HOW TO PREPARE PROSPECTIVE TEACHERS TO TEACH MATHEMATICS – SOME REMARKS

Ewa SWOBODA
Rzeszow University, Poland,
eswoboda@univ.rzeszow.pl

Jerzy TOCKI
Rzeszow University, Poland,
jjtocki@univ.rzeszow.pl

ABSTRACT

The reform of the Polish education system initiated in 1998 is a new challenge for teachers of mathematics, especially at the early teaching level and the primary school level. Since that time mathematical content has been bound up with other items of education. It was necessary to prepare a different way of teacher training – so as to prepare teachers to go through the new content of the subject "math". There was a chance to:

- Create a new approach to teaching early geometry.
- Create new connections between arithmetic and geometry, keeping the essence of arithmetical and geometrical cognition.

Keywords: early geometry, teaching, proportions

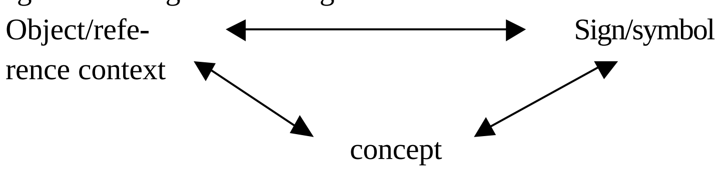
1. Introduction

The reform of the Polish education system iniciated in 1998 created a new challenge for teachers of mathematics, especially at the early teaching level and primary school level. According to the reform concept, mathematics at the early teaching level is a part of the integrated educational block. It causes a real danger of losing mathematical content in the amount of information and topics. The teachers of primary level are not good enough at mathematics – they are not sure whether they have sufficient knowledge of mathematics, they are afraid to look for their own didactical proposals.

It was necessary to prepare a different way of teacher training – so as to prepare teachers to go through the new content of the subject "math". This has created a chance to extend an offer for students – prospective teachers - in the framework of their professional preparation. Among others, there is a chance to:

- Create a new approach to teaching early geometry.
- Use a new tendency (based on recent didactical research) in teaching arithmetic.
- Create new connections between arithmetic and geometry, keeping the essence of arithmetical and geometrical cognition.

In the primary education system we try to adhere to Dienes' "deep end" idea, adjusting to the contemporary trends. It means that we think not only of issues emended in the current teaching programme. We also try to organise some of mathematical activities to create a wide intuitional basis for concept, which formally will appear in the further levels of education (Fischbein, 1987). Intuitions are built gradually. We give every student a chance to make their own individual investigations through participation in some real situations. Wide and various context, real materials and tools help to create a rich reference context. This is accompanied by a special mathematical or informal language. This model of work – in our opinion – corresponds not only to Freudenthal's idea (1973) that "mathematics is a human activity", but also to the epistemological triangle by Steinbring's (1997) model establishing the meaning of knowledge.



2. Theoretical background for didactical proposals

One of the main concepts, which we worked-out theoretically and tried to realise practically are *proportions*.

For a long period of time proportions have been the centre of interest among didacticians of mathematics (Researchers from Freudenthal Institute in Utrecht: van den Heuvel – Panhuizen M. 1990, 1991, Treffers 1991, Streefland, 1985). At international conferences the new aspects of understanding of this concept are still being referred (ICME 9 – Nunes. T, 2000, PME 25 – de Boeck et el, 2001, van den Valk T. at el, 2001).

Proportion can have a geometrical as well as arithmetical aspect. In our work we tried to realise the idea that the early beginning of geometrical and arithmetical learning should not be connected with each other – the way of learning and teaching of each type of concept is different and very specific, so making links between those two domains too early can destroy the notion itself (Tall, 1995). On the other hand – there is a huge need to enrich the geometric substance for pupils at early educational level. Pupils at this level shouldn't limit their geometrical knowledge only to geometrical figures such as square, triangle, or circle. Geometrical world, which has emerged from children's surroundings, is a lot richer.

In the book "International Handbook of Mathematics Education"(1996) three perspectives of teaching geometry are discussed:

1. Interacting with real shapes and space,
2. Shape and space as the fundamental ingredients for constructing a theory,
3. Shapes or visual representations as a means for better understanding of concepts, process and phenomena in different areas of mathematics and science (pp.161)

In this perspective shapes appear in a dynamic aspect. Students from the very beginning should not only distinguish shapes but they should also be able to perceive the position of one shape in relation to the other, and be sensitive to relations between shapes. One possible relation between geometrical objects is the relation of similarity, and similar figures can be used as the visual representation for better understanding of proportions.

The decision about the creation of the proportion supported by similar figures gave rise to the necessity of detailed phenomenological analyse of this concept.

Similarity as a transformation can be defined in two ways:

1. as a composition of isometry and homothety – this is geometrical description;
2. as a transformation changing all distances in the same way – this is an arithmetical description, based on proportions.

But the way of creating the concept of similarity cannot be directly guided by a final mathematical product.

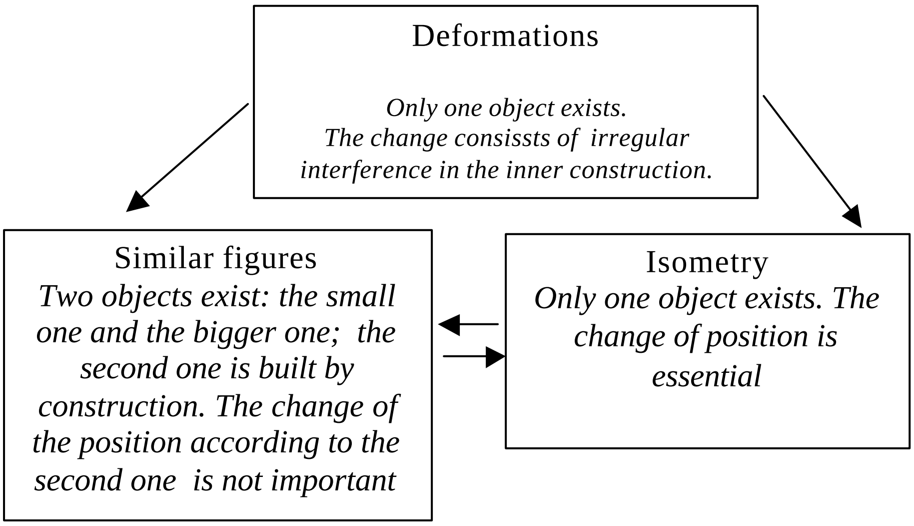
From a didactical point of view it is important that similarity is the equivalence relation, and its' abstraction class is "shape". Shape is seen visually and in such a way is closer to a geometrical way of creating the similarity. But because of actual and future student's matehematical knowledge it is important to use "shape intuition" to create a proportional description.

For several years the understanding of proportion in similar figures is the subject of our own empirical research. Results from this research brought a lot of important information about the process of forming geometrical concepts in childrens minds, and about differences between sources of geometrical and arithmetical acquaintance.

- One of the results from the research is an empirically confirmed fact, that children recognize similar figures visually. This statement concerns not only children from early educational level –in the same way react older students, who already know the formal definition of similar figures.
- The next result is that students' activities related to isometries are different than activities related to similarity. In both situations an utterance "the same shape" appears in a different meaning related to the performed action:
    - The basis of the activities related **to isometries** is a physical movement of the whole figure. Action on the object consist of moving, turning, reflecting. The object does not change itself, it changes only its position. So – the object all the time has "the same shape", because it is not changed as a whole at all.
    - The base for activities related **to similarity** is the existence of two separate objects. These objects have "the same shape" – only one of the figures is smaller and the other – bigger.
- Also onother type of transformation exists: deformations. Deformations are not a topic in the curriculum at all, but they exist in children's minds as a spontaneous concept (in Wygotski' sense, 1987). By deformations the object is changed by interference in its inner construction.

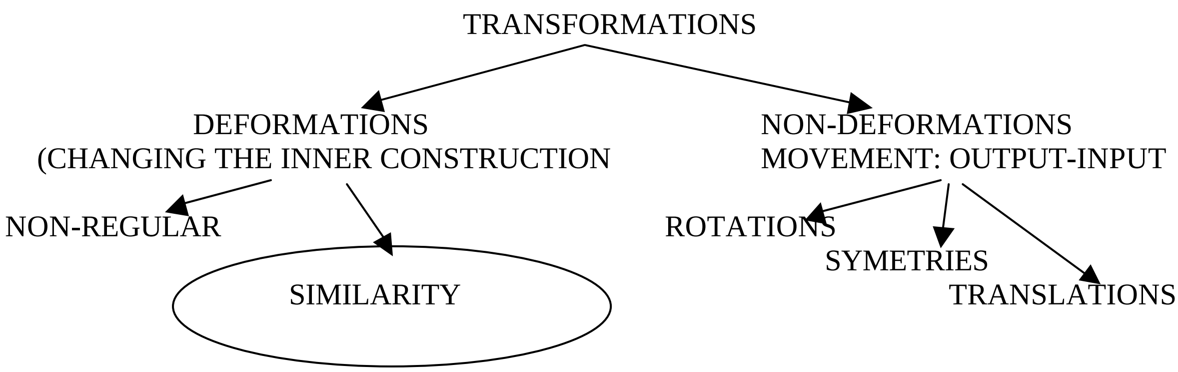
The figure is changed; it looks different than at the beginning. The change is irregular. Otherwise – the inner proportions are changed.

The diagram below shows the differences between each of these transformations:

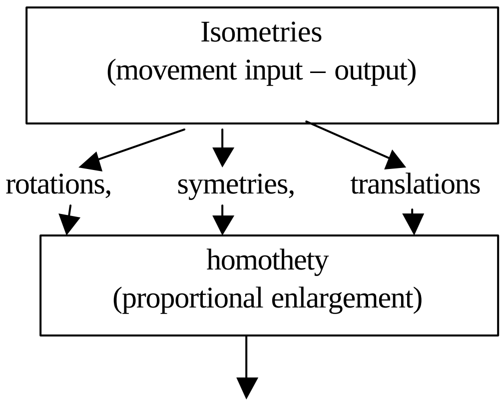


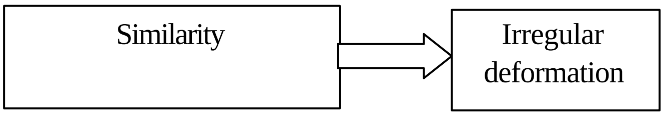
Although each of these transformations are totally different, given the specific understanding of the utterance “the same shape” it seems that similarity has more in common with deformations than with isometries. Comparing similarity with deformations gives a chance to differentiate “the same shape – the changed shape”. In the area of isometries we can only differentiate “the same shape – the other shape”. Comparing similarity and isometries can lead to misunderstanding of the statement “figure persists the shape”.

Due to such conclusions the hypothetical students’ way to similarity can be shown as follows:



It is more convinient for a student to notice the preservations of the shape of the figures by the regular deformations (that means by using proportions) than doing this by composition of isometries and homothety





Teaching similarity through the composition of transformations is directed at the structure of mathematics as the science, but it is not guided by the psychological aspect of learning mathematics. Integration of the final results achieved in these two ways (due to regular deformations or due to composition of transformation) is the teachers' assignment in older classes. This task will be easier when both these ways become clearly mathematicaly established on appropriative level. In cognitive psychology, it is said, that we all have an assortment of mental models connected with the mathematical concept. Conceptual structures are the major factor of progress in understanding mathematics, mainly by testing the reality. Though mathematically and psychologically different, both ways of building up the meaning of the similarity enable the complex understanding of this concept. It is so, because in both these ways the concept is created in the following aspects:

- connected with real situations;
- dynamic, as a construction process of a final result (the figure is similar to the second one $\Leftrightarrow$ the figure with the same shape);
- giving a chance to descript the basic, structured relation (inner proportion of the length of segments $\Leftrightarrow$ external scale of similarity).

We try to implement the conclusions of these analyses in didactical proposals directed to teachers and students. Here are the basic assumptions of a didactical line concering teaching proportions from a geometrical aspect:

1. The starting point is the intuition about preserving the shape of a figure.
2. First assesment of changing the shape or preserving the shape are done visually, without any metrical aspect.
3. The activities concerned with similar figures are based on the construction of the figure, which is similar to the second one. Different tools are used and a different reference context is given.
4. Description of the mathematical properties is focused on preserving the inner proportions, mainly as the relation between the segments, which have the same length.
5. Numeral relations between lengths of the related segments are based on the intuitional understanding "the rule of three".

As can be seen, the basic assumptions of these proposals are the visual essesment of a shape. This is the core of student's individual work. The context is the construction of similar figures. The basic mathematical relations, which are discovered and described by a student in his own language, are the inner proportions of the figure (see Duval 1998, p.38). This proposal is new for teachers and students in our country.

Acceptance of these assumptions has clear didactical consequences. It seems that paying attention to the inner proportions in similar figures can help to understand the following concepts:

- **Fractions**. One of the aspects of understanding fractions is seeing it as a ratio between two quantities. In this aspect not only a magnitude of the numerator and the denominator determines the value of the fraction, but mainly their ratio.
- **Irrational numbers**. Number $\sqrt{2}$ or π are defined as the ratio of the length of eligible elements of some geometrical figures. $\sqrt{2}$ is as the ratio the length of the diagonal of a square to the length of its side, π is the ratio of the circle to the diameter. All squares are similar, and all circles are similar. A student who is familiar with the properties of similar figures is ready to

accept the fact that mentioned ratios are independent from the size of figures, and that such ratio have only one value.

- **Trigonometric functions.** At school the sine, cosine, tangent and cotangent is defined as some ratios in right-angle triangles. Also in this case, the knowledge about the inner proportions in similar triangles is very helpful.

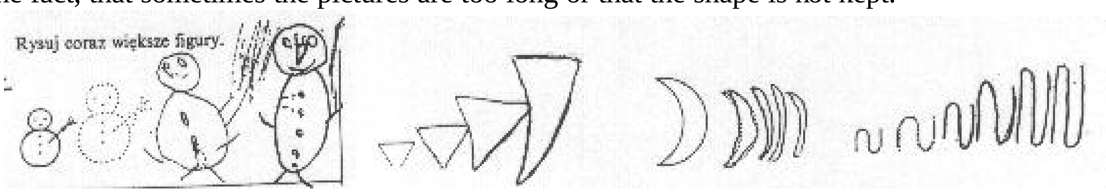
3. Didactical propositions

The following examples affirm the realisation of our proposition. Tasks were prepared for students from the lower educational level (6 – 9 years). In our opinion, learning about proportions at this level can cause more controversies. We do not show all the possible forms and methods of work – there are only examples reinforcing of our proposition.

3.1. Intuition of preserving the shape of the figure

3.1.1 Series of tasks with commission: draw figures bigger and bigger, draw figures smaller and smaller.

These series usually are placed at the beginning of the booklet for children from first grade, in the chapter "size relations" (Semadeni, 1992). But in analysing student's work it is good to pay attention to the fact, that sometimes the pictures are too long or that the shape is not kept.



3.2. Preparation to the understanding of an utterance „the same shape".

3.2.1 Deformations

The meaning of this preparation is the situation of the contrasting the change of the shape. During various activities with physical objects, children observe the change of the shape. For example:

- reflection of own face in water, in disturbed water;
- reflection of own face in the glass Christmas-tree bauble;
- looking at different things through water in a jar, through water in bottles of different shapes;
- stretching rubber with a picture;
- blowing a balloon with the picture.

During these exercises children describe in words the observed changes. The teacher encourages individual statements, which express the point of changes in the best way, eliciting things like: is very tall and thin, is too fat, the nose is too big in relation to the whole face.

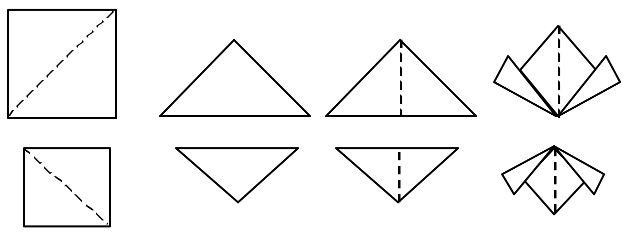
Continuation of these exercises may be with plastic works: 'my carituture", "bad witch", the world reflected in a puddle"... Drawing is here a form of transformating the information achieved during the observation of the deformed shapes. The next step in the coding, as in the mathematisation, should be the children's conversation about their own work.

3.2.2 Proportional enlargement

A: We draw advertisements. There is a huge number of situations, in which we can prepare a big poster giving information about a (for example) celebrity. It is a perfect occasion to make a spontaneous enlargement with an intuitional idea of preserving the shape

(For example fluorisation action – we draw a big toothbrush ....)

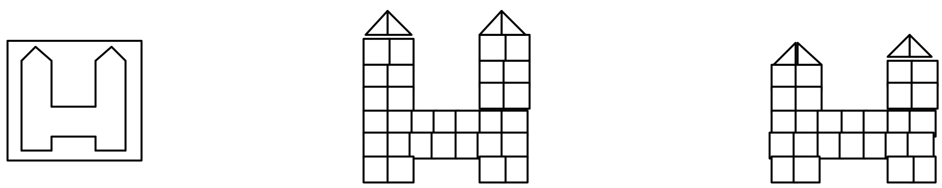
B: Origami – activities concerned with creating the same shape in two different sizes (a big flower – crocus, a small one - snowdrop. During this exercise the teacher guides the children's work to underline the fact, that the construction (the following foldings in origami) is the same for the big and small figure (the same angles are created, the lines are shared in the same proportions). After finishing work, the children code the information about the figure construction, making "a letter" for friends from other classes. In this letter they code the algorithm of making the origami (Wollring 2000, Karwowska at al. 2001).



3.3 Description of the metrical properties of similar figures

It is not easy to cross the path between a visual perception and a mathematical description of the numerous relations between lengths of the segments. Visual perception is spontaneous, natural. The mathematisation process needs a concious act of abstraction, and the ability of paying attention to the isolated parts of the figures (instead of a "Gestalt" perception). We can ease children work by preparing the tools of work, which force the preservation the proportions. That's why our next proposals lead our students to preserve the inner figure proportions.

A: *Jigsaws*. Children get squares 3cm x 3cm and a small picture. They work according to the tasks: make the same shape as in the picture.

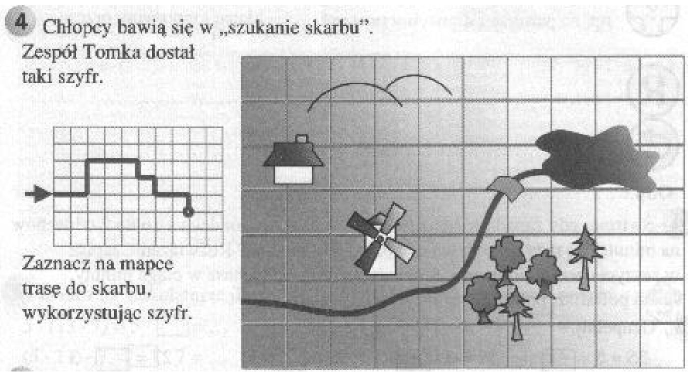


(correct example) (incorrect example)

Preserving the shape depends on preserving the inner proportion - changing the shape is done by changing the inner proportions. The teacher encourages students to talk about those facts (in informal language), during the work as well after. Child may notice that the second example is incorrect because the towers are not high enough. The towers should be as high as the width between these two towers. The tool – square – is used as the unit and forces the enlargement of the figure. Children work spontaneously, changing one small (non-existing) unit from a picture on to a bigger one.

B: *Tasks on the grid*

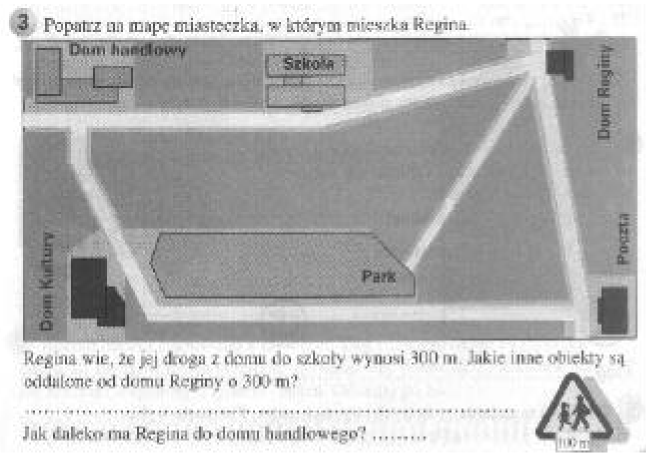
(*Task: Boys play the game "searching for treasure". Tomek's team has the key-code. Find the way to the treasure on the map.)*



In this task (Wilk – Siwek, Swoboda, 1996) units are given. A child can see that there are small units and big units. To solve the task, children have to count the units very carefully, because they enlarge a special type of figure: the open broken line.

3.4. Tasks paying attention to the relations between lengths of the related segments based on the intuitional understanding “the rule of three”

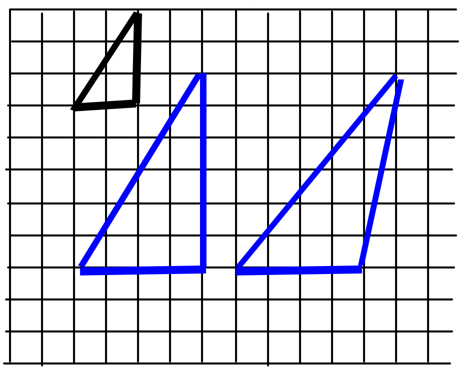
(*Task: Look at the map ofRegina’s village. Regina knows, that the distance between her house and the school is 300 m. Are there any more buildings laying in the same distance from Regina’s house? How far is the shopping centre?).*



Building the similar figure segments having the same length changes into segments having the same length; if any segment is two times longer as the other one then the dependence is kept on the enlarged figure. The distance on the picture between Regina’s house and the school is 6 cm, in reality it is 300m. Each segment having a 6 cm length determinates 300 m. in reality. This is why the school, the park, and the post office are 300 m from Regina’s house. The shopping centre is 600 m. further, because it is two times further than to the school.

3.5. First mathematisation in direction of external proportions (scale)

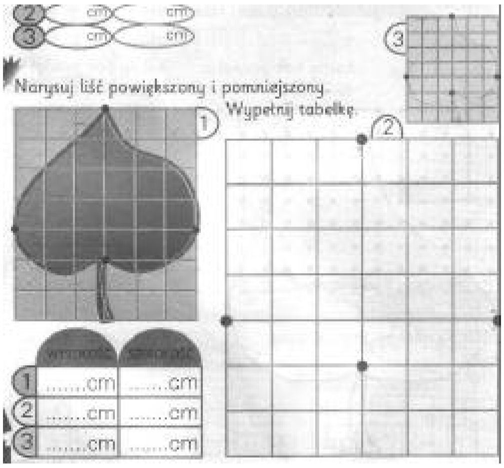
3.5.1. Tasks on a geobord



(Task: Build the triangle two times bigger)

If a child works on the one geoboard, he must count units very carefully and multiply lengths by the enlarger’s factor. In addition to that, this task gives the opportunity to discuss the connections between the inner and external proportions: not every triangle which has the base and the height which is two times longer than the original is similar to the given triangle, because not all of the inner proportions are preserved and not all of the angles are the same in both triangles.

3.5.2. Tasks for "re-counting"



(Task: Draw the enlarged and the reduced leaf. Fill the table)

This is a "scale" drawing . The realization of the work is a little different than traditionally is. The child does not need to count the "new" length of the segments, because the grid is already changed. He/she can concentrate on the shape of the enlarged figure. After drawing (and after visual estimation of the received shape), the pupil measures some segments and fills the table. It is worth discuss the results. Reduction is two times in the size – what does it mean? Which segments are two times shorter? Are there any other shorter segments? Enlargement is "strange". Shape is correct, but the numbers do not fit, the figure is not two times, not even three times enla rged ...Children can measure squares from grids and compare the results.

4. Final remarks

Proposition, presented in this paper, is a part of the project of preparation of the prospective teachers for primary educational level. We tried to connect two streams: mathematical preparation (as the answer for the question: what kind of mathematics should university students learn), and didactical preparation.

Working with students we try to show philosophy of mathematics, differnt from that they usually know. Matematics is not a set of facts for learning. It is a knowledge build individually by pupils, and the teacher's task is to create activities, which are the base for mathematical ideas. Maths on this level at school is one of the elements of integrated teaching to young learners. The process of teaching is based on "thematic areas" which allow children to study the reality in a complex way. Children store their mathematical knowledge in various cognitive situations (observation of real world, creating imaginative worlds). In this way the children create a network of associations, which let them creatively use their mathematical experiences for solving mathematical problems. The teacher's task is to organize the process in such a way that the children who study the world and regularities which exist in the world, can describe them in mathematical language Teacher should help a child to make links between the problem, the procedure of solving it and the solution.

For this reason the teacher have to be sensitive for mathematics emerged from the real world. His/her mathematical preparation has to go over of the narrow frames of traditional topics prepared for primary education. He/she has to see mathematics in a very large perspective.

The teacher's practical preparation depends on drafted theoretical establishments of the whole proposition. Tasks, commonly found in existing books help in its realisation, by projecting activities for children at school.

REFERENCES:

1. De Bock D., van Dooren W., Verschaffel L., Janssens D., 2001 Secondary school pupils' improper proportional reasoning: an in-depth study of the nature and persistence of pupils' errors, *Proceedings of the 25th Conference of PME, Utrecht, the Netherlands,* pp.2-313 – 320.
2. Duval R., 1998. Geometry from a cognitive point of view, *Perspectives on the Teaching of Geometry for the 21st Century,* An ICMI Study, ed. Carmelo Mammana and Vinicio Villani, Kluwer Academic Publishers

3. Fishbein E., 1987, Intuition in Science and Mathematics. An Educational Approach. Dortrecht: D.Reidel Publishing Company.
4. Freudenthal H, 1973 *Mathematics as an Educational Task,* D. Reidel Publ. Co., Dordrecht.
5. Hershkovitz R., Parzysz B., Van Doormolen J. (1996) :Space and Shape", in: ed. Bishop A.J.et al *International Handbook of Mathematics Education,* Part 1, Kluver Academic Publishers.
6. Karwowska-Paszkiewicz K., £yko A., Mamczur R., Swoboda E., 2001, Activities about similar figures in primary education *Proceedings of the International Symposium Elementary Maths Teaching,* Prague, the Czech Republic, pp.85-90.
7. Nunes T., 2000, How mathematics teaching develops pupils' reasoning system, *Abstracts of Plenary Lectures and Regular Lectures, ICME 9,* Tokyo/Makuhari, Japan.
8. Semadeni Z, 1992, *Matematyka 1, Zeszyt æwiczeñ dla klasy pierwszej szko³y podstawowej.* WSiP Warszawa.
9. Skemp, R 1987, *Inteligence, Learning, Action,* J.Wiley&sons.
10. Steinbring H, 1997, Epistemological Investigation of Classroom Interaction in Elementary Matheamatics Teaching, *Educational Studies in Mathematics 32 (1997)* p.49-92.
11. Streefland, L.1985, Search for the roots of ratio: some thought on the long term learning process (towards ....a theory) *Educational Studies in Mathematics 16.* pp.75-94.
12. Swoboda E., 1994, *Development of concept of similar figurs* (non published doctoral thesis), Kraków
13. Swoboda, E. 2000 On the development of the mathematical concept – case study *„Dydaktyka Matematyki", Annales Societatis Mathematicae Polonae, Seria 5,* (No. 22), pp.109 – 145.
14. Tall D. 1995, Cognitive Growth in Elementary and Advanced Mathematical Thinking, *Conference of the International Group for the Psychology of learning Mathematics,* Recife, Brasil, vol.1, pp.161 – 175.
15. Tocki J., Turnau S., 1998, Geometry in the Polish school: present state and perspectives, *Perspectives on the Reaching of Geometry for the 21st Century,* An ICMI Study, ed. Carmelo Mammana and Vinicio Villani, Kluwer Academic Publishers.
16. Tocki J., 2000, *Struktura procesu kszta³cenia matematycznego,* Wydawnictwo Wy¿szej Szko³y Pedagogicznej, Rzeszów.
17. Treffers A., 1991 Didactical background of a mathematics programme for primary education, *Realistics Mathematics Education in primary school*, ed. L. Streefland, Freudenthal Institute, Utrecht University, The Netherlands.
18. van den Heuvel-Panhuizen M., 1990, realistic Mathematics/Mathematics Instruction and Tests, *Context Free Productions, Tests and geometry in Realistic Mathematics Education,* pp.53 – 78, OW&OC, Utrecht, The Netherland.
19. van den Heuvel-Panhuizen M., 1991, Ratio in special education, *Realistics Mathematics Education in primary school*, ed. L. Streefland, Freudenthal Institute, Utrecht University, The Netherlands.
20. van den Valk T., Broekman, H, 2001, Science teachers' learning about ratio tables, *Conference of the International Group for the Psychology of learning Mathematics,* PME 25 Utrecht, the Netherlands, pp.4-351-358.
21. Wilk – Siwek H., Swoboda E, 1996, *Podrêcznik matematyki dla klasy III serii B³êkitna Matematyka,* Wydawnictwo Kleks Bielsko – Bia³a.
22. Wollring B. 2000, *Faltbilderbucher, Faltgeschichten und Faltbildkalender – Arbeilsumgebungen zur ebenen Papierfaltgeometrie fur die Grundschule, „Grundschulzeitschrift" 14 (2000) Heft 138.*
23. Wygotski L.S., 1989, Myœlenie i mowa, PWN Warszawa.