today's mathematics classes are much more interactive; students engage in a wide range of mathematical activities. A more contemporary view might be that the responsibility of the mathematics teacher is to provide students with a range of activities (possibly including lecture, individual or small group work, whole class activities, the use of manipulative materials, and more) that allow students to engage with the mathematical ideas at hand, and to learn as a result.

This does indeed sound contemporary. The point to recognize, however, is that everything that I said about the "lucid explanations" perspective applies to this more contemporary view as well. Here the master teacher might be viewed as the teacher with a large bag of tricks, including a large range of activities that support multiple approaches to the mathematics. This certainly covers more territory than the first perspective. But, like the other, it creates a clear division of responsibilities. The teacher now has a larger set of responsibilities – the pedagogical tool kit is expected to be much larger. But here too, faculty are given a warrant for abdicating responsibility for some student learning: if classroom activities have been field tested and are thought to be of high quality, then it's the students' fault if they don't learn the material.

A third view is that effective teaching (defined as "things the teacher does that lead to successful learning") is teaching that helps students to negotiate the terrain between what they bring to the learning environment and what one wants them to learn. Of necessity, this kind of teaching calls for understanding and building upon what the students bring – predispositions and understandings, habits of mind, patterns of engagement, patterns of communication (including norms of social interaction and linguistic patterns), and more. It should be obvious that many of these are shaped by the student's experiences outside classroom boundaries – that is, they are shaped culturally. From this perspective, then, effective teaching must be responsive to what the students bring with them to the classroom – in Ladson-Billing's (1994) words, pedagogy must be "culturally responsive."

If one accepts the notion that one has to "meet students where they are," the next set of questions to address concerns how to understand what the students bring to the classroom, and how to foster productive interactions between students and mathematics. As will be elaborated below, there is reason for optimism about what can be achieved. Indeed, there are some suggestions of the kinds of conditions that might, in concert, sustain positive change. These will be reviewed, albeit briefly. But even given these, I find myself confronted with a series of questions about the kind of research I would like to produce.

The question I would like to address is this:

- Suppose one wants to do classroom-based research – that is, one's work will be grounded in observations of what takes place in middle school mathematics classrooms in which there are diverse populations of students. What kinds of data should one gather in order to determine which practices support the learning of mathematics by diverse groups of students, which do not, and how they work? What theoretical frame will provide the best purchase on these issues?

As simple as these questions may seem, the answers are anything but simple.

Context

Why this might matter to people outside the United States.

The United States has often gone its own way in curricular matters. For example, the traditional U. S. mathematics curriculum consists of a year's study of elementary algebra in 9th grade, Euclidean geometry in 10th grade, and a return to more advanced algebra and trigonometry in 11th grade. In the traditional curriculum, geometric problems are not dealt with in the algebra courses, and vice-versa; applications are few and far between. This course configuration, along with the nature of topic coverage in the U.S. ("a mile wide and an inch deep"), are somewhat anomalous internationally (see, e.g., Schmidt, McKnight, and Raizen, 1997). Given the atypical nature of the curriculum, and the somewhat atypical history of race relations in American society, why might the study of issues of diversity and mathematics education in the U.S. be relevant anywhere else?

I shall answer by assertion – but someone else's rather than mine. In a paper written for the International Commission on Mathematics Instruction, Robyn Zevenbergen writes the following:

The international phenomenon of expansion of the higher education sector has resulted in greater diversity in the intake of students. No longer is higher education the domain of the elite, but now more students can access it than in any previous times. . . . Students who, in earlier times would not have gained access to (or even considered enrolling in) tertiary mathematics, are now coming to classes. These students have very different needs and expectations of their study and are likely to encounter difficulties… (Zevenbergen, 2001, p. 13).

In short, the democratization of higher education worldwide will result in more diverse groups of students in tertiary mathematics classes, and a concomitant set of pedagogical issues. And such issues will not appear for the first time at the post-secondary level; they will appear in the mathematics "pipeline," as students are being prepared for the further study of mathematics.

100 years of American curricular history in a few paragraphs

The 20th century can be seen as a century of mathematical "democratization" in the United States. As the century began, mathematics was the province of the elite. As it ended, arguments were being made that all citizens need to be quantitatively literate in order to participate fully in the American democracy.

In 1890 only 6.7% of the 14 year-olds in the United States attended high school, and only 3.5% of the 17 year-olds graduated (Stanic, 1987). The purpose of schooling was to provide the vast majority of students with workplace skills and little else. Schooling for the masses focused on what were called the three R's: Readin', Ritin', and Rithmetic." Education for the elite was reserved for high school and beyond.

Over the course of the 20th century there were continuing pressures for additional schooling. By mid-century almost three-fourths of the children of age 14 to 17 attended high school, and 49% of the 17 year-olds graduated. (Stanic, 1987, p. 150). These enrollment changes resulted in the pressures identified above by Zevenbergen: courses once designed for a select group of students were being studied by increasing numbers of students. These demographic trends continued through the end of the century. A part of the American ethos is that education is a pathway to social and financial advancement: the "G.I. Bill," for example, provided soldiers returning from World War II with incentives to take courses at the post-secondary level. General social goals included high school graduation and access to further study for all students. By the end of the century, more than half of the high school graduates in the U.S. had enrolled in some form of post-secondary education.

Outside the classroom the world had changed in significant ways. Inside the classroom, however, the mathematics curriculum was largely unchanged: for most students grades 1-8 consisted of the study of arithmetic. In grade 9 they studied algebra. Half the students stopped taking mathematics at that point, and half went on to geometry in grade 10. Half the students stopped taking mathematics at that point, and half went on to “advanced algebra/trigonometry” in grade 11. The attrition rate from the mathematics pipeline continued at 50% per year as students proceeded through pre-calculus and then calculus, either in their senior year in high school or in their first year of post-secondary education.

1989 and beyond: New curricular goals

In 1989 the U. S. National Council of Teachers of Mathematics issued the *Curriculum and Evaluation Standards for School Mathematics*, a volume that proposed significant changes in mathematics teaching. This was followed in 1991 by the *Professional Standards for Teaching Mathematics* and in 1995 by the *Assessment Standards for School Mathematics*. I shall refer to these three volumes collectively as the *Standards*, while noting that the first volume, published in 1989, is the one that had the greatest influence. Part of the reason for creation of the *Standards* and the changes they suggested was dissatisfaction with the then-current curriculum, including the huge attrition rate from the mathematics pipeline described in the previous paragraph. But equally important was a reconceptualization of the underlying goals and purposes of mathematics instruction. The curriculum had been inherited from a time when mass education was for limited purposes of general literacy, and advanced education was for the elite. The *Standards* specified new instructional goals for *all* students: "New societal goals for education include (1) mathematically literate workers, (2) lifelong learning, (3) opportunity for all, and (4) an informed electorate" (NCTM, 1989, p. 3).

The publication of the *Standards* catalyzed a large (and not uncontroversial) change in mathematics instruction, which came to be known as “reform.” Desired reforms (which were grounded in contemporary research, but had not been empirically tested on a large scale) included the following:

"We need to shift –

- toward classrooms as mathematical communities—away from classrooms as simply a collection of individuals;
- toward logic and mathematical evidence as verification—away from teacher as the sole authority for right answers;
- toward mathematical reasoning—away from merely memorizing procedures;
- toward conjecturing, inventing, and problem solving—away from an emphasis on mechanistic answer-finding;
- toward connecting mathematics, its ideas, and its applications—away from treating mathematics as a body of isolated concepts and procedures." (NCTM, 1991, p. 3)

The *Standards* emphasized mathematical processes as well as content. Specifically, there was a focus at all grade levels on problem solving; on reasoning; on connections within mathematics and from mathematics to ideas outside mathematics; and on communicating using mathematical ideas. In the years from 1989 to the present, there has been some slow implementation of reform, along with a fair amount of experimentation[3]. After the publication of the *Standards*, some groups (sometimes with funding from the U. S. National Science Foundation) began the development of

[3] The *Standards* did not specify curricula, but rather a set of learning goals for students. Thus it was possible to develop very different approaches to instruction that were “in the spirit of the standards.”

curricula aligned with (their authors' interpretation of) its goals. These curricula became available in the mid-to-late 1990s. Reliable data on their use, discussed later in this section, is just beginning to accumulate.

Toward the end of the 20th century, NCTM realized that it needed to re-examine the contents of the *Standards*. Part of the reason for this reconsideration was political: the original document had been interpreted in so many different ways that some clarification was in order. More importantly, a lot had been learned in the years since the *Standards* had been issued. Ideas that had been speculative (that is, research-based but not extensively field-tested) had since been examined in practice, and methods, ideas, and materials had been significantly refined over the ensuing decade. Equally important, there had been important changes in the world outside of school. When the *Standards* were written, its authors took a bold stance, arguing that all high school students should have access to (and use) graphing calculators. Just a few years later, computers and the World Wide Web became accessible resources. Numbers no longer had to be "nice"; machines could do number crunching. Large data sets were available on the web, meaning that students didn't have to work with "faked" data. Graphing packages were available, as were various modeling tools. With such tools and data available, the nature of the mathematics that could be done in classrooms changed considerably. And, the threshold of mathematical competence for full participation in America's participatory democracy kept rising.

All of these reasons led NCTM to issue *Principles and Standards for School Mathematics* in April 2000. (Full disclosure: I was a member of NCTM's Commission on the Future of the Standards, which decided that a new vision was needed, and a member of the writing team that produced *Principles and Standards*.) *Principles and Standards* represents an evolutionary change from its antecedent, in that it is informed by a decade's experience working toward the content and process goals of the *Standards*. But there are ways in which *Principles and Standards* is itself revolutionary. Just as the original *Standards* represented a vision statement – a set of goals for the future – so do *Principles and Standards.* Perhaps one of the strongest positions in the document is that all students should study a basic core set of mathematics courses each and every year that they are enrolled in secondary school. The expectation is that this common core will prepare all students for quantitatively literate citizenship, entry into the workplace upon graduation, and the pursuit of mathematics at the university level if they desire.

This expectation flies in the face of 100 years' curricular tradition in the United States. It is also a bold (and some would say impossible) cry for social justice, given the data that I shall soon describe.

Part of the rationale for the recommendation is as follows. There are basically two audiences to consider: those who (for the time being at least) see themselves as having no mathematical needs beyond those required for a good job and literate citizenship, and those who will pursue the further study of mathematics. A good case can be made that the needs of these two groups are converging. The threshold for quantitative literacy has been rising. Today one expects people to be able to model and understand real-world phenomena using quantitative tools, to analyze and understand (and even make) complex logical arguments; to make decisions about social issues; to use technological tools appropriately when necessary; and to communicate effectively orally and in writing. Such skills are required for decision-making in one's personal life (e.g., when choosing mortgages or telephone plans), for interpreting information in newspapers (which is increasingly given in graphical or tabular form), for making informed choices regarding public policy (just how dangerous is a pesticide suspected of causing damage, or living near power lines?), and on the job (e.g., making predictions using spreadsheets and other software, defending one's choices or line of argument in a memo).

Many of these skills were given scant attention in the traditional curriculum. They can be seen not only as part of the foundation for quantitatively literate citizenship, but also as part of the foundation for mathematical and scientific careers. Let me describe my own background. My Ph.D. is in mathematics. Through secondary school and well into my collegiate career I studied no statistics and learned nothing about analyzing data. (I first studied statistics when I had to teach it.) I never did any "real world" modeling, or had practice at representing real world phenomena in mathematical terms. With the exception of a rather stilted form of writing up proofs in 10th grade geometry, I was not asked to make mathematical arguments of any sort until I was asked to *reproduce* proofs in calculus, then write them in a linear algebra course. I was rarely if ever asked to communicate using the language of mathematics; more often than not, producing a string of symbols and the right number at the end of my computations sufficed to get full credit for working a problem. In sum, my preparation as a mathematician-to-be would have been far richer had I been asked to develop the skills that are now relevant for all citizens. A common core can serve both groups (with the mathematically inclined studying additional mathematics if they wish.

That being the goal, what is the reality?

The data speak: Diversity and equity must be major concerns with regard to mathematics education.

As a mathematician, I value mathematics for myriad reasons: its beauty, its clarity and coherence, its power as a way of thinking, its role as the "language of science," its contributions to our intellectual heritage, and more. As an educator, I realize that access to high quality mathematics instruction – the kind of instruction that will enable students to develop mathematical competency – is a matter of social justice.

Everybody Counts, a 1989 report from the U. S. National Research Council, made the case this way:

More than any other subject, mathematics filters students out of programs leading to scientific and professional careers. . . . Mathematics is the worst curricular villain in driving students to failure in school. When mathematics acts as a filter, it not only filters students out of careers, but frequently out of school itself. . . .

Low expectations and limited opportunity to learn have helped drive dropout rates among Blacks and Hispanics *much* higher -- unacceptably high for a society committed to equality of opportunity. It is vitally important for society that *all* citizens benefit equally from high quality mathematics education. (National Research Council, 1989, p. 7)

This last statement situates mathematics instruction firmly as an equity issue. The "gender gap" in mathematics performance and the role of mathematics as a "critical filter" for women have been documented for some time (see, e.g., Sells, 1975, 1978). Similar data exist for under-represented minorities (specifically African Americans, Latinos, and Native Americans). In 1990, the U. S. National Research Council published *A Challenge of Numbers,* which synthesized a great deal of data regarding the mathematical trajectories of various sub-populations of the United States. Here in tabular form are data regarding the percentage of students enrolled at various levels in mathematics in the late 1980s.

	8th Grade	12th Grade	B.S. in math	M.S. in Math	Ph.D. in Math
Asians	2	2	6	8	8
White Male	40	41	45	55	70
White Female	39	39	40	33	17
Black	12	11	5	2	2
Hispanic	7	6	2	2	2

Percentage of students at various points in the mathematics pipeline.
Data drawn from Figure 4.2 of NRC, 1990.
(Rounding results in some column sums not being 100)

Reading each row from left to right provides documentation of increasing or decreasing participation in mathematics, from eighth grade on. Since schooling is essentially universal at eighth grade, the first column represents the approximate proportion of each demographic group in the U.S. population. One sees a substantial percentage increase in mathematics participation among Asians and White males, and a substantial decrease among White females, Blacks, and Hispanics. These data represent just the tip of the iceberg, for they fail to capture the "performance gap" between various demographics groups (in terms of scores on various standardized exams) at all levels of the educational system. A synthesis of current performance and demographic data has just been published in the *Educational Researcher* by Jaekyung Lee. Lee's (2002) findings are not encouraging. They suggest that the progress toward narrowing racial and ethnic achievement gaps in the 1970s and 1980s (as reflected by scores on a range of standardized tests) may have slowed or reversed in the 1990s. In what follows, NAEP refers to the U.S. National Assessment of Educational Progress, a federally funded national sampling of student performance in core subject areas. The SAT is a "high stakes" examination taken by a large percentage of students applying for post-secondary study. Among Lee's findings were the following.

- Black-White average score gaps on the NAEP mathematics tests tended to diminish from 1971 through 1990, but then stabilized or increased through 1999. In 1999 these differences were between 25 and 35 points at all grade levels. (NAEP defines five "performance levels" of mathematical proficiency corresponding to of 150, 200, 250, 300, and 350. The average differences of 25 points represent a very large and significant difference.)
- Hispanic-White average score gaps on the NAEP mathematics tests showed a similar trend, in that they tended to diminish from 1971 through 1990, but then stabilized or increased through 1999. In 1999 these differences were between 20 and 30 points at all grade levels.

- Black-White average score gaps on the SAT mathematics exams followed a similar pattern over the period from 1977 to 2000, with a steady decrease in the score gap from 123 in 1977 to a low of 91 in 1990, but then very slow increases to a difference of 94 in 2000. (SAT scores are on a 200-800 scale, with a mean of 500 and a standard deviation of about 110. These are very large and significant differences.)
- Hispanic-White gap trends on the SAT mathematics exams were similar, although the magnitude of the gaps has been a bit smaller (as it was on NAEP). There was a steady decrease in the average score gap from 80 points 1978 to a low of 57 points in 1989, but then a steady increases in differences from then on, to an average difference of 69 points in 2000. These too are very large and significant differences, with the trend moving away from equality.

Lee also offers comparative data on trends of selected measures of socioeconomic, cultural, and educational conditions among Blacks, Whites, and Hispanics from 1970 through 1998. These data offer few reasons for cheer, other than the fact that, generally speaking, things do tend to be better now than they were thirty years ago. Here are some of the relevant data. Data are given in terms of ratios of proportions of the populations being compared. In 1998,

- The likelihood of a Black family living in poverty was 2.5 times that of a White family; the likelihood of a Hispanic family living in poverty was 2.3 times that of a White family.
- The likelihood of a Black family being headed by a single parent was 2.5 times that of a White family; the likelihood of a Hispanic family being headed by a single parent was 1.3 times that of a White family.
- The high school dropout rate for Blacks was 1.8 times that for Whites, and the high school dropout rate for Hispanics was 3.8 times that for Whites.

These statistics are troubling – and, of course, data summaries capture the realities in rather dry ways. Kozol's (1992) *Savage Inequalities* brings them to life in dramatic (and much more disturbing) fashion.

It should be noted that while the data portray some of the harmful realities that need to be addressed at both the social and school levels, they do not at all paint a clear picture of precisely how they are related. Indeed, some trends such as high school dropout rates differed substantially for Blacks and Hispanics, while many of the trends regarding socioeconomic and cultural conditions looked remarkably similar. Lee summarizes his presentation of the data with the following comment: "In brief, this analysis of schooling conditions and practices shows that none of the conventional indicators examined above fully accounts for the bifurcated racial and ethnic achievement gaps trends that I have described" (Lee, 2002, p. 10).

Despite the absence of a clear causal (or in some cases, correlational) mechanism, aspects of the problem are clear. There are huge performance gaps in mathematics. There is differential access to mathematical resources, with poor and underrepresented minority students less likely than others to have access to high quality instruction. (See Kozol, 1992, for graphic descriptions of educational inequities in the United States; see Secada, 1992, for a broad characterization of racial, ethnic, and class issues in mathematics education.) The legal term for guaranteed access to educational opportunities is “opportunity to learn (OTL).” OTL has become a major civil rights issue in the U.S.

Generally speaking, a lack of credentials or poor performance in mathematics is likely to lead to decreased opportunities. Assuring high quality instruction, and moving toward a high level of performance for all students, is an issue of social justice.

This point has been highlighted by Robert Moses, civil rights leader and founder of the Algebra Project (a project intended to help provide disenfranchised minority students access to mathematics). Moses notes that algebra has come to take on a powerful filtering role in school curricula: those who will “make it” do so by passing algebra, while the rest will have severely limited opportunities. In *Radical equations: Math literacy and civil rights*, Moses writes:

> Today … the most urgent social issue affecting poor people and people of color is economic access. In today’s world, economic access and full citizenship depend crucially on math and science literacy. I believe that the absence of math literacy in urban and rural communities throughout this country is an issue as urgent as the lack of Black voters in Mississippi was in 1961. (Moses, 2001, p. 5)

Focusing in on the classroom: Some of what we know

Let me begin this section by reiterating two points. The first is my emphasis on examining classroom instruction – albeit with the recognition that factors outside the classroom walls obviously play a powerful role shaping what can and does take place inside them. The second is my notion of teaching as a set of actions that help students negotiate the terrain between what they bring to the learning environment and what one wants them to learn. The question for me in thinking about focusing on the classroom is deceptively simple: What can we know, and how can we know it?

There is a clear policy context, which I shall summarize in brief. And there are suggestions (both in terms of findings and methods) from research on gender; on language; on attempts to teach “mathematics for all”; on individual agency; on classroom community; and in fine-grained analyses of learning.

Policy Assumptions

As noted above, there have been some dramatic changes in American mathematics curricula since the issuance of the NCTM *Standards* in 1989. These changes have not been uncontroversial. Curricula constructed in line with the *Standards* tended to emphasis “process” to a significant degree: the first four standards at each grade level concerned problem solving, mathematical reasoning, making connections, and communicating mathematically. There has been a concomitant de-emphasis on practicing basic skills and on the mastery of procedural algorithms (e.g., the procedures for long division and multiplication of multi-digit numbers). This raised for some the concern that students would lose foundational mathematical skills, without which they would be seriously handicapped. For some years the controversies lay primarily in the political arena, since there were no hard data to make the case one way or another. The first volume of *Standards* was published in 1989, and “standards-based” curricula were developed in the mid-1990s. They were first implemented on a large scale in the late 1990s, and data concerning their implementation have only begun to be available over the past year or two. Those data suggest the following:

The alignment of curriculum, student assessment, and professional development (enhancing the capacity of teachers to implement curricula as intended) is essential. When a standards-based curriculum is implemented in a stable context and when assessment and professional development are consistent with that curriculum, there can be significant improvements in student learning. Those improvements include:

- scores on measures of skills that meet or exceed the scores of students who study traditional (U.S.) mathematics curricula. (In other words, fears that less direct attention to basic skills would result in an absence of those skills are not warranted.)
- tremendously enhanced performance on measures of concepts and problem solving, in comparison with the test scores of students who study traditional curricula. (This, of course, should come as no surprise; traditional curricula give much less attention to concepts and problem solving than do reform curricula.)
- a significant decrease in racial "performance gaps." In one well-documented case, Black/White racial differences essentially vanished on measures of skills; they dropped substantially on measures of concepts and problem solving.

Data supporting these assertions may be found in Schoenfeld (2002). These data provide a warrant for looking at contexts where students are encouraged to engage with meaningful mathematics – that is, with mathematics curricula consistent with NCTM's *Principles and Standards* or the earlier *Standards.* The data also point to the fact that such engagement is much more likely to be successful in the right "policy surround" – one in which teachers are supported in their efforts to make the mathematics accessible to students, both by means of assessment policies and by professional development.

Issues of Context

Though they are not the focus of the classroom analyses I propose to discuss here, one must keep in mind the variety of contextual factors that shape the opportunities made available to students. These include differential opportunities due to unequal distribution of resources and tracking or "curriculum differentiation." Secada (1992) documents relationships between various contextual factors (race, ethnicity, social class, and language) and mathematics achievement (typically measured on standardized achievement tests); Lee (2002) updates some of these. As noted above, Kozol (1992) portrays the stark realities that lie behind some of those data. Oakes, Gamoran, and Page (1992) describe the effects of tracking:

> "Curriculum differentiation works against the success of academically deficient students: By the end of the year, they tend to fall even further behind. Even in the best of cases, in which ability grouping benefited low-ability as well as high-ability students in certain elementary school studies, high-group students tended to gain more, so that the gaps still widened." (Oakes, Gamoran, & Page, 1992, pp. 599-600)

Putting aside for the time being the problematic nature of constructs such as "high ability" and "low ability" students[4], this does suggest some issues that could be examined in classrooms, e.g., the uses of grouping and the consequences thereof. Of particular interest to me is explanation at the level of mechanism. Such studies exist in reading, for example: "At the elementary level, low reading groups spend relatively more time on decoding activities, whereas more emphasis is placed on the meanings of stories in high groups" (Oakes et al., 1992, p. 583). This serves as an explanation of why the rich get richer, in that the more advanced students are presented more opportunities to learn the things that all students need to learn. Similarly in high school mathematics, teachers of "low ability" classes tended to emphasize mathematical procedures, while teachers of "high ability" classes gave much greater emphasis to inquiry skills, problem solving, and the preparation for further study (Oakes et al., 1992, p. 584).

Issues of Differential Treatment

The previous section focused on differential treatment at the group level. Classroom analyses have also focused on differential treatment at the individual level (aggregating the individual data). Some studies with the best potential for the detailed examination of classroom practices regarding differential treatment were gender studies, which have a tradition that goes back some 30 years. After examining patterns of classroom interactions, for example, Good, Sikes, and Brophy (1973) concluded that "male and female students are not treated the same way" (p. 85; quoted in Koehler, 1990). Typical studies examined the frequency of the questions teachers asked boys and girls, and their nature – whether questions were at high or low content levels, how often they were focused on disciplinary issues, and how often teachers' comments focused on substantive content issues or superficial aspects of work such as neatness.

In early work on classroom practices, in the 1970s, achievement scores were not examined. As a result, systematic patterns of interactions could not be related (even statistically) to outcomes. Also, the scope of processes covered was rather narrow. Hence it is not clear what would correlate with what (or even if the right variables had been chosen), even if outcome measures had been used.

A next generation of studies in the late 1970s and 1980s, called differential effectiveness studies, employed the "process/product" paradigm, which attempted to link differential teacher and student behaviors to differential performance outcomes. Such studies rapidly revealed unexpected complexities. First, correlational patterns were not what one might naïvely expect. Differential patterns of engagement did not consistently produce differential scores, raising hypotheses that some teacher behaviors might be appropriate for some students, and not others. (In the language of the time, there might be an "aptitude-treatment interaction" that confounded the relation between teacher actions and student outcomes.)

Leder (1992) reviews a broad spectrum of gender studies in mathematics. A jaundiced summary of Leder's summary might be "there are lots of interesting things to look at, but very few if any clear-cut conclusions that one can draw." Environmental variables listed by Leder

[4] Such classifications are often made on the basis of standardized tests, which tend not to make accommodations for linguistic skills. The use of such tests can thus lead to the classification of a mathematically talented student who is taking the test in his or her second (or third)language as being "low ability."

included school variables, teacher variables, the peer group, the wider society, and parents. Learner-related variables included intelligence, spatial abilities, confidence, fear of success, attributions, and persistence.

The process/product paradigm died pretty much a natural death, and for good reason. There were two main difficulties regarding such studies. The first is that the work was correlational – and as indicated above, the correlations did not provide much by way of insight. The second is that outcome measures were almost all mathematically superficial. Standardized tests were typically employed. These gave little attention to the complex processes of mathematical thinking and learning that are now central to educational discourse. Thus, while such studies suggest interesting things to look for in patterns of teacher-student interactions, a new (and much more fine-grained) perspective is required. Such a perspective would attend much more to the mathematical richness of the interactions, and would try to link the character of the interactions more directly to student performance.

Looking more closely at teacher practices

One lens through which one might examine teacher practices is that of "culturally relevant pedagogy," as described by Gloria Ladson-Billings (1994). Ladson-Billings (1997) abstracts some principles of productive pedagogies for *all* students as follows:

- Students treated as competent are likely to demonstrate competence.
- Providing instructional scaffolding for students allows students to move from what they know to what they do not know.
- The major focus of the classroom must be instructional.
- Real education is about extending students' thinking and abilities beyond what they already know.
- Effective pedagogical practice involves in-depth knowledge of students as well as subject matter.

Ladson-Billings goes on to note that researchers face serious theoretical (and methodological) challenges in trying to frame productive "next steps" in research – the job being to confront the necessary complexity of classroom interactions and characterize it in ways that allow for building productively on what students know. That is indeed the challenge.

It is worth noting that culturally relevant pedagogy need not be "culturally specific." Some programs, such as the Algebra Project (Moses, 2001; Moses, Kamii, Swap, & Howard, 1989) and the Jaime Escalante Math Program (Escalante & Dirmann, 1990) are designed to address the perceived needs of specific groups of students. Other programs, such as Cognitively Guided Instruction, or CGI (Carey, Fennema, Carpenter, & Franke, 1995) and QUASAR (Silver, Smith, & Nelson, 1995), or many of the standards-based curricula, were not designed for implementation with specific populations of students. The key desideratum is that they were designed to meet students "where they are."

Additional factors to consider follow.

Issues of Language and Discourse

In recent years there has been a significant change in perspective regarding the mathematics instruction of "English language learners" and/or those students whose cultural backgrounds are from other than mainstream U.S. culture. Older studies tended to look upon mathematics learning as the acquisition of vocabulary and of skills; English language learners were often thought of as having language (and other) "deficits" and instructed narrowly in terms of vocabulary. Today it is understood that engaging in mathematics involves a form of sense-making that far transcends the acquisition of a technical vocabulary; also that deficit models are not a productive way to address the educational needs of students with non-mainstream backgrounds. Echoing the comments of Ladson-Billings summarized above, for example, Garcia and Gonzales (1995) note the following characteristics of teachers considered successful with linguistic and cultural minority students: high expectations for all students; a rejection of models of their students as intellectually disadvantaged; commitment to students' academic success; commitment to student-home communication; and willingness to modify curriculum and instruction to meet the specific needs of their students.

The new emphases in standards-based curricula on mathematical processes – on problem solving, reasoning, connections, and communication – call for a much higher level of mathematical discourse.

> "Research in mathematics education documents a variety of perspectives regarding what it means to learn mathematics. Learning mathematics can be seen as learning to carry out procedures, develop hierarchical skills, solve mathematical problems, or mathematize situations. Recent theoretical perspectives have focused increasingly on mathematics learning as a process that intrinsically involves the use of language. Such notions include descriptions of mathematics learning as sense-making (Lampert, 1990; Schoenfeld, 1992), as participation in communities of practice (Lave & Wenger, 1991; Brown, Collins, & Duguid, 1989), as developing socio-mathematical norms for participating in the discourse of mathematics classrooms (Cobb, Wood, & Yackel, 1993), and in general as learning to participate in mathematical discourse practices such as modeling and argumentation (Brenner, 1994; Forman, McCormick, & Donato, 1998; Greeno, 1994)." (U.C. ACCORD Mathematics working group, October 2000, p. 10).

As Brenner (1994) observes, *Standards*-based curricula typically call for discussing and analyzing problem situations, choosing the relevant analytical and representational tools, solving problems, and communicating the results. In comparison with traditional curricula, this requires the increased use of language in the service of mathematical sense making. Hence classrooms in which these curricula are employed run the risk of placing English language learners at risk – unless their teachers can find ways of taking advantage of the first language resources the students bring with them to instruction. This will call for mediating between the linguistic resources that the students come with – typically everyday language in their first language and some mastery of English – and the specialized use of the "mathematics register" (Halliday, 1978), a precise technical form of expression using mathematical terms that has its own specialized syntax and meanings (see, e.g., Khisty, 1995; Moschkovich, 1999, 2000; Pimm, 1987; Warren & Rosebery, 1995). More generally, an argument can be made that teachers (and researchers on teaching) need to be familiar with a range of issues pertaining to language, language development, and language acquisition (See Fillmore and Snow, 2000). In terms of classroom research, this will call for fine-grained analyses to see how interactions among students and between the students and the teacher work to support or inhibit students' meaningful engagement with the rich conceptual aspects of mathematics.

To make this discussion concrete, let me give some examples of how an inappropriately high linguistic threshold can impede English language learners' participation in mathematics and other subjects, and paint a distorted picture of the students' competencies. Lily Wong Fillmore has investigated the language demands in "high stakes" contexts such as high school exit examinations in various states. Fillmore (2002) points out that the tests examine not only subject matter mastery, but students' command of academic English. Here is a sample problem from the Arizona exit exam.

> If x is always positive and y is always negative, then xy is always negative. Based on the given information, which of the following conjectures is valid?
>
> A. $x^n y^n$, where n is an odd natural number will always be negative.
>
> B. $x^n y^n$, where n is an even natural number, will always be negative.
>
> C. $x^n y^m$, where n and m are distinct odd natural numbers, will always be positive.
>
> D. $x^n y^m$, where n and m are distinct even natural numbers, will always be negative.

Fillmore writes:

"What's difficult about it? Nothing, really, if you know about, can interpret and use—

- exponents and multiplying signed numbers;
- the language of logical reasoning;
- the structure of conditional sentences;
- technical terms such as *negative, positive, natural, odd,* and *even* for talking about numbers.
- ordinary language words and phrases such as *if, always, then, where, based on, given information, the following, conjecture, distinct,* and *valid.*" (Fillmore, 2002, p. 3.)"

Fillmore continues with sample questions from the tenth-grade Massachusetts Comprehensive Assessment System (MCAS).

1. *Which of the points below is not collinear with the others?*
   M (3, -2) N (-5, 6) S (-9, 10) T (10, -21)

   A. N only
   B. S only
   C. T only
   D. They are all collinear.

2. *The amplitude, frequency, and shape of an electrical signal can be displayed and measured using*

   A. a signal generator.
   B. a multimeter scope.

C. an oscilloscope.
D. an odometer.

3. *The Petition of Right that the English Parliament forced King Charles I to sign in 1628 included the principle of habeas corpus, which means that*

A. only a legislative body can collect taxes in time of peace.
B. civil law cannot apply to the clergy.
C. martial law can only be applied by the head of the government.
D. no one can be imprisoned unless charged with a specific crime within a reasonable time.

These examples are clearly problematic, if one takes seriously the idea that assessment should help reveal "what students know and can do." These examples, taken from formal assessments, also highlight potential linguistic issues in the acquisition of mathematical understandings. It will not be terribly difficult, I suspect, to find evidence of unhelpful discourse practices in mathematics classrooms. The question is, how does one document what are likely to be productive practices, and provide meaningful evidence of the relationship between the practices and their impact?

Issues of Participation and Agency

Active engagement (of a mathematically appropriate and productive kind) is likely to be a major factor contributing to students' mathematical success. There are various ways one can look at issues of engagement, at both the collective and individual levels. One can examine participation structures, both whole class and small group. Are all students "invited" to participate fully? Are there moves by teacher and/or students that enfranchise various students, or that disenfranchise them? Analyses of this type, combined with analyses of the kinds of comments made by individual students, can paint a good picture of local engagement – of what students are doing and how they engage with the material. But then there are at least three other issues that need to be considered, if one is to have a chance of seeing the "big picture."

First, there is the issue of linking participation and engagement to outcomes. In the past, some of my explanatory work has been at the aggregate level. For example, I was able to argue on the basis of classroom observations that particular practices in high school geometry classrooms led to the development of particular student beliefs regarding the nature of the mathematical enterprise. It is not clear to me whether the study of aggregate or individual trajectories is more promising for linking participatory experiences with student perceptions and behavior.

Second, at what point in students' mathematical histories is it most profitable to start looking at interactive and engagement patterns? To give a specific example: a few weeks ago my research group viewed a videotape of a group of students working together on an applied problem. The interactions were nothing short of wonderful; the three students (two girls and one boy, all of different ethnicities) all contributed in substantive ways to the solution of the problem they were addressing. In terms of the methods discussed above, it would be straightforward to do a discourse analysis indicating how each was enfranchised by the others, what their contributions were, and so on. And that's essential. But looking at this tape raised more questions than it answered. How in the world did the students learn to interact like that? How typical were the interactions? How far back do you have to go to trace the ways these students learned to interact with each other, to describe the role of the teacher in shaping the group's interactions? A comprehensive data analysis would take a huge amount of time. What strategies are there for targeting the "right" things for the "right" kinds of analysis?

Third, it must be recognized that in-class interactions are shaped in myriad ways by events that take place outside of class. To name one essential feature of the interactions, consider the issue of students' mathematical agency and mathematical identities. Whether students will engage mathematically and how they will do so is a function of how they see themselves, how they see the instrumentality of the mathematics they are studying, and how they see themselves fitting in with their environment (Eckert, 1989; Martin, 2000). Martin (2000), for example, describes interviews with African Americans who felt that, now matter how well they did mathematically, they would never be given job opportunities that would use such skills – so why bother? Other interviews reveal that parents, by underestimating the specific mathematical prerequisites for progressing through the educational system, can limit their children's opportunities. How far back in time, and how far outside the classroom, must one go to trace such things appropriately? Another issue has to do with beliefs. For example, the typical American belief that one is either born good or bad at mathematics (in contrast to the typical Japanese belief that one's performance in mathematics is directly related to the amount of work one puts into studying) clearly shapes how students engage mathematically.

Issues of meaningful mathematics (in and out of the classroom)

The question here is: what is meaningful to students, in what ways; what unexpected territory might one enter when trying to introduce students to rich mathematical terrain? This is, in a broad sense, a curricular issue. (I take "curriculum" to mean both the materials that students study and the ways in which they are brought together to study them.) Horn (2002; in preparation) has studied the concept of "group-worthy" activities used by one reform-oriented mathematics department. These are mathematical problems and activities that can be accessed from multiple starting points and that can engage students with diverse mathematical backgrounds. Group-worthy activities provide affordances for classroom interactions that can enfranchise and support a wide range of students. Teasing out the interaction of such curricular materials with the kinds of interactions that can and do take place in the classroom adds yet another level of complexity to the task of seeing "what counts."

Curricular choices intended to "reach the students where they are" can raise issues that are not encountered when one teaches more traditional mathematics. Silver, Smith, & Nelson (1995) describe one such example. Teachers in the QUASAR program had administered the following open-ended task to students:

> Yvonne is trying to decide whether she should buy a weekly bus pass. On Monday, Wednesday, and Friday she rides the bus to and from work. On Tuesday and Thursday, she rides the bus to work, but gets a ride home with her friends. Should Yvonne buy a weekly bus pass? Explain your answer.
>
> *Busy Bus Company Fares*

One Way	$1.00
Weekly Pass	$9.00

Teachers were surprised by the number of students who responded that the weekly pass was a better buy, given that the one-way fares described in the problem statement added up to only $8.00 per week. When they discussed their answers with students, "many students argued that purchasing the weekly pass was a much better decision because the pass could allow many members of a family to use it (e.g., after work and in the evenings) and it could also be used by a family member on weekends." (Silver, Smith, & Nelson, 1995, p. 41) This makes good sense – it's a real-world solution to a "real world" problem. It points to the complexities one faces in designing and implementing curricula that try to bridge meaningfully to children's lives, and to the subtleties that one faces in assessing issues such as student thinking and what it means for a curriculum to enfranchise students.

Concluding Comments

I have argued for some years (see, e.g., Schoenfeld, 1999) that the state of the art is such that educational researchers can now conduct research in contexts that really matter. For me, that means mathematics classrooms. I also have my own personal standards for what constitute well-warranted claims in education. Those have to do with explanation at a level of mechanism, where one is obligated to explain how things fit together and why things happen. My research on problem solving and on teaching has typically been at a very fine-grained level of analysis: a typical claim has been that the student or teacher behaves in particular ways because he or she has very specific knowledge, goals and beliefs. Looking for causality has often caused me to expand the scope of inquiry, and to expand the theory within which the empirical work that characterized the behavior was situated. For example, my analysis of student problem-solving protocols revealed that students routinely made conjectures in contradiction to things they "knew" (and had proved just a short time before). This led to studies of beliefs – e.g., the idea that some students "believe" that proof-related knowledge is not relevant or useful when working "discovery" problems of a particular type. That raised questions about the origins of such beliefs – which turned out to be the mathematical practices in which the students had engaged, over time, in their mathematics classes. The chain of causality for "simple" behavior in a twenty-minute problem solving session in the laboratory reached back to formative experiences, over a period of years, in mathematics classrooms.

The challenge of the problem solving research pales in comparison to the challenges of developing a coherent frame within which to examine issues of diversity and mathematics learning. It should be clear that the brief summary of some of what is known about issues of "algebra for all" given in this paper raises far more questions than answers. Each of the arenas addressed – context, differential treatment, teacher practices, language and discourse, participation and agency, and meaningful mathematics in and out of the classroom – is itself complex and not well understood. Interactions among them are that much more complex. Painting the "big picture" while maintaining a focus on detail and a predilection for explanation at a level of mechanism will be an interesting challenge.

Acknowledgment: I thank Lily Fillmore, Charles Hammond, Ilana Horn, Sue Magidson, Ed Munnich, Ann Ryu, Annie Selden, and Joe Wagner for conversations and comments that helped to shape this paper.

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