*Panel "**On the role of the history of mathematics in mathematics education**"*

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ABSTRACT

In recent years, important works on the relationship between history and mathematics education have appeared:

(a) The Proceedings of the "European Summer University on History and Epistemology in Mathematics Education" (Montpellier, France, 1993, Braga, Portugal, 1996, and Leuven/Louvain-la-Neuve, Belgium, 1999),

(b) Two books based on the elaboration of papers which were presented during the satellite meetings of HPM (History and Pedagogy of Mathematics, one of the ICMI affiliated international groups), the first edited by R. Calinger (MAA 1996), and the second edited by V. Katz (MAA 2000),

(c) The ICMI Study book on "History in Mathematics Education", edited by J. Fauvel and J. van Maanen.

(d) Journals for Mathematics Teachers and/or Mathematics Education Researchers have published special issues on the History of Mathematics in Mathematics Teaching (e.g. *For the Learning of Mathematics* in 1991, *Mathematics in school* in 1998 and *Mathematics teacher* in 2000). The re-born newsletter of HPM (International Study Group on the Relations between History and Pedagogy of Mathematics) is becoming (we hope) a forum where piece of information and ideas are shared.

These material and the experiments carried out all over the world make further discussion on the *role of the History of Mathematics in Mathematics Teaching* both possible and necessary. In recent discussions the expression "integration of History in Mathematics Teaching" appears frequently. Which ideas are behind this expression? The main idea is that of using History as a mediator to pursue the objectives of Mathematics Education. This means that, these objectives, together with the study of the historical evolution of concepts should be analysed. This work has to be carried out by educators and historians in a collaborative way. Among the benefits, which are expected to result from this work, is the new perspective offered by History to consider students' difficulties in learning Mathematics. To make teachers active actors in this process we need to give a convenient place to the History of Mathematics in pre-service and in-service teacher education.

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ON THE ROLE OF THE HISTORY OF MATHEMATICS IN MATHEMATICS EDUCATION

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ABSTRACT

In recent years important works on the relationship between history and mathematics education have appeared. Some of them, such as the proceedings of the European Summer Universities in History and Pedagogy in Mathematics education, the HPM satellite meetings of ICMI Conferences, the French publications of IREM, are evidence of rather regular activities in the field. The re-born newsletter of HPM (International Study Group on the Relations between History and Pedagogy of Mathematics) is becoming (we hope) a forum where piece of information and ideas are shared.

These materials and the experiments carried out all over the world make it possible to go further in the discussion about the role of the history of mathematics in mathematics teaching. In the recent discussions a word is appearing frequently: integration [of history in mathematics teaching]. What behind this word? The main idea is that of using history as a mediator to pursue the objectives of mathematics education. This means to develop an analysis of these objectives together with the study of the development of concepts in history. This work has to be carried out by educators and historians in a collaborative way. In the present paper we show how the preceding ideas may be applied in introducing a concept of infinitesimal analysis.

1. Introduction

In the recent years important works on the relationship between history and mathematics education have appeared. Often they are the results of initiatives particularly addressed to teachers, such as the proceedings of the European Summer University (held in 1993, 1996, and 1999). Other times they are the output of meetings among researchers (historians, mathematicians, educators), such as the two books originated by the HPM satellite meetings of ICMI conferences (1996 editor R. Calinger, and 2000 editor V. Katz), the ICMI Study book edited by J. Fauvel and J. van Maanen (2000), the book *Learning from the masters!* edited by F. Swetz, J., Fauvel, O., Bekken, B., Johansson, & V. Katz (1995), the proceedings of the Brazilian meetings *Encontro Luso-Brasileiro de história da matemática & Seminario Nacional de história matemática*, the book *History of mathematics and education: ideas and experiences* edited by H. N. Jahnke, N., Knoche and M. Otte (1996),

Journals for mathematics teachers have published special issues on the history of mathematics in mathematics teaching (e.g. *For the learning of mathematics* in 1991 and 1997, *Mathematics in school* in 1998 and *Mathematics teacher* in 2000).

Particularly impressing is the net of publications (mainly in French) edited by the French University Institutes for teacher education (IREM): they constitute a kind of common thread in the development of the subject "The history of mathematics in mathematics education".

The *Newsletter of HPM* (International Study Group on the Relations between History and Pedagogy of Mathematics, affiliated to ICMI) informs three times a year the readers about a range of initiatives (conferences, meetings, exhibitions) and publications concerning the history of mathematics in mathematics education.

Eventually I like to point out the importance of the new information and communication technology in establishing a new relationship with history, especially for those people as teachers, who had difficulties in finding the suitable materials. As illustrated in (Barrow-Green, 1998), the access to historical sources, to biographical information and references is now more available than in the past to everybody.

In the publications that I have mentioned we may find attempts of answering the central question "What is the role of the history of mathematics in mathematics education?". This question may be split into more focused sub-questions:

- which educational benefits are introduced by the history of mathematics?
- which teaching strategies are to be applied?
- how mathematics teachers are prepared to this introduction?

In theory, these issues are the same as those faced by researchers in mathematics education or curriculum developers when introducing innovations in mathematics teaching. I am thinking, in particular, at the introduction of information and communication technology. I may explain this similarity reminding the view of historiography as a "literary artifact" expressed by Hayden White, as reported by Eco (1994, p.161). Extending this concept we may say that as the technology, history too is an artifact which intervenes in teaching. As an artifact it may play the role of mediator in the process of teaching/learning. Of course in these similarities there are differences specific to the specific object of study, but at a first level we may take the plan in Fig.1 as the common path when using different mediators (e.g. history, technology).

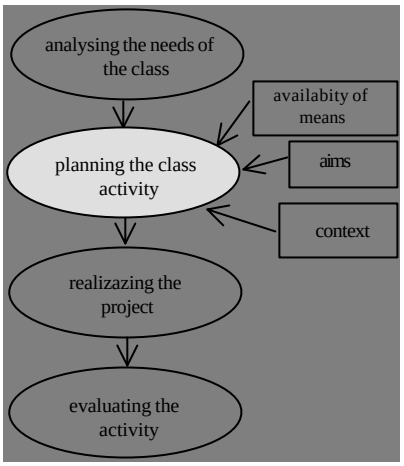

Figure1. Plan for the implementation of a teaching sequence

In the case of history the striped zone has to be specified according to the plan of Fig.2.

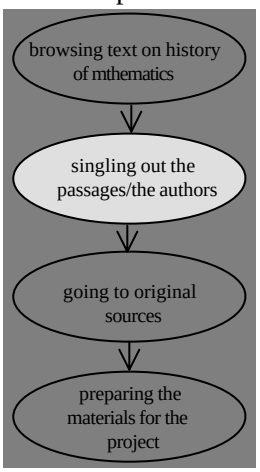

Figure 2. Plan for introducing the history of mathematics in a teaching sequence

Of course, there are variations to this plan such as going directly to the sources, if one has a suitable knowledge of the history of mathematics. The point is that the choice of passages/authors (striped zone of Fig.2) has to be carried out in the light of the educational needs.

2. An example

I give an example of this way of working by outlining the features of a project on which I have worked myself with two secondary teachers. The subject was the introduction of derivative. Our main concern was the poor concept images held by undergraduate students. To focus on the elements that may intervene in the formation of this concept image we designed a questionnaire addressed to students. The questionnaire consisted of 14 questions related to the derivative, each question containing four options plus an option allowing comments. The full work is reported in (Boggiano, Furinghetti & Somaglia, submitted). The questionnaire was given to the students of the scientific lycei of Genoa (big town) and two little towns near Genoa. All together we analyzed 434 questionnaires. The findings show that the students answer in a satisfying manner when they resort to prototypes, but fail in facing new situations. Moreover the questions containing graphics bring to light the weakness of concept images held by students, since graphics require an *active* and *aware* construction of mathematical objects. Also it emerges the students' weakness in passing

from the algebraic to the geometric domain and vice versa. We may say that derivative is one of the mathematical object to which students connect manipulation of formulas, but not mathematical meaning.

Thus the problem is to recover the mathematical meaning. The plan illustrated in Fig.3, taken from (Furinghetti & Somaglia, 1998), shows the steps we use to bridge the gap between informal and formal mathematics.

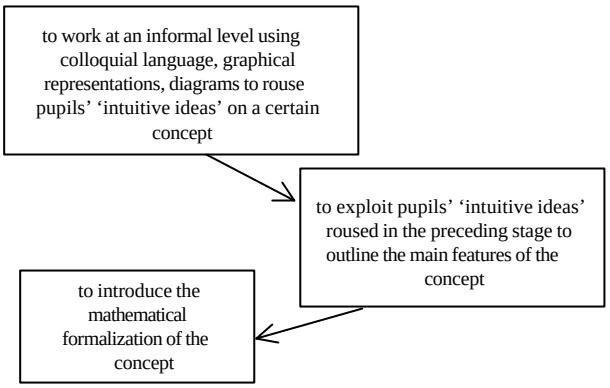

Figure 3. Steps from informal to formal mathematics

To make students work at an informal level before tackling a given topic formally allows the reification of concepts. Sfard (1994) ascribes a central role to the birth of metaphors, as explained in the following passage

> If the meaning of abstract concepts is created through the construction of appropriate metaphors, then metaphors, or figurative projections from the tangible world onto the universe of ideas, are the basis of understanding. [...] the leading type of sense-rendering metaphor in mathematics is the metaphor of an ontological object. (p.5)

For us to work at the informal level means to work in a world which is close to the students' experience, i.e. the "tangible world" mentioned by Sfard. My position is in line with Freudenthal's ideas on the efficacy of context problems as an opportunity to let formal mathematics emerge. As explained in the paper (Gravemeijer & Doorman, 1999), context problems have to be intended in a broad sense as "problems on which the problem situation is experientially real to the student" (p.111).

In this framework to use history may reveal itself fruitful and sense-carrier. In our project it was considered the pioneering period at the beginning of calculus, where the roots of the mathematical entities in the world of material objects are more visible. The tangent line to a curve was taken as the first step in the construction of the derivative. Other authors have tried this way, see, for example, (Grégoire, 2000; Villareal, 1997).

Passages from original sources were proposed to the classroom. One was taken from *Observations sur la composition des mouvements et sur le moyen de trouver les touchantes des lignes courbes* by Gilles Personne Roberval (1602-1675). Already in 1644 Marin Mersenne informed the scientific community about a method by Roberval to find tangents based on kinematics. The manuscript containing the method was written by a pupil of Roberval (Du Verdus) and presented to the *Académie des Sciences* by Roberval only in 1668.

The method holds if the kinematic generation of the curve is known, and thus only particular curves may be treated with this method. The author assumes that the direction of the movement of a point on a curve is the tangent to the curve in any position of this point. The parallelogram law for the addition of constant velocity vectors was already known. Roberval applied this law to

instantaneous velocity vectors. From the specific properties that define the curve Roberval finds the components of the movement and afterwards the tangent as the composition of them, see Fig.4. A discussion of the Roberval's method may be found in many texts, see, for example, (Edwards, 1979).

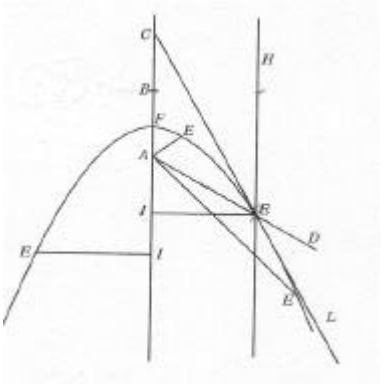

Figure 4. Kinematic construction of tangents (Roberval)

There are two aspects of the chosen extract that make it close to the ideas about the use of history that I explained before: geometry and movement. Both these aspects are part of students' experience: geometry mainly belongs to school experience, movement to everyday experience. The construction may be applied to other curves. We have chosen the second order parabola, since we wish that students work on a well known curve, applying its definition in an operative way.

The passage by Roberval was available in Italian in a reliable translation taken from one of the few readers published in Italian (Bottazzini, Freguglia, Toti Rigatelli, 1992). Thus we bypassed the problem of translation, which is one of the main problems in using the history of mathematics in teaching. Jahnke et al. (2000) distinguish at least two types of translation:

> translation into modern mathematical language, and translation from one language into another. While the former serves in particular to reconstruct a mathematical argument, the latter has promising educational advantages insofar as it initiates students and trainees into mastering a language and to conceptual analysis. (p.316)

Usually to have to deal with a foreign or dead language (Latin, Greek) is a great difficulty which takes teachers away from using history.

3. Conclusions

I have outlined the basic ideas that I see behind the use of history in mathematics teaching. To simplify my discussion I made the choice to skip the big problem of teacher education in history. I'm aware that this problem exists: it is not by chance that a full chapter of the ICMI Study on the use of the history in mathematics teaching is devoted to this subject (Fauvel & van Maanen, 2000). As far as I know the related problem of teachers' attitude is less investigated, see (Philippou & Christou, 1998). I think that the discussion on my model may be a starting point both to encourage teachers to approach history as a mediator in their work and to make plans for teacher training (pre-service and in-service).

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