

# DEVELOPMENTAL MATHEMATICS

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## ABSTRACT

The appearance of developmental mathematics in the United States has brought low-key but still intensive debate about its place in higher education, its content, its methodology, and its costs. This paper presents the reasons for such reaction and examines developmental mathematics programs in post-secondary and higher education institutions. This examination addresses the nature of developmental mathematics, the reasons for its emergence, the philosophical principles on which it is based, its historical background, and the way it is practiced in American institutions. The material presented comes from review of some of the relevant literature, anecdotal information from the writer's colleagues around the country, and the thirty-five-plus years of experience of the writer as a mathematics educator and administrator in public secondary schools, two-year colleges and four-year colleges and universities, as well as from the last five years he has spent as instructor in and supervisor of the developmental mathematics program of Northern State University.

## Terms Applying to American Education as Used in this Paper

- Campus.** The total physical parts of an institution, but it is also used to imply the collection of every entity of the institution (including students, faculty and staff).
- College vs. university.** Interchangeable terms; college, however, is also used to designate a university school (i.e. college of medicine), and the term college, by itself, may refer to any postsecondary institution, from community college to university.
- Community colleges.** Two-year comprehensive (academic, technical/vocational, and community service programs) institutions granting associate degrees.
- Freshman.** First year college student.
- General education.** A body of courses, in the basic disciplines, required of every college student.
- Higher education.** Education provided by four-year colleges and universities granting at least bachelor degrees
- Open admissions.** Matriculating any high school graduate in a college.
- Postsecondary education.** Education/training beyond high school, but normally used to designate associate degree-granting institutions (community colleges, technical/vocational schools).
- Private education.** Private and denominational schools and colleges are independent of state control.
- Public education.** Primary and secondary schools are controlled and funded on the local level, under the supervision of each state. Colleges and universities are funded and controlled on the state level.
- Secondary education** (high schools). Normally, the last four years of pre-college education.

## **Introduction**

The American education scene is a paradox.

On one hand, the need for well-educated citizens has been constantly increasing since the end of World War II. This is especially true in mathematics and science, because of the demands that galloping technological advances have placed on such education. Yet, on the other hand, the preparation of American high school pupils for entering college is progressively deteriorating—especially in mathematics.

The underpinnings of the American educational philosophy are essentially based on the credo of egalitarianism—all citizens should have an equal opportunity for access to education. In post-secondary education this has been translated by many higher education institutions as open admission. Philosophically, most colleges and universities believe that the poorly prepared high school students should be given a second opportunity. And, in the five-year experience of the writer, such a second chance works for some students.

Besides philosophy, however, there are also practical reasons for this situation. Student body size has direct relationship to financial resources colleges may have. This is especially critical for public institutions, as many states base their financial support on their number of students. Additional students bring more tuition income. Often there is economy in size (larger classes) and possibly some gain in higher prestige and political influence, as the number of students' increases`.

American collegiate institutions require their students to successfully complete general education, which includes at least one mathematics course. Institutions found out that large sections of entering freshmen are not able to complete successfully the study of even college level algebra.

This situation has been disturbing to mathematics and science educators as well as to officials of both state and federal governments. Efforts have been made by the National Science Foundation, the Mathematical Association of America, the National Council of Teachers of Mathematics and others to improve the teaching of mathematics in public schools but, by and large, their results have been spotty at best.

## **Developmental Education**

During the social and civic upheaval in the 1960s and 1970s, concerns were raised about the manner in which higher education institutions were treating their students. Universities reacted by enacting student-friendly policies. Some such policies centered on students with special needs or, as they came to be known, “students at-risk academically.” Through the years the list of students at-risk expanded and today it includes students with physical disabilities and emotional, psychological, mental, and learning difficulties, as well as students from minorities and students who lack preparation for college academic work, lack study skills, and have difficulty fitting into the campus environment.

In the past, remedial academic work was given rather haphazardly and with inconsistent efforts. What colleges and universities have done is to reform such work by expanding its scope,

sharpening its focus, codifying its application, and consolidating its efforts to the overall goal that the assistance to the student at-risk is effective. Thus, they elevated efforts for the college educational development of the student at-risk to official professional status, and remedial education was metamorphosed into Developmental Education. Since lack of preparation in mathematics has become one of the most critical educational needs in the nation, it provided impetus to facilitate remedy to it; thus, the emergence of Developmental Mathematics.

In the United States, if there is a constituency there must also be a national organization representing that constituency. It's hardly surprising, then, that the National Association for Developmental Education (NADE) emerged. NADE provides information to developmental educators through electronic mail, newsletters, and a journal. It also sponsors workshops and seminars, holds an annual meeting for its membership at large, and lobbies Congress and the Executive Branch in the interests of developmental education. It has also created regional subdivisions to facilitate communications of professionals across the states, as well as networks for each discipline called Special Professional Interest Networks (SPINs). Thus, the mathematics educators have the MathSpin network through which they communicate, mostly by electronic mail.

## **Causes and Extent of the Problem**

About 29% of students entering four-year institutions need to enroll in developmental mathematics. In some institutions that figure comes higher than 40%, while in two-year institutions the average is 43% (Carriuolo, 2001). The causes of this situation are complex. And as it normally happens, when difficulties occur that cut through and reflect on the society at large, one entity or constituency blames the other for contributing to this conundrum.

Part of the problem for high school graduates is not necessarily that they did not receive instruction in mathematics, but that instruction was not adequate. A few years back, the writer read a study done by the National Council of Teachers of Mathematics (if his recollection is correct) that many high school mathematics teachers openly criticize pupils for not doing well and especially female pupils. Criticism when constructive is helpful, but the criticism referred to in the study was malicious.

It is no surprise then that many of these students have developed "mathematics anxiety." The term was coined by Professor Sheila Tobias and is accepted as a psychologically induced phobia. Such a phobia renders the student incapable of tackling mathematics. The student believes in earnest that he/she does not possess the intelligence peculiar to understanding mathematics. The writer had students that just thinking that they had to take mathematics made them physically ill.

Another reason for the mathematics problem is the increasing graying population of college students, commonly known as "non-traditional students." These aging adults come to higher education in such numbers that they are fast replacing the traditional college students (Carriuolo, 2001). They have not attended school or college for ten, twenty or more years, and most of them are deathly afraid to tackle mathematics.

This writer also considers that the affluence of American society and the uneven distribution of that affluence contribute to the fact that both high school pupils and college students avoid difficult academic subjects as mathematics and science.

## Theory

Developmental mathematics differs from remedial mathematics, according to developmental mathematicians and behaviorists, in the fact that remedial education addresses “student weaknesses or deficiencies” and carrying the connotation that the student needs “fixing” in a specific area. Developmental mathematics, on the other hand, addresses the problem by a “comprehensive process focussing on the intellectual, social, and emotional growth” of the student (Kinney, 2001).

Part of the general problem with graduating high school pupils is that they have been conditioned to be followers rather than leaders, both in their studies and social behavior. Thus, part of the responsibility of mathematics education on the collegiate level is to develop the students to be original thinkers. Stahl, Simpson, and Hayes have found that developmental mathematics instructors should “strive to help students to become independent learners: autonomous, self-regulated, and good strategists.” (Kinney, 2001)

Generally, high school pupils, especially in inner city schools as well as in small rural schools, are not trained to respond positively to demands of their studies and their environment. The position of mathematics educators is that a developmental mathematics program should include the following elements of demandingness and responsiveness, which in turn should help the student to reach a responsive and responsible self-regulation (Wambach, 2000)

### A. Demandingness

1. Standards for excellence and expectations for appropriate behavior are clearly stated and enforced.
2. Skills courses are challenging and clearly connected to the curriculum.
3. Content competence is demonstrated by required reading, writing, and computation.

### B. Responsiveness

1. Responsiveness is exhibited by delivering timely and useful feedback.
2. Responsiveness is exhibited when the development of self-regulation is intentionally fostered.
3. Responsiveness is exhibited when a wide variety of learners are accommodated.
4. Responsiveness is exhibited when the program staff gets to know the learners as individuals..

## Content

The type and extent of the content of developmental mathematics programs is normally organized by mathematics faculty of each institution. Thus, developmental mathematics programs differ from institution to institution in form, extent of the material covered, and the level of difficulty the material is treated.

Normally, the program includes two (or more) courses in sequence. Typically, a first course includes arithmetic, concentrating on the set of rational numbers and their properties and operations. Then, it moves to a rather light treatment of basic algebra. This includes linear

equations and inequalities, systems of linear equations, introduction to Cartesian Geometry, some treatment of Euclidean Geometry, polynomials and functions and possibly algebraic fractions or even further. The second course raises the difficulty of the algebraic materials covered in the first course, and moves on to cover as much of algebra as time and the students' caliber allow.

## **Student Placement**

In order for the student to succeed in his efforts in mastering mathematics, the institution's intervention must be proactive, and measures taken must be done in a positive manner. The institution, therefore, must intervene in the student's studies as early as possible. As a rule, students entering college must take a mathematics placement test. According to results of that test, students are placed in one of the courses in developmental mathematics, or in one of the more advanced courses. In some universities, such placement is done according to scores in the national college admission tests.

In the past, the student's high school performance in mathematics, as reflected by the student's grades, was the criterion for placement in the appropriate college mathematics course. It was found, however, that high school grades were unreliable for predicting the student's achievement in college mathematics. This is also indicated in two research studies done in Utah. It was found that students who had taken high school Algebra I and Algebra II (and some geometry) still had to take a developmental course in college (Hoyt 2001).

## **Delivery**

Some universities have established their own schools, usually named "General College" or "University College." They are charged with the first two-year college education for students who have not decided their field of study or do not meet the standard academic criteria for admission to regular programs, and with providing developmental education. The majority of the universities, however, have left that to their mathematics departments or to specially developed administrative units. Finally, the community colleges provide developmental education to their students who transfer to four-year institutions..

## **Delivery Techniques**

According to research, the lecture method is not the most effective way for the delivery of information in the classroom. It is, however, the most efficient way and, therefore, the most economical—which budget-makers love. It is also the technique in which most of us were raised with, from grade one to doctorate level. Additionally, it offers the most expedient use of our time and caresses our ego the most. So most of us prefer it. And our institutions unabashedly endorse it.

Team teaching offers students a better approach than the lecture method. In such a case, besides the mathematician in charge of the course other instructors or even behaviorists are actively present and partaking in the instructional process.

Increasingly, universities are moving into computer-mediated instruction. The effectiveness of such instruction seems to be supported in a study that found that those students in developmental mathematics courses using the lecture technique were more likely to withdraw from the course than the ones in computer-mediated courses (Kinney, 2001).

In sparsely populated areas, the advent of distance learning, mainly through electronic media, is becoming popular and promising (Kinney, 2001). Such an area is South Dakota's northern tier, which covers almost half of the state, with dispersed ranches, farms, small towns and villages, and Native Indian Reservations, all large distances from each other. Here, the schools are small, with limited resources and often inadequate staff. Northern State University (NSU) is presently preparing distance education modules in mathematics to assist these schools in becoming effective in the remediation of their pupils. This is in conjunction with a federally funded program (Upward Bound) that provides the resources for NSU to assist high school pupils in understanding the academic demands of a collegiate institution and familiarizing themselves with the campus environment.

The best results occurred, however, when the students were allowed to move in the course at their own speed. In such individualized, self-paced instruction, students move at their own choice of speed from module to module and decide when they are ready to be tested in the modules given to them for study (Kinney, 2001).

Part of the delivery process is the Supplemental Instruction. Here, graduate assistants or other qualified staff members hold special sessions with small groups of developmental mathematics students and go over the same material already covered by the instructor, but modified according to the needs of each group.

Tutoring is an integral part of developmental mathematics. It is done both by peers of the students and by tutors paid by the institution. It is a one to one process and each student receives the full attention of the tutor. Some institutions contract tutoring to outside concerns whose expertise is in tutoring.

Electronic media tutoring in developmental mathematics is gaining ground. Some use of videotapes takes place, but interactive computer programs are becoming popular. Additionally, more and more publishing houses have developed their own tutoring programs that correspond to their developmental mathematics texts and are offered free to students through the Web

Complementing tutoring is mentoring. This is a two-pronged process. First students are trained to be mentors of other students. Second, mentors assist in the training of students wishing to be tutors and supervise them at the beginning of their tutoring duties. Mentors may also be assigned to specific students in order to assist them in their studies and campus department. .

## **Learning Communities**

Learning communities use the resource of faculty from various disciplines, so that various types of expertise come to bear on a specific unifying theme Carriuolo (2001). Some students performing poorly in mathematics because of a combination of poor study habits, social problems, family difficulties, low self-esteem, lack of motivation, or even poor health. Such students who have similar academic, professional or social interests are grouped together to constitute a learning community. This is especially facilitated if students are housed in close

proximity to each other, as in a dormitory. A staff member is in charge of the group and qualified staff members work with them to resolve their problems. The staff member that is in charge of the group monitors the progress of each student in the courses he/she is taking and meets individually with students whose performance does not measure up to expectations.

#### Reaction of the Campuses

Many campuses, especially in small to medium institutions, did not react kindly to the introduction of developmental education. The various campus constituencies are not happy to share their resources with other constituencies. So, it is not unusual that mathematics departments were not overjoyed when they were asked to fit the expenses for developmental mathematics into their budgets. And the reaction of mathematics faculties was typical of the general reaction of faculty at large not participating in the developmental program.

Aside from finances, some mathematics faculties seem to feel that students who come to the campus without a solid high school grounding in mathematics do not belong there. And the general student population seems to reflect the professors' feeling—so much so that students in developmental mathematics in small institutions are often reluctant to attend the tutoring laboratories or supplemental instruction group sessions.

Some of this animosity comes from feelings of snobbery, possibly feelings of embarrassment to have such students on campus, or from deep philosophical conviction that does not include second chances. The writer once listened to the remarks of another mathematics professor who said “We don't want these students in our university. If this [having developmental mathematics students on campus] continues it will eventually lead to lower academic standards.” The feeling that the institutions' academic integrity is being violated by the presence of students who have to be specially “cuddled and pampered” runs strong among both faculty and regular students. The resources could be spent more profitably; the common logic goes, by helping the good students.

To minimize this behavior, the title of developmental education in some institutions is Transitional Academic Studies (thus, Transitional Studies in Mathematics), perhaps giving a clearer picture of what developmental education does.

## **Meeting the Cost of Developmental Mathematics**

Besides the campus community, taxpayer groups object to developmental education programs. They have brought forth the argument that developmental education programs supported by state or federal funds constitute double taxation for American citizens. They state that they pay taxes for the student's mathematics education in high school, and then they are taxed again to have the same student take essentially the same mathematics courses in college (Saxon 9001). Because of that, in some states public colleges and universities expect the students to pay special tuition for the developmental programs.

## **Summary and Conclusions**

In the United States, philosophical position for education and practical reasons have led universities and colleges to practice open admission. As a result, a large number of students cannot successfully study college level courses, especially in mathematics and science, because

they lack adequate background for them. Additionally, some students are not ready socially and psychologically to adjust to the demanding academic environment and the social department prevalent on collegiate campuses. These two situations contribute to a large number of students dropping out of college or failing in their studies. And to correct or minimize this, developmental education became a part of the curriculum of the American collegiate campuses.

Since mathematics is one of the most critical disciplines for the country, and also one apparently largely contributing to students' academic difficulties, mathematics became part of the developmental education programs of colleges and universities. Developmental mathematics involves going back to mathematics (albeit with higher level of difficulty) that should have been mastered during high school.

The purpose of developmental mathematics, therefore, is accustoming students to mathematics work ethic and allowing them success in the study of at least elementary mathematics required by general education. Because of that, developmental mathematics programs are multifaceted and, besides mathematicians, faculty from other disciplines as well as behaviorists are involved.

That fact that developmental mathematics has firmly established its place among college mathematics curricula was accomplished because of critical national need for mathematics education rather than the support it received from the campuses at large.

There is plethora of research for developmental education. Specific research for developmental mathematics is wanting, however. There is need for research that is well designed, for both longitudinal and short-term studies. The value of developmental mathematics as an instrument of introducing laggard students into mathematical thought must be established, and the most effective approach to its delivery should be identified.

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