

WEBWORK
Generating, Delivering, and Checking Math
Homework via the Internet

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ABSTRACT

The WeBWorK system delivers homework problems to students through standard web browsers, giving them instant feedback as to whether or not their answers are correct. It has been developed and used extensively for calculus instruction and physics courses at the University of Rochester over the last six years and is and is currently in use at over 30 other universities.

WeBWorK provides an individualized problem set for each student and, as with standard homework, students are allowed to work on each assignment until the due date. When students submit an answer, WeBWorK analyzes their answer and informs them whether or not it is correct, but does not give the correct answer. Students immediately know their status. They have succeeded or they can find and correct a careless mistake, review the relevant material before attacking the problem again, or seek further help with this problem (frequently via e-mail) from friends, the TA or the instructor. With this system, nearly all of our students, after some work, complete almost all of their homework assignments 100% correctly. Our surveys indicate that they are very happy with the instant feedback and the resulting control they feel over their education.

WeBWorK's large collection of existing problems and its extensible macro framework (modeled on TeX) for posing questions *and* checking the answers, allow each instructor to ask the mathematical questions they *should* as opposed to the questions they *must* because of machine limitations.

By focusing on checking homework answers alone rather than also supplying guidance and instruction, WeBWorK plays to the strengths of computers, and avoids some of the difficulties inherent in trying to build "intelligence" into a computer program. WeBWorK collaborates well with existing educational practices such as traditional lectures, reform calculus, workshops, and expository writing.

See <http://webwork.math.rochester.edu> for more information.

1. Introduction

A longstanding truism of the mathematical community is that "mathematics is not a spectator sport." Proficiency in calculation and in problem solving come only through practice and that means doing your homework. But what if you are doing the homework wrong? While some students can tell whether an answer is "coming out right," many have little sense of whether their answer is reasonable or not. Once they have an answer, any answer, to a homework problem, it is up to the TA or professor to check the answer and respond before they need to think about it again. In these circumstances, homework collected, graded and returned a week later simply does not provide sufficient feedback to be of much help.

WeBWorK changes this situation dramatically by providing instantaneous feedback for the mathematics homework problems encountered in pre-calculus and calculus courses using an internet-based method for delivering these problems to the students. It provides students with instant feedback about the correctness of their answers and, in the process, creates a learning climate in which students continue to work at their homework until they "get it right."

2. Description

Each WeBWorK problem set is individualized in that each student has a different version of each problem (see, for example, the two versions of the problem presented in Figure 2). Students complete the assignment, log onto the internet and enter their answers into a web browser. The WeBWorK system responds telling them whether a specific answer (or set of answers) is correct or incorrect; students are free to try problems as many times as they wish until the due date. As they make entries, the system records the correctness of each attempted answer so that instructors may easily monitor their students' progress.

By focusing on checking homework answers alone rather than also supplying guidance and instruction, WeBWorK plays to the strengths of computers, and avoids some of the great difficulties inherent in trying to build "intelligence" into a computer program. WeBWorK allows the new technology to collaborate with existing educational practices rather than to replace them. Assigning and grading homework is more effective, more efficient and more uniform when using WeBWorK than when using traditional paper-and-pencil problem sets. This allows the TAs and instructors to devote more time to helping students with the conceptual and problem solving aspects of the course material.

In using WeBWorK at the University of Rochester, we have chosen to follow the homework paradigm in which a student is given a fixed assignment and a fixed period of time in which to do it. The computer allows us to change the traditional model in these important ways:

- We can give slightly different versions of the problems to each student and still effectively check them.
- We can efficiently give instant intermediate feedback, at any time of the day or night, as to whether a student's answer is correct or not. We do not usually give answers or hints via the computer, but the student is allowed to attempt the same question again without penalty. After the due date the correct answers, and often complete solutions to the problems, are made available over the internet.
- In addition, we can efficiently and accurately grade every homework problem of every student, even in large introductory classes; we can easily extend the due date of assignments for

individuals as appropriate without causing added bookkeeping burdens, and we can easily and efficiently monitor class performance on the homework problems.

3. Coverage and Availability

At Rochester, WeBWorK is used in our pre-calculus, first year calculus, multidimensional calculus, and differential equations courses as well as in various courses in physics and astronomy. We are just starting to experiment with using it in statistics and finite math courses. In addition, other institutions have used WeBWorK for college algebra (e.g. Howard University, University of Utah), finite mathematics (e.g. Arizona State University, Computer Science at Stony Brook), actuarial science (Georgia State University), and financial mathematics (University of Virginia). It is also being used at the high school level (e.g. Detroit Country Day School) and Ken Appel at the University of New Hampshire is experimenting with using it in an abstract algebra course. Table 1 gives the URL's for over 30 institutions currently using WeBWorK including pointers to many of the sites cited above.

WeBWorK is distributed with a collection of over 2000 problems covering pre-calculus, standard first year calculus, vector calculus, differential equations and elementary statistics.

A list of the current problem collection is available at:

<http://webhost.math.rochester.edu/webworkdocs/ww/listLib?command=setsOnly> .

WeBWorK is freely available to educational institutions and can be downloaded from <http://webwork.math.rochester.edu>. (Follow the "download WeBWorK" links.)

4. Technical comparisons

Since the inception of WeBWorK in 1996, many course management systems and gateway testing programs have been made available for use at universities and colleges. WeBWorK's greatest strength, in comparison to these other systems, lies in the variety of mathematically oriented questions it can successfully present and grade and *the ease with which this capability can be extended*. In addition to numerical answers, short answer questions, and a wide variety of matching questions, it is possible to *effectively check answers that are functions or equations*. Because of its modular design, it has been possible in the last year to add new problems containing graphs generated "on demand," vector field graphs, and problems involving complex numbers, all without changing the basic underlying WeBWorK program.

Another significant core feature of WeBWorK is the ability for instructors to specify new methods of checking the answers at the same time that they write the problem. If an instructor can specify an algorithm for checking an answer, then that algorithm can be implemented within WeBWorK. This has allowed the easy extension of the function evaluator to functions of several variables, and allowed differential equations problems to be written in which the answer is not a single equation, but a family of equations, any one of which is a correct answer to the problem.

Every medium limits the kind of questions that can be realistically asked and effectively checked, and WeBWorK, or any computer-mediated teaching method, is no exception. However, within its focused objective of checking homework answers, WeBWorK's design goal is to enable the instructor to "ask the questions they should, rather than the questions they can" to the greatest extent possible and with the least possible hassle. With this goal in mind, WeBWorK was designed so that the instructors can use it in simple ways without mastering the entire system. If

their desired homework problems are similar to existing problems, then they can quickly and easily modify these template problems to achieve their goals.

At the other extreme, instructors are not blocked from making innovative and creative use of WeBWorK in line with their own educational philosophies. WeBWorK is built for extensibility, so instructors with programming experience can extend WeBWorK's functionality without rewriting the entire system from scratch. This modular design is achieved by creating macro packages that simplify the creation of each homework problem. Since it is possible to use the full power of the text processing language Perl in writing the macro packages, this provides great flexibility for experimentation and improvement without altering the basic WeBWorK framework.

The extensibility of WeBWorK, and our experience so far with the institutions currently using WeBWorK, lead us to predict rapid growth in the variety of types of mathematics questions available through WeBWorK, from traditional mathematics questions to reform calculus questions and the rapid development of WeBWorK problem sets in many other scientific fields.

5 Assessment

Having designed WeBWorK software and acquired some experience in using it effectively, we are currently working on more formal assessment activities to determine more precisely how WeBWorK affects student learning, and simultaneously to suggest improvements that would make it work even better. The WeBWorK system remains a moving target as we constantly incorporate improvements, and instructors at the University of Rochester and at other institutions add new problems on an ever-growing variety of topics.

Our goals during the initial phase of assessment were to understand student opinion of the WeBWorK and to collect information about how students engage with the system. Our assessment tools have included interviews, an electronic survey (developed at the University of Rochester from local interview responses), observations of students using WeBWorK, and the data captured by the system regarding the level of homework completion, number of attempts per answer, and level of overall correctness of final answers.

The positive responses collected during the interviews and observations and on the electronic surveys can be grouped in several key categories: students appreciate WeBWorK because it eliminates paper-based homework, it provides unlimited attempts for solving problems, it promotes continued efforts toward completing all problems, and it serves as an aid in test preparation. Survey and interview responses also indicate that WeBWorK is regarded as an encouragement for legitimate collaborative study among students since the system creates individualized problem sets. The most strongly endorsed benefit of WeBWorK, however, is the immediate feedback provided to students. For example, in our Fall 1999 survey, completed by almost 90% of the students enrolled in WeBWorK-supported courses, the strong majority of the respondents valued the rapid feedback provided by the system, as noted in the following sample comment:

The part that I like about WeBWorK is that as soon as I submit my answer I know if I got it right or wrong. I can enter the answers as many times as I want to. All the assignments are right there for me if I ever want to go back and study over those problems.

In general, students report value in WeBWorK. A Fall 2001 survey respondent explained how WeBWorK supported his/her calculus learning:

I appreciate the wide variety of material it covers....If I find that I have difficulty with the webwork then I go to the textbook or the instructor for understanding and guidance. I enjoyed webwork because it helped to show me my strengths and weaknesses as a student and where I stand with the material that has been taught.

On the other hand, some respondents report ongoing difficulty in entering answers correctly into the system (which we term “syntax problems”), and express the desire for hints or partial credit when a wrong answer is submitted:

I don't like that it's not possible to receive partial credit. Since assignments that you hand [in] are graded by a human being, you can receive partial credit and know where you went wrong in doing the problem. Sometimes with webwork problems you may have made only a tiny mistake but it's still wrong and you don't know why.

Other students found that the precision required by the system led them to further investigation and connection with the overall course:

Occasionally the syntax of complicated problems caused the answer to not come out correctly. This often made me wonder whether I was doing the problem correctly, or if I just typed it in wrong. However, this problem is remedied by the feedback option, the recitations, and general class notes. So although it was at times frustrating, it wasn't a huge problem.

In terms of student usage, we found a number of different patterns; some students print the problem sets and work them out on paper first, while others work only on the computer and scratch paper. Many begin the assignment several days in advance of its due date; others wait until a few hours before the deadline. Here at the University of Rochester, a highly residential campus, most students complete their WeBWorK assignments in their dorm rooms; a smaller number used campus computer facilities. This finding is of special interest to those who are responsible for designing and maintaining student computing resources.

Of particular interest to math faculty, however, is the information captured by WeBWorK about student persistence. Since the system records all student attempts, we have been able to document a remarkable thoroughness towards full and accurate completion of homework: nearly all students using the system here at the University of Rochester completely virtually all of their homework sets until their answers are nearly 100% correct. Student comments lead us to believe that the immediate feedback feature drives this persistence:

It is nice to have the opportunity to end up with all correct answers and not be penalized for having tried the problem ten times because it encourages me to not give up and actually LEARN how to do the problem. If it were pencil and paper homework, I would try it once, hand it in, receive a wrong answer, and then just be upset about it, but not DO anything about it.

Math has never been my strong subject and I get discouraged very easily when doing problems. I like how I can find out if I am doing my work right on WebWork. Unlike paper and book assignments, I know if my method is wrong and can try and fix it right away. This is much better than book work, which I sometimes, unknowingly, can do an entire assignment wrong and waste many hours in the process, because I may have mixed up a rule or step. Here, I know right away and can fix my problems. I now can use my time more efficiently, which is a big bonus. Plus, when I get the 100% on my problem, it is like a small victory. I don't get that encouragement with paper/book work. I also like how I can email my professor if I need to. That's nice to know I have that option.


Because WeBWorK captures so much information about students as they complete their problems sets, it is possible for us to examine possible connections among various approaches to homework and eventual performance in the course. We are seeking to analyze the data to determine if there are certain patterns which indicate that early intervention -- extra help, encourage or tutoring -- would be beneficial. Preliminary analysis indicates that requiring many attempts to answer questions in the third week correlates positively with low midterm exam scores. We are continuing to refine our analysis, but if these patterns are robust, we believe that WeBWorK will provide instructors with an easy means by which to identify students early in the semester who could benefit from intervention, such as more assistance from the instructor, teaching assistant, or tutoring center staff.

Table 1
Educational Institutions Currently Using WeBWork

Alfred University	http://cs.alfred.edu/webwork/mat120/
Arizona State University	http://hobbes.la.asu.edu/119/
California State at Long Beach	http://bosna.natsci.csulb.edu/webwork
Cleveland State University	http://webwork2.math.rochester.edu/csu-mth181/
Columbia University	http://www.math.columbia.edu/~achter/2a/help/studentintro.html
Dartmouth College	http://www.math.dartmouth.edu/webwork
Detroit Country Day School (high school)	http://gauss.dcds.edu:5127/webwork/honalg2/
Georgia State University (actuarial science)	http://webwork2.math.rochester.edu/LifeCon/
Gustavus Adolphus College	http://www.gac.edu/oncampus/academics/mcs/webwork/
Harvard University	http://calculus.math.harvard.edu/
Howard University	http://webwork2.math.rochester.edu/howard-mth156/
Hobart and William Smith Colleges	http://math.hws.edu/webwork/math131
Indiana University	http://www.indiana.edu/~mathwz/
Johns Hopkins University	http://xena.mat.jhu.edu/webwork/
McGill University	http://msr01.teaching.math.mcgill.ca/webwork
National Chiao Tung University (Taiwan)	http://calculus.nctu.edu.tw/webwork/
Ohio State University	https://webwork.math.ohio-state.edu/
Penn State at Altoona	http://webwork.aa.psu.edu:8080/
Radford University	http://ruacad.radford.edu:8080
Rochester Institute of Technology	http://webwork2.math.rochester.edu/rit-mth305
Rochester Institute of Technology (Physics)	http://spiff.rit.edu/webwork/phys311_w2002/
Rutgers University	http://www.math.rutgers.edu/courses/135/135-f01/WWindex.html
SUNY at Stony Brook	http://webwork.ams.sunysb.edu/ams161/
Syracuse University	http://webwork.syr.edu/webwork/
Union College	http://omega.math.union.edu
University of Akron	http://golovaty.math.uakron.edu/webwork/calc2/
University of California at Irvine	http://homework.ps.uci.edu/webwork/
University of Hartford	http://zeus.hartford.edu:3142/webwork/
University of Michigan	http://instruct.math.lsa.umich.edu/classes/215/webhw/
University of Rhode Island	http://webwork.math.uri.edu
University of Rochester	http://webwork.math.rochester.edu/
University of Toledo	http://www.utoledo.edu/~klesh/3860/
University of Utah	http://webwork.math.utah.edu:8080
University of Virginia	http://webwork.math.virginia.edu/
Victor Valley College	http://webwork2.math.rochester.edu/VVC-math26B/

Screen shots of some integration problems:

▲ Prob. List Next ▶



Our records show problem 1 of set 6 has not been attempted.

(1 pt) Evaluate the indefinite integral.

$$\int 91 \cos^2(69x) dx$$

Show Correct Answer

Note: it is after the due date. Answers available.

Display Mode: text formatted-text typeset

Problem Set Version Number: 583664
Page produced by script: /ww/webwork/system/cgi/cgi-scripts/processProblem8.pl

(1 pt) For each of the indefinite integrals below, choose which of the following substitutions would be most helpful in evaluating the integral. Enter the appropriate letter (A, B, or C) in each blank. DO NOT EVALUATE THE INTEGRALS.

A. $x = 7 \tan \theta$

B. $x = 7 \sin \theta$

C. $x = 7 \sec \theta$

1. $\int \frac{dx}{(49 - x^2)^{3/2}}$

2. $\int \sqrt{x^2 - 49} dx$

3. $\int \frac{dx}{(49 + x^2)^2}$

4. $\int (x^2 - 49)^{5/2} dx$

5. $\int \frac{x^2 dx}{\sqrt{49 - x^2}}$

Another student would see this version of the same problems:

▲ Prob. List Next ▶

Our records show problem 1 of set 6 has not been attempted.

(1 pt) Evaluate the indefinite integral.

$$\int 35 \cos^2(22x) dx$$

Show Correct Answer

Note: it is after the due date. Answers available.

Display Mode: text formatted-text typeset

Problem Set Version Number: 726751
Page produced by script: /ww/webwork/system/cgi/cgi-scripts/processProblem8.pl

(1 pt) For each of the indefinite integrals below, choose which of the following substitutions would be most helpful in evaluating the integral. Enter the appropriate letter (A, B, or C) in each blank. DO NOT EVALUATE THE INTEGRALS.

A. $x = 4 \tan \theta$

B. $x = 4 \sin \theta$

C. $x = 4 \sec \theta$

1. $\int \sqrt{x^2 - 16} dx$

2. $\int \frac{dx}{(16 - x^2)^{3/2}}$

3. $\int \frac{dx}{(16 + x^2)^2}$

4. $\int x^2 \sqrt{16 + x^2} dx$

5. $\int \frac{x^2 dx}{\sqrt{16 - x^2}}$

Figure 2