In this paper we propose a program of pre–service and in–service teacher of Mathematics’ training in teaching with the use of computer software programs. The program consists of a) the presentation of the most characteristic theories of learning, teaching methods and models and models of using computers in the teaching–learning environment and b) the training of teachers in the use of computer software programs that are being chosen as appropriate to offer more than the traditional instruction.

Moreover we present the results of the application of the program in the Mathematics’ Department of the University of Athens during the academic years 1999–2000 and 2000–2001. The computer software program used in the application is Mathematica® in point of the possibilities it offers in the negotiation of mathematical subjects in Secondary Mathematics’ Education. This paper studies the evaluation of the use of the program by the pre–service and in–service teachers that participated in the research in point of the aims, the operation and the use of the program in the teaching–learning process of Secondary Mathematics’ Education. Moreover it studies the differences between pre–service and in–service teachers’ opinions and the relation between their opinions and their interest and experience in the use of computers.

**Keywords**
Adult learning; Computers; Constructivism; Discovery Learning; Improving classroom teaching; Media in Education; Mathematics; Postgraduate Education; Software; Teaching/learning strategies; Training; Undergraduate Education
1. Introduction

Contemporary “society of information” offers the possibility for self-education and self-training, but also requires the competence of using and making worthy of the possibilities offered by the new technologies by everyone. The generalized use of new technologies and especially computers in almost everyone’s working and personal life, could not leave untouched the sensitive field of Education which is the “mirror” of society. In that context the role of the teacher is significantly modified, since the education of students which affects and provokes, is necessary to include all these elements that will make them competent to participate, act and operate in the society of today and tomorrow.

The introduction and appropriate use of computer software programs in the teaching – learning process of Mathematics is proposed by contemporary research as able to cure some of the weaknesses of traditional teaching [7], [8], [9]. In particular the use of computers seems to provoke the students’ interest and attain their attention. Moreover it seems to help students develop an inquiring attitude towards mathematical concepts and ideas through the experimentation with the program, the formation and checking of their conjectures and hypotheses. Lastly it seems to help the transfer of their knowledge (concepts and ideas) in other domains, transforming it to functional knowledge.

In this paper we propose a program of pre–service and in–service teacher of Mathematics’ preparation in teaching with the use of computer software programs. Moreover we present the results of the application of the program in the Mathematics Department of the University of Athens during the academic years 1999-2000 and 2000-2001. The computer software used in the application is Mathematica® [3], [12], [13], in point of the possibilities it offers in the negotiation of mathematical subjects in Secondary Education. This paper studies the evaluation of the use of the program by the pre-service and in-service teachers that participated in the research. Also the differences between pre-service and in-service teachers’ opinions and the relation between their opinions and their interest and experience in the use of computers.

2. A Theoretical Framework

According to Discovery Learning, proposed by J. Bruner, the basic role of the teacher is to help and encourage his students to discover the mathematical concepts and ideas; moreover to help them develop a general attitude of exploration and experimentation towards mathematical concepts and ideas [1], [2]. According to Constructivism, based on J. Piaget’s ideas and developed by various theorists and researchers of Education, the teacher through the preparation of appropriate activities and problematic situations, should provide his students with an environment where they can construct knowledge actively, using their preexistent knowledge [10], [11].

The possibilities that the contemporary computer software programs have to offer make computers the ideal tools that Discovery Learning and Constructivism are describing in theory. Computer Aided Learning (CAL) includes all the activities via which computers contribute in the process of learning. In the emancipatory paradigm of CAL computers are used as accommodating tools that are partially engaged in the process of learning [5]. In that context, Mathematica® and other programs with similar possibilities, that does not presuppose efficient programming skills, yet have function–based structure, can be used effectively in the visualisation of concepts, in the quick and precise plotting of graphs and in difficult and complex calculations [9], [3], [10].
The teacher must adjust to the formatted new settings and learn how to make worthy of the new technological means and especially computers in his lesson. The conditions in order for him to live up to this new role [5] are:

i. **To acquire a positive attitude towards the value of the new technologies**

Teacher training as much as any vocational training has one significant difference from any other form of education: In vocational training the trainees are directly interested in the subject of learning, having as a basic objective to making worthy of the result in the vocational level. In that context the teachers must be motivated and convinced for the necessity and the value of the use of computers in the teaching–learning process.

ii. **To learn how to organize his teaching effectively**

The teacher must be aware of the theories of learning, teaching methods and models and models of using the technological means in the teaching-learning process, in order to be able to select the appropriate technological means and to introduce them in his teaching appropriately.

iii. **To have as a priority his pedagogical role**

The teacher, being free from everyday time wasting, tiresome tasks, is able to dedicate more time to the special difficulties of his students and help them overcome these difficulties. Also he is able to spend more time with each student, to adjust his answers to the students’ individual skills, to evaluate, help, encourage and guide them appropriately.

iv. **To be educated in the effective use of the means of technology and especially computers and to be trained constantly**

The teacher must be aware of what is considered as the most appropriate way of using the means of technology in order to make worthy of the possibilities they have to offer so as the learning and educational results to be maximized.

### 3. The Methodology of Research

The research was designed for pre-service and in-service teachers of Mathematics. The research was realized in the Mathematics’ Department of the University of Athens during the academic years 1999–2000 and 2000–2001. Two groups participated, a group of 75 undergraduate students of the Mathematics Department (pre–service teachers) and a group of 29 postgraduate students specialized in Mathematics Education (in–service teachers), a total of 104 teachers. A program of teacher of Mathematics’ training in teaching with the use of computer software programs was designed and realized in the classrooms and computer laboratories of Mathematics’ Department.

A questionnaire was designed and developed in order to evaluate the use of the computer software Mathematica® by the teachers that participated concerning the aims, the operation and the use of the program in the teaching-learning process of Secondary Mathematics Education. The questionnaire was given to the teachers after the completion of the program of training.

The data that was gathered by the encoding of the questionnaires was analyzed with the statistical programs SPSS®. The percentages are counted to the whole of the students that answered each question, given that it is not significantly lower than the whole of students that participated. The three methods used in the statistical analysis are tests of hypotheses. In particular the methods used are $X^2$ – Testing for homogeneity, $X^2$ – Testing for independency and Mann-Whitney (U) [8].

The results that arise by the statistical analysis form only conjectures about the tendencies of the students and the relations between their characteristics and not safe conclusions, since the
teachers that participated in the research were not selected via one of the Sample Survey methods of Statistics.

4. The program of training

The program of training consists of two parts. The first part is theoretical and includes the presentation of:
B. The classification of teaching methods and models to teacher–centered, student–centered and interactive [4]. A selection of methods and models that support the principles of the theories mentioned above presented adjusted so as to make efficient use of computers.
C. The change in the role of the teacher and the interactions that are taking place between the teacher, the student and the computer in the contemporary educational environment [5].
E. Propositions about the evaluation of educational software programs [6].

The second part concerns the training of teachers in the use of computer software programs in the Emancipatory model of CAL; in particular Mathematica® or Maple® [9], [3], [10]. It includes the presentation of problematic situations and activities in the teaching of:
A. The geometric quantities of functions and the change of the graph of a function according to the change in its parameters, using multiple traces on a graph and animation.
C. The definition of plane curves (cycle, parabola, ellipse, hyperbola), as cone intersections, as locus and the geometric quantities of curves.

5. The teachers that participated in the research

The in–service teachers that participated in the research (Group A) were 29, with ages from 23 to 49, with mean age 39,14 years and std. deviation 9,20 years. As for the gender 41,4 % were males and 58,6 % were females. The pre–service teachers (Group B) were 75, with ages from 20 to 25, with mean age 22,58 years and std. deviation 1,09 years. As for the gender 74,6 % were males and 25,4 % were females.

The pre–service teachers show a greater tendency to use computers at the University (76 %) than the in–service teachers (44,8 %) ($X^2$–Homogeneity, $X^2 = 9,235$, $P$–value = 0,002, Df = 1). The teachers posses and use computers at home, as 93,1 % of the in–service and 77,3 % of the pre–service teachers stated, but we cannot come to a safe conclusion about their homogeneity ($X^2$–Homogeneity, $X^2 = 3,483$, $P$–value = 0,062, Df = 1) (see Table 1).

Both in–service and pre–service teachers have experience in the use of computers ($X^2$–Homogeneity, $X^2 = 3,841$, $P$–value = 0,147, Df = 2); in particular 77,8 % and 83,1 % respectively
use computers for more than 1 year and 55,6 % and 39,4 % respectively for more than 3 years (see figure 1).

The in–service and pre–service teachers are interested in the use of computers ($X^2$–Homogeneity, $X^2 = 1,654$, P–value = 0,198, Df = 1), as the whole of in–service teachers and 94,5 % of the pre–service teachers stated.

A series of extra questions were posed to the in–service teachers about their previous experience in Education. The statistical analysis showed that 93,1 % of them is working or has worked in Education, mainly in Secondary Education. The teachers have worked mainly in High Schools (75,9 %), Junior High Schools (69 %), Tutorial Schools (48,3 %) and private lessons (86,2 %). The teachers are experienced in educational work, as 93,1 % of them has been working as teachers more than 3 years and 62,1 % more than 10 years.

72,4 % of the in–service teachers state that the teaching approaches they use in their lesson are partly in accordance with what they consider as appropriate; only 20,7 % stated full accordance. That is supported by the statistical analysis that showed the independency of the variables of the teaching approaches they use and the approaches they consider as more appropriate ($X^2$–Independency, $X^2 = 6,694$, P–value = 0,153, Df = 4).

The in–service teachers do not use computers in their lesson as 93,1 % state. On the contrary the whole of them believe that computers would give an aid to the lesson, with 51,7 % of them in a great extent. Moreover 86,2 % believe that there is a need for the introduction of computers in their lesson.

6. Evaluation of the use of Mathematica® in the teaching–learning process

6.1 Aims of the program

The program is regarded as suitable to be used primarily in High School (90,4 %) and in Higher Education (84,6 %); secondarily in Junior High School (61,5 %) and in Training (55,8 %).

The program can offer more than the traditional instruction mainly in subjects of Geometry (90,4 %) and Analysis (90,4 %), secondarily Algebra (39,4 %).

6.2 Evaluation of the operation of the program

The program is considered to start easily by 86,2 % of in–service and 97,3 % of pre–service teachers; we cannot come to a safe conclusion about their homogeneity though ($X^2$–Homogeneity, $X^2 = 4,580$, P–value = 0,032, Df = 1).

The program is considered to be easy to use by both in–service (76 %) and pre–service teachers (69,9 %) ($X^2$–Homogeneity, $X^2 = 0,344$, P–value = 0,558, Df = 1) (see figure 2). The opinion of the teachers about whether the program is easy to use is independent to their interest ($X^2$–Independency, $X^2 = 1,719$, P–value = 0,190, Df = 1) and experience in computers’ use ($X^2$–Independency, $X^2 = 3,259$, P–value = 0,196, Df = 2).

The prerequisite skills for the use of the program by the teacher are mainly knowledge of the program’s commands (80,4 %) and experience in the use of computers (66,7 %); programming skills are regarded as accommodating skills (24,5 %).

The prerequisite skills for the use of the program by the students are also knowledge of the program’s commands (69,4 %) and experience in the use of computers (67,3 %). 12,2 % of the teachers state that there are not prerequisite knowledge and skills for the students.
The clarifications and explanations that are given by the program when there are mistakes in the input of commands or programs, are characterized by both in-service and pre-service teachers mainly as good (34.3 %) or adequate (34.3 %) (Mann–Whitney, U = 1032, P–value = 0.836). The help browser of the program is also regarded by both in-service and pre-service teachers mainly as good (43.6 %) or adequate (34 %) (Mann–Whitney, U = 836, P–value = 0.541).

The teachers disagree on how the teacher can learn how to operate the program, (X²–Homogeneity, X² = 14,026, P–value = 0.003, Df = 3). In–service teachers believe that mainly the specialist’s help is required (48.3 %), while pre–service teachers believe that either direct operation in association with the help browser (40.8 %) or the use of a manual (50.7 %) would do. The teachers seem to agree that the teacher’s help is required when the students is learning how to operate the program (in–service: 75 %, pre–service: 57.7 %); we cannot however come to a safe conclusion (X²–Homogeneity, X² = 6,506, P–value = 0.089, Df = 3).

6.3 Evaluation of the use of the program in the teaching-learning process

The whole of teachers believe that the use of the program in the lesson would provoke the students’ interest for the lesson; indeed 72.4 % and 65.3 % respectively in a great extent (see Table 2). In–service and pre–service teachers are homogenous (X²–Homogeneity, X² = 0.476, P–value = 0.490, Df = 1). Their opinion is independent to their interest (X²–Independency, X² = 2.082, P–value = 0.149, Df = 1) and their experience in the use of computers (X²–Independency, X² = 20.68, P–value = 0.356, Df = 2).

A noteworthy result is that the whole of in–service teachers and 89.3 % of pre–service students believe that the possibilities of the program would provoke the students’ interest for Mathematics as a science; 48.3 % and 46.3 % respectively in a great extent. In–service and pre–service teachers are homogenous (X²–Homogeneity, X² = 3.385, P–value = 0.184, Df = 2). Their opinion is dependent to their interest in the use of computers (X²–Independency, X² = 11.576, P–value = 0.003, Df = 2). On the contrary it is independent to their experience in computers’ use (X²–Independency, X² = 5.468, P–value = 0.243, Df = 4).

93.1 % of in–service teachers and 81.7 % of pre–service students believe that the use of the program would enable the students’ active participation in the lesson; 70.4 % and 55.2 % respectively in a great extent. In–service and pre–service teachers are homogenous (X²–Homogeneity, X² = 3.970, P–value = 0.137, Df = 2). The opinion of teachers is independent to their interest (X²–Independency, X² = 0.920, P–value = 0.631, Df = 2) and their experience in the use of computers (X²–Independency, X² = 6.249, P–value = 0.181, Df = 4).

93.1 % of in–service teachers and 87.7 % of pre–service students believe that the use of the program would enable the students’ self–action, exploration and experimentation; 63 % and 65.6 % respectively in a great extent. In–service and pre–service teachers are homogenous (X²–Homogeneity, X² = 0.697, P–value = 0.706, Df = 2). Their opinion is independent to their interest (X²–Independency, X² = 0.916, P–value = 0.633, Df = 2) and their experience in the use of computers (X²–Independency, X² = 0.715, P–value = 0.949, Df = 4).

The teachers have different opinions about the teaching approaches that support the most effective conditions for the use of the program (X²–Homogeneity, X² = 25.946, P–value < 0.001, Df = 3). Although both pre–service and in–service teachers support discovery learning approaches, in–service teachers present a greater percentage (100 % to 88 % respectively).

52.1 % of pre–service teachers believe that the students would be able to use the program as means of self–instruction, opposed to 13.8 % of in–service teachers (X²–Homogeneity, X² = 12.499, P–value < 0.001, Df = 1).
72.4% of in-service teachers believe that teaching with the use of the program saves time compared to the traditional teaching, opposed to 48% of pre-service teachers (see figure 3). We cannot, however, come to a safe conclusion about their homogeneity ($X^2$–Homogeneity, $X^2 = 5.033$, P-value = 0.025, Df = 1).

Teachers present differences in point of the optimum distribution of students per computer ($X^2$–Homogeneity, $X^2 = 22.729$, P-value < 0.001, Df = 5). Although they both propose 2 students (72.4% and 74.7%) and 1 student per computer (44.8% and 10.7%), pre–service teachers propose also 3 students (13.4%) and 1 computer operated by the teacher (4%).

The whole of in-service and 94.4% of pre-service students state that they would select to use Mathematica® in their lesson and / or propose it to others; 65.5% and 46.3% in specific subjects.

The selection of Mathematica® by in-service teachers is independent to their beliefs about computers giving an aid to the lesson ($X^2$–Independency, $X^2 = 0.419$, P-value = 0.518, Df = 1) and the need for the introduction of computers in the educational environment ($X^2$–Independency, $X^2 = 0.495$, P-value = 0.482, Df = 1).

7. Conclusions

The teachers that participated in our research were pre-service and in-service teachers of Mathematics. They presented great interest and experience in the use of Computers. The in-service teachers also presented experience in Education with many years of work and service in many degrees and forms of Education.

The teachers proposed the use of the program primarily in High School and in Higher Education, where the students’ abilities are harmonized with the function–based structure of the program. The program can offer more than traditional teaching mainly in Geometry and Analysis, where the graphic negotiation of subjects can essentially aid in the understanding of concepts and subjects studied in general.

The program starts easily and is easy to use as both in-service and pre-service teachers stated. The teacher and the students, in order to use the program, must be familiar with the program’s commands and have some experience in the use of computers; programming skills are not required, yet can accommodate the teacher.

The clarifications and explanations, given by the program when there are mistakes in the input of commands or programs and the help browser of the program, are sufficient.

The teacher can learn how to operate the program according to in-service teachers mainly with the help of a specialist, while according to pre-service teachers via direct operation in association with the help browser or the use of a manual. The students in order to learn how to operate the program need the teacher’s help.

The use of the program in the lesson can provoke the students’ interest for the lesson. Moreover the possibilities of the program can provoke the students’ interest for Mathematics as a science. The students that come in contact with a strict, static, inspired yet untouchable form of Mathematics usually lose their interest since they regard Mathematics as a structure that can be handled only by inspired minds.

The use of the program can also enable the students’ active participation in the lesson and the students’ self-action, exploration and experimentation, making the lesson of Mathematics an energetic, exploratory, collaborative, social process.

The teachers disagree about whether the students would be able to use the program as a means of self-instruction; pre-service teachers believe they would, in-service teachers they would not. If
the teacher is present in the laboratory when students are using the program by themselves, the students can use the program as means of self-instruction, yet ask for the teacher’s help when and if they need it.

The majority of in-service and approximately half of pre-service teachers believe that teaching with the use of the program saves time compared to traditional teaching. An important question that arises though is that even if it does not save time, should the teaching-learning benefits of the use of the program to be sacrificed in the altar of presenting one more theorem or solving one more exercise?

The students should work on computers in groups of two students or individually. If the number of computers in the laboratory is not sufficient, the groups of students could include but not exceed three students. If the laboratory cannot be used or there is a limitation of time, 1 computer operated by the teacher could be used.

The teachers are very positive towards the use of Mathematica® in their lesson. They state that they would select to use that program and / or propose it to others. Mathematica® is a powerful, promising tool, a tool in the service of educators who want to provide their students with an environment in which they are able to develop higher order and transferable skills, skills they will use in the society of today and tomorrow.

REFERENCES
Table 1: Evaluation of pre-existent skills of the students and views-attitudes relative to Computers and Education

<table>
<thead>
<tr>
<th>Question</th>
<th>In–service teachers</th>
<th>Pre–service teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Are you using computers at the University?</td>
<td>Yes 44,8 %</td>
<td>76 %</td>
</tr>
<tr>
<td></td>
<td>No 55,2 %</td>
<td>24 %</td>
</tr>
<tr>
<td>2. Do you have and use a computer at home?</td>
<td>Yes 93,1 %</td>
<td>77,3 %</td>
</tr>
<tr>
<td></td>
<td>No 6,9 %</td>
<td>22,7 %</td>
</tr>
<tr>
<td>3. Do you find the use of computers interesting?</td>
<td>Yes 100 %</td>
<td>94,5 %</td>
</tr>
<tr>
<td></td>
<td>No 0 %</td>
<td>5,5 %</td>
</tr>
<tr>
<td>4. Do you work or have you been working in Education?</td>
<td>Yes 93,1 %</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>No 6,9 %</td>
<td>–</td>
</tr>
<tr>
<td>5. Are the teaching approaches you use at school in accordance with the teaching approaches you consider as more appropriate?</td>
<td>Fully 20,7 %</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Partly 72,4 %</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>No 6,9 %</td>
<td>–</td>
</tr>
<tr>
<td>6. Do you use computers in your lesson?</td>
<td>Yes 6,9 %</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>No 93,1 %</td>
<td>–</td>
</tr>
<tr>
<td>7. Do you think there is a need for the introduction of computers in the teaching–learning environment?</td>
<td>Yes 86,2 %</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>No 13,8 %</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 2: Evaluation of the use of the program in the teaching-learning process

<table>
<thead>
<tr>
<th>Question</th>
<th>In–service teachers</th>
<th>Pre–service teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Do you think that the use of the program would provoke the students’ interest for the lesson?</td>
<td>Yes, in a great extent 72,4 %</td>
<td>65,3 %</td>
</tr>
<tr>
<td></td>
<td>Yes, in some extent 27,6 %</td>
<td>34,7 %</td>
</tr>
<tr>
<td></td>
<td>No 0 %</td>
<td>0 %</td>
</tr>
<tr>
<td>2. Do you think that the use of the program would provoke the students’ interest for Mathematics as a science?</td>
<td>Yes, in a great extent 48,3 %</td>
<td>41,3 %</td>
</tr>
<tr>
<td></td>
<td>Yes, in some extent 51,7 %</td>
<td>48 %</td>
</tr>
<tr>
<td></td>
<td>No 0 %</td>
<td>10,7 %</td>
</tr>
<tr>
<td>3. Do you think that the use of the program would allow the students’ active participation in the lesson?</td>
<td>Yes, in a great extent 65,5 %</td>
<td>45,1 %</td>
</tr>
<tr>
<td></td>
<td>Yes, in some extent 27,6 %</td>
<td>36,6 %</td>
</tr>
<tr>
<td></td>
<td>No 6,9 %</td>
<td>18,3 %</td>
</tr>
<tr>
<td>4. Do you think that the use of the program would allow the students’ self-action, exploration and experimentation?</td>
<td>Yes, in a great extent 58,6 %</td>
<td>57,5 %</td>
</tr>
<tr>
<td></td>
<td>Yes, in some extent 34,5 %</td>
<td>30,1 %</td>
</tr>
<tr>
<td></td>
<td>No 6,9 %</td>
<td>12,3 %</td>
</tr>
</tbody>
</table>
Fig. 1. “For how long have you been using Computers?”

Fig. 2. “Is the program easy to use?”

Fig. 3. “Does the program save time compared to traditional teaching?”