INTEGRATING TI-92/CAS IN TEACHING CONCEPS FROM CALCULUS: HOW IT EFFECTS TEACHERS' CONCEPTIONS AND PRACTICES

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Abstract: Although there are many efforts today, trials and experiments in many countries, there are no clear answers how to teach various concepts effectively in mathematics, in particular pre-calculus and calculus, by using hand-held personal technology (H-hPT) and CAS. To inform and train a group of prospective mathematics teachers (PMTs) and practicing teachers in Turkey we have attempted to organize a series of seminars and workshop on H-hPTs for the last few years. In the present study, we report our experiences at the certificate courses for a group of PMTs in Selcuk Uni-Konya, and show the sample of materials designed to teach various basic concepts in calculus. In the study, we concentrate on how the PMTs can apply their knowledge from mathematics and pedagogical courses in teaching of mathematics and use TI-92/DERIVE, share our experience how to improve the curricula by designing new instructional materials and implementing new strategies in teacher education and training.

Keywords: TI-92/Derive supported/aided teaching, Teacher education, Teaching calculus, Linearity and Proportionality, Guided discovery learning

1. Introduction

Teaching mathematics is a complex endeavor and dynamic process, and the changing role of mathematics teachers for the contemporary society requires new tasks and a rather different training (Ersoy, 1991, 1992a). In such an endeavor, teachers are important figures in changing the ways in which mathematics is taught/learned in schools and they should use cognitive tools properly for effecting teaching. Because, computer and the hand-held personal technology (H-hPT), namely graphing and advanced calculators, is profoundly changing various aspects of teaching and learning of mathematics, as well as doing mathematics, and they are considered as cognitive tools. Therefore, considerable attention must be paid to the pre-and continuing (in-service) education of teachers and the integration of such tools into mathematics/science education (Ersoy, 1992a, b). Then, we enrich teaching and learning environment, and may have chances to improve the quality of teaching mathematics/science al all level of schools. Although there are many efforts, trials and experiments in many countries, there are no clear answers how to teach various concepts effectively in mathematics, in particular pre-calculus and calculus, by using computers, H-hPT and software, e.g. computer algebra systems (CAS) today.

To inform and train a group of prospective mathematics teachers (PMTs) and practicing teachers in Turkey we have attempted to organize a serie of seminars and workshop for the last few years. One of them was hold for a group of PMTs who got their BSc degree from department of mathematics in various universities in Turkey, on August 2001 as an integral part of ongoing projects at the Middle East Technical University (METU) in Ankara which is guided and directed by the researchers (Ersoy, 2001) and of the teaching certificate for becoming high school mathematics teachers. In the present study, we report our experiences at the certificate courses for a group of PMTs, and show the sample of materials designed to teach various basic concepts in calculus. In the study, we concentrate on how the PMTs can apply their knowledge from mathematics and pedagogical courses in teaching of mathematics and use TI-92/DERIVE, share our experience how to improve the curricula by designing new instructional materials and implementing new strategies in teacher education and training. In fact, the present study is part of ongoing project about the calculator-supported/aid mathematics teaching in Turkey. In the study, we introduce some samples of instructional materials on the concept of linearity, and reflect the response of a group of PMTs about the use of TI-92/Derive as a cognitive tool. Thus, we would like to share our experience with other experts as well as improve the curricula by designing new instructional materials and implementing new strategies in teacher education and training.

2. Background

Although there are many efforts today, trials and experiments in many countries, there are no clear answers how to teach various concepts effectively in mathematics, in particular pre-calculus and calculus, by using hand-held personal technology (H-hPT) and CAS. Here a short overview about the background of the present study is given.

2.1. Teachers Teach Mathematics with Technology

The use of technology in either doing or teaching/learning mathematics has influenced and changed many aspects (e.g. Howson & Kahane, 1985). Many authors have investigated the issue and reported the influences, impacts and research the findings. The degree and aspects of the impacts depend upon several variables and various factors e.g. implemented H-hPts and software, backgrounds, views and teaching styles of teachers, etc. Any way they may lead to change in teachers' views and their use of computerized tools (Ford & Ford, 1992). Therefore, it is important to prepare teachers for the future needs of their students (Zehavi, 1996). In the training of teachers, technology is not the focus of learning but its use in teaching. Rather, it empowers teachers and students to explore mathematical concepts with real world data and simulations of real world events.

When technology is used in the way prescribed above, interdisciplinary and real world connections become a natural and powerful way for students to make sense of mathematics (Drier, Dawson, and Garofalo, 1999). Because, both CAS calculators, i.e. H-hPTs, and computers are valuable cognitive tools which enable us trivialization, visualization, experimentation and concentration (Kuzler, 2000). Therefore, there are many groups in developed and industrialized countries collaborate and come together to discuss the issues and share their experience and the instructional materials developed. Among them, T^3-US and T^3-Europe are well known, and they design technology-rich and H-hPT-supported/aided curricula and train many teachers each year. Because, technology-rich curricula can meet the demands of the new standards for more inquiry based learning and new content, and can support more sweeping change that goes far beyond what is envisioned in the NCTM standards (Tinker, 2001)

The average rate of use of information and communication technology (ICT) for instruction and of instructional materials in the education system of Turkey is less than ten percent, but the percentage is increasing gradually. However, few mathematics teachers and some instructors are trying to use ICT, implement TI-92/Derive-Cabri in teaching various mathematics topics (Ersoy, 2001). The present study is part of ongoing project in Turkey called *"T^4: Teachers Teach with Technology in Turkey"*. Activities on research and training teacher began at the METU early 1990s and continue with collaboration with other experts in Turkey and abroad, in particular US, UK, France, Austria, etc. During the last decade we have accumulated some experiences and presented our research findings in the national and international congress and symposiums. We are still doing various researches on the technology supported/ aided mathematics teaching and learning, and organizing seminars and workshops for teachers.

2.2. Design of Study and Implementation of Technology

Before the experimental study, we interviewed several PMTs first, and administered a questionnaire to get PMTs' attitudes and opinions about the advanced graphing calculators (AGC), namely TI-92 and the CAS-software Derive as cognitive tools. We found out that none of the PMTs (n1 = 67) had any idea about AGC and CAS; and were reluctant to learn and use the cognitive tools in learning and teaching mathematics[1]. Therefore, we decided to inform and train the PMTs, who participate in the teaching certificate courses (a special program) taken place at the Selcuk University in Konya for a couple of weeks. Thus we scheduled a new program on the operation system of TI-92 and main features of CAS-Derive and tried to find out the PMTs' needs to learn how to teach mathematics in TI-92/Derive environment.

We continuously have the impression that if a learner performs the requested tasks carefully he/she will explore the concept him/herself. Then he/she may discover a new relationship between concepts in TI-92/Derive environment and understand it deeply. After the introduction of new trends and innovations in mathematics education in the seminar and give information on the use of TI-92, we interviewed with some PMTs. We assigned five activities to the PMTs who would understand the concepts deeply and discover certain rules by answering the posed questions. The assignments were designed by the researchers in the form of worksheets but the PMTs in pairs

[1] The gathered data is the process and the results will be reported later. Here we only reflect a few results and some personal views on the issue.

would find out relations among certain concepts learned in pre-calculus or calculus before and understand the potential of the technology.

In the course, the PMTs worked in-groups of two ore three; but each had to access calculator. The activities were purely concerned with the concepts decay and growth, linearity, local linearity, limit by approximation, uniform continuity, direct proportionality, arithmetic and geometric sequences. In the end of these activities it was asked the PMTs (n = 24) to construct a concept map showing the relations among the concepts on which they worked out in detail. Finally, we requested them to prepare a lesson plan for the implementation of the modified version of the instructional materials or new ones in their own mathematics classroom later.

3. Worksheets and guidelines

3.1. Aim and General Features of Activities

Aim: The aim of the present study is two fold: training a group of PMTs and reflecting their views on the use of H-hPTs. More specifically, one of our aims is that the groups of the PMTs explore some features of the cognitive tools, namely TI-92/Derive, and discover themselves the effective ways of teaching calculus concepts in schools.

General Features of Activities: The researchers designed a set of activities by using the guided discovery approach and basic ideas of constructivism. It is important to notice that these activities help student teachers comprehend basic concepts and integrate H-hPT in teaching mathematics. A set of some activities prepared for training the PMTs is below. Although there are more activities, we present six of them here only. In the training periods, the PMTs should work in-groups, and follow the guidelines/instruction explained in each activity and answer all questions therein. Of course, depending upon the needs the materials presented here can be modified and transformed into other forms, used for some other purposes and may be in new contexts.

3.2. Activities and Guideline

Activity 1 (Linear motion): A body of mass has 4 m/s velocity and is moving along a straight line. Examine the rate of change in distance by the change in time. Explain this rate of change by means of certain concepts, e.g. linearity, direct proportionality etc.

Guideline/Instruction: Read the following statements carefully and do the operation without skipping any step.

1. Write a function for the distance (x) as a function of the velocity (v) and time (t).
2. To calculate the first-degree differences in time and distance and the rate of change of them, use TI-92 and sketch the graph of the rate as follows.

- Press Diamond+TblSet and give the initial value of time (t) as Tblstart: 0, ΔTbl:1.
- Press Diamond+[Y=] and write the distance function, rate of first degree differences in distance/ first degree differences in time and distance/time as y1=4x, y2=4(x-1), y3 = y1(x) - y2(x), y4 = y1(x)/x.
- Press Diamond+Table and see the numerical values and press Diamond +GRAPH to see the gaphs of the functions.
- Fill in the table below using numerical values from the table on the display.

X	Y=f(x)	1st degree differences in x	1st degree differences in f(x)	Rate of change
0	0			
1	4	1= -0	4= 4-0	= 4/1
2	8	1=2-1	4=8-4	=8/2
3	12	1=3-2	4=12-8	=12/3

Result 1: From the ratio at the last column of the table can you find out the that the points (x,y) are Linear or not?

Result 2: Using the ratios $f(x_i)/x_i$, $i\in N$, can you find out the that x_i and y_i are directly proportional? Why?

Result 3: If the x variable forms an arithmetic sequence then does the range values of $y = 4.x$, $x\in R$ are forms an arithmetic sequence?

Fill in the blanks in the following propositions.

- The directly proportional quantities are ………………………………….
- The directly proportional quantities form both ………. …….. and ………..…………………
- If the ratio of the first degree differences is constant for a function then it is called ………………….

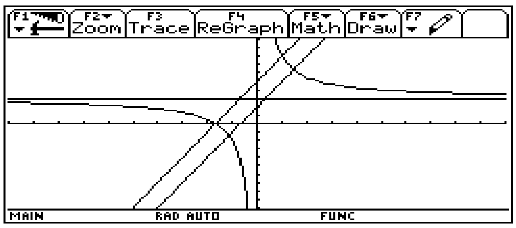
<u>Activity 2</u> (Symbolic and numerical and graphical representation)**:** Using a TI-92 calculator, study whether the set of points on a straight line are directly proportional or not.

<u>Guideline/Instruction:</u> Read the following instruction carefully and do the operation without skipping any step.

1. The set of the points (x,y) on the graph of the function $y = 3x + 5$, $x\in R$ are linear.
2. Test the values of x and y are proportional or not and construct a TI-92 model for the problem.

- Press Diamond+TblSet and give the initial value of time as Tblstart: 0, ΔTbl:1.
- Press Diamond+[Y=] and write the function, changes in function in time as a function and the ratio of the changes by the time as $y1 = 3x + 5$, $y2= 3(x-1) + 5$, $y3 = y1(x)-y2(x)$, $y4 = y1(x)/x$.
- Press Diamond+Table and see the numerical values and press Diamond +GRAPH to see the gaphs of the functions y1(x), y2(x), y3(x), y4(x) which are all linear.
- Fill in the table below using numerical values from the table on the display.

x	y1(x)	y2(x)	y3(x)=Δy/Δx	y4(x)=y1(x)/x
0	5	2	3	undefined
1	8	5	3	8/11
2	11	8	3	11/2
3	14	11	3	14/3
4	17	14	3	17/4



- What have you perceived from the last column of the data table for the ratio y/x?
- Do the quantities x and y are right proportional? Why?
- What is the relation between the concepts set of linear points and directly proportional quantities?

<u>Activity 3</u> (Change in Population)**:** Consider the data in Table 1 for the population of Mexico in the early 1980s. Study the changes in population increase and calculate the rate of changes by time. Using this ratio, can you explain the linearity of the population function? Do the quantities, population and year, directly proportional? Do the population data forms a geometric sequence? Study the relations among the concepts linearity, direct proportionality and geometric sequence.

Table 1. Population of Mexico, 1980-1983

Year	n	Population (p(n))	Ratio of Changes ($\Delta p/\Delta y$)	p(n)/p(n-1),n = 1,2,3,4	P(n)/n, n=0,1,2,3,4
1980	0	p(0) = 67.38		1.026	Undefined
1981	1	p(1) = 69.13	1.75	1.026	69.13
1982	2	p(2) = 70.93	1.80	1.026	35.465
1983	3	p(3) = 72.77	1.84	1.026	24.257
1984	4	p(4) = 74.66	1.89	1.026	18.665

- Calculate the first-degree differences in population and time and the rates of change in population by the change in time using TI-92 calculator and fill the forth column of the table. What have you perceived? Does the population function linear or exponential? Why?
- Calculate the ratio p(n)/p(n-1), n =1, 2, 3, 4 and fill the fifth column of the table. Write the population function p(n) related with the initial population p(0). Do the population data form a geometric sequence?
- Calculate the ratio p(n)/n, n = 0, 1, 2, 3, 4 and fill the sixth column of the table. Do the quantities p(n) and n are right proportional? Why?
- Find out the relations among the concepts linearity, direct proportionality and geometric sequence.

Note: Get data for the population of Turkey in early 1950s, 1970s and 1990s. Then find out the changes of population of Turkey in each given period and in the last 50 years.

Activity 4 (Quadratic expression)**:** Calculate the ratio of first-degree differences for the function $f(x) = x^2$, $x \in R$ in the neighborhood of x =1 by the change in x using TI-92. Study the global linearity and local linearity of the function f(x) on R and in the neighborhood of x =1 respectively. Do the quantities x an y are directly proportional? Study the relations among the concepts global linearity, local linearity and directly proportional quantities.

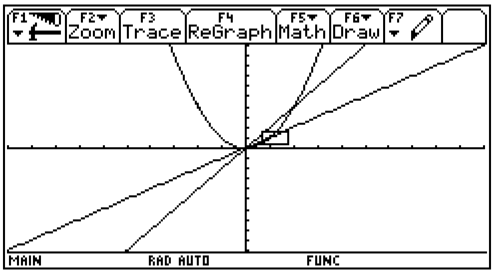
Guideline/Instruction: Read the following instruction carefully and do the operation without skipping any step.

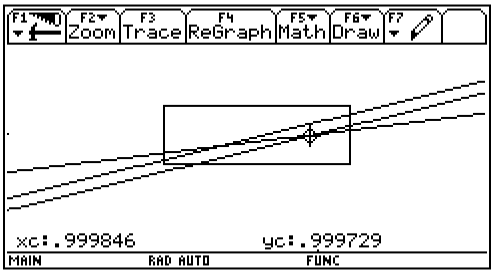
- Press Diamond+TblSet and set the initial value of x as Tblstart: 0,999, ΔTbl:0,001.
- Press Diamond+[Y=] and write the function, rate of first degree differences in f(x)/ first degree differences in x and the rate of change in f(x) by change in time as $y1= x^2$, $y2 = (x+0,001)^2$, y3 =(y1(x) - y2(x))/ 0.001, y4 = y1(x)/x.
- Press Diamond+Table and see the numerical of the functions.

X	Y1	y2	y3 = Δy/ Δx	y4 = y1/x
0.999	0.998001	0.996004	1.997	0.999
1.000	1.000	0.998001	1.999	1.000
1.001	1.002	1.000	2.001	1.001
1.002	1.004	1.002	2.003	1.002
1.003	1.00601	1.004	2.005	1.003

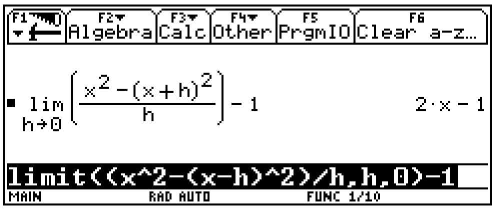
- What have you perceived from the forth column of the data table? Do the function $y = x^2$ global linear on R ?

- What have you perceived from the last column of the data table? Do the quantities x and y are directly proportional? Why?
- Press Diamond+GRAPH and display the Graph screen, lets you draw a box that defines a new viewing window, and updates the window. The display after defining Zoombox by pressing $\boxed{\text{ENTER}}$ you will see the graphs consecutively.





- What have you perceived from the Zoombox? Does the function [Y=] x^2 local linear at x = 1?
- Write a new function for the points local linear in the neighborhood of x = 1. On the HOME screen calculate the following limit.



- Do this result same with the new function? Why?
- Compare the rate of change y3 = (y1(x) - y2(x))/ 0.001 with the result of the limit process. What have you perceived?
- Construct a map showing the relations among the concepts global linearity, local linearity, right proportional quantities, first-degree differences and the limit?

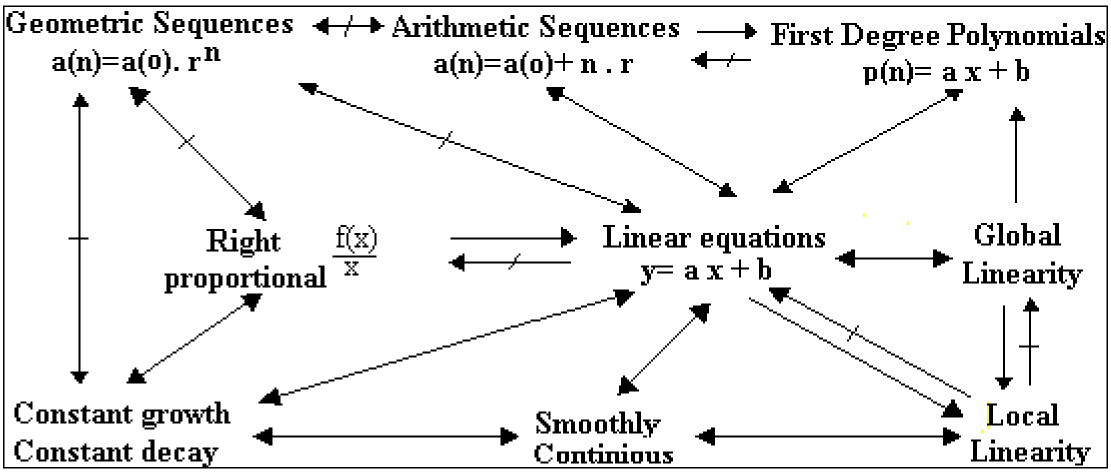


Figure 1. Concepts Maps on Linearity

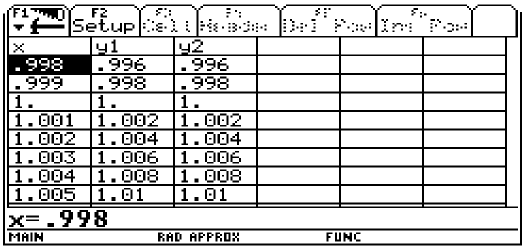
<u>Activity 5(</u>Local linearity and square root):

1. Display the MODE dialog box. For graph mode, select FUNCTION.

2. Display and clear [Y=] editor. Then define y1 = x^2 and press $\boxed{\text{F2}}$ 6 to see the graph of the function.
3. From the graph screen, press $\boxed{\text{F5}}$ and select A:Tangent. Set the tangent point. Either move the cursor to the point or press $\boxed{\text{A}}$ and type its x values as 1
4. Press $\boxed{\text{ENTER}}$. The tangent line is down, and its equation is displayed.
5. Repeat the process at the step 3 for the x values 2, 3, 4 etc and then fill the table below.

x	y= $\mathbf{x^2}$	Tangent y = m x + n	Slope of Tangent	Slope of Tangent / x	y / n
0	0	y = 0 x + 0	0	0 / 0	undef
1	1	y = 2 x – 1	2	2 / 1	1 / -1
2	4	y=4x-4	4	4/2	4/-4
3	9	y=6x-9	6	6/3	9/-9
4	16	y=8x-16	8	8/4	16/-16

6. Compare the range values of the function and tangent function for the x values in the very close neighborhood of x = 1.



x	y1	y2
.998	.996	.996
.999	.998	.998
1.	1.	1.
1.001	1.002	1.002
1.002	1.004	1.004
1.003	1.006	1.006
1.004	1.008	1.008
1.005	1.01	1.01

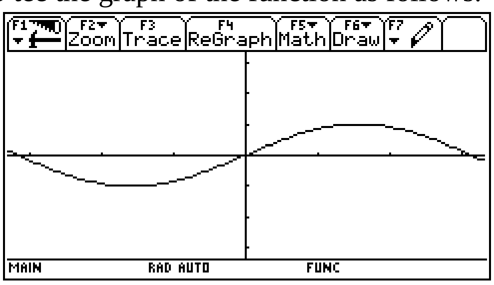
x=.998
MAIN RAD APPROX FUNC

7. Using this data table, study the following questions.
   - The tangent line approximation is $x^2 \cong 2x - 1$, for every x=1±Δx, Δx tends to zero. Why?
   - Using the above approximation we write $1 \pm \frac{\Delta x}{2} \cong \sqrt{1 \pm \Delta x}$, Δx tends to zero.
   - How can you calculate the square root of 1.00015 with the six digits after the point?
   - Calculate the square root of 3 with the error less than 0.001 using the approximation above and compare the direct result of calculator. Use the formula (error = First degree differences-slope of the tangent) to calculate the error.

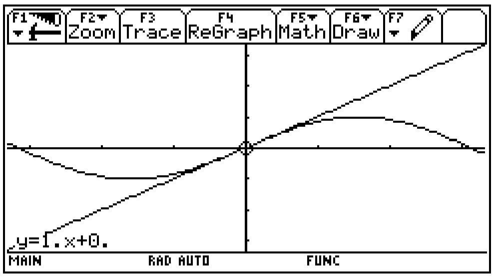
Activity 6.(Local linearity and limit process): Use TI-92 calculator and the local linearity in activity 5 to find $\lim_{x \to 0} \frac{\sin(x)}{x}$ and compare with the traditional limit process.

Guideline/Instruction: Read the following instruction carefully and do the operation without skipping any step.

1. Press Diamond+TblSet and set the initial value of x as Tblstart:-0.006, ΔTbl:0,001.
2. Press Diamond+[Y=] and define the function, y1 = sin(x) and y2 = x .
3. Press Diamond +GRAPH to tee the graph of the function as follows.

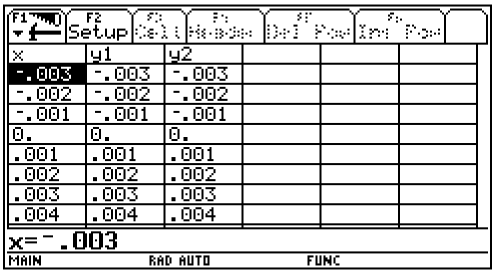


4. From the graph screen press [F5] and A:Tangent. Set the tangent point. Either move the cursor to the point or press $\boxed{\text{A}}$ and type its xc value as 0. Press ENTER. The tangent line is drawn, and its equation is displayed.



5. Now, calculate the range values of the function and the tangent line in the very close neighborhood of zero.

- Press Diamond+Table and see the numerical values of the functions.



x	y1	y2
⁻.003	⁻.003	⁻.003
⁻.002	⁻.002	⁻.002
⁻.001	⁻.001	⁻.001
0.	0.	0.
.001	.001	.001
.002	.002	.002
.003	.003	.003
.004	.004	.004

x=⁻.003

MAIN RAD AUTO FUNC

- What have you perceived from the second and third column of the data table? Do the function y1 = sin(x) have local linearity near 0? For a constant change in x of 0.001, there is a nearly constant change in sin(x) of 0.001. Thus, near x = 0 the sin(x) function appear nearly linear with slope 1. So, the local liberalization of sin(x) near x=0 is sin(x) ≈ x. When x tends to zero, also sin(x) tends to zero. Thus, local linearity tells us that $\frac{\sin(x)}{x} \approx \frac{x}{x} = 1$.

The traditional way to find this limit is to use circle and the sin(x)< x < tan(x) inequality. The local liberalization is more meaningful than the traditional way to find limit in functions.

4. CONCLUDING REMARKS

The available H-hPTs in the market have changed the practice of doing research in mathematics education and is profoundly changing the teaching and learning of mathematics at all levels of schools, teacher training colleges and universities. All mathematics teacher, regardless of age and experience need training on the use of H-hPTs in teaching of mathematics in almost all countries. In this process, teacher educators in the developing countries face on more problems and constraints and do not have enough funds to meet basic needs. The project on both research and training PMTs is the first experimental study in mathematics teaching in the Selcuk University, Konya and it is still going on. We presume that we have achieved our goal and work out the details by considering the following results and impressions.

The PMTs think of that the designed worksheets were very valuable for them and the H-hPTs should be used in teaching various concepts in calculus as well as other topics. Most members in the groups of the PMTs stated that their background and previous education were not suitable for to use such cognitive tools in mathematics teaching at the beginning, but it gradually changed in the end of the course. All groups constructed the concept map shown in Figure 1 and they said that they liked Activities 5 and 6 very much. The most important the PMTs' view was that the activities were more meaningful than the traditional examples thought in calculus before. Thus, it is our personal impression during the training periods that the PMTs become aware of various teaching strategies, benefits of group study, visualization and the power of TI-92/Derive, i.e. CAS calculators (Ardahan & Ersoy, 2002).

REFERENCES

Ardahan, H. & Ersoy, Y. (2001). "Issues on integrating CAS in teaching mathematics: A functional and programming approach to some questions". ICTMT-5 Derive and TI-89/92 Session. 8-10 Aug 2001 Uni. of Klagenfurt, Austria.

Ardahan, H. & Ersoy, Y. (2002) " A group of PMTs' views on the uses of H-hPTs in teaching mathematics". The Fifth National Congress on Science and Mathematics Education (UFBMEK-5). Sep 16-18 2002, METU, Ankara (to be submitted-in Turkish).

Ersoy, Y. (1991). "An overview of maths teachers' education in Turkey: A general perspective, progress and problems'. Paper presented in IACME-8, Aug 3-8, 1991, Uni. of Miami, Florida.

Ersoy, Y. (1992a). "Mathematics education in Turkey: Challenge, constraints and need for an innovation". Paris: UNESCO Pub (ED-92/WS-11), 156-158.

Ersoy, Y. (1992b). "A study on the education of school mathematics and science teachers for information society" Education Report, vol 1, 39-54. Ankara: METU Pub.

Ersoy, Y. (2001). HeMaDME: Applied Research Project Report, AFP-00/01.05.01.01, METU, Ankara (in Turkish).

Howson, A.G. & Kahane, J.P (Eds) (1985). School Mathematics in the 1990s. Cambridge: Cambridge Uni. Press.

Drier, H. S., Dawson, K. M., & Garofalo, J. (1999). Not your typical math class. *Educational Leadership 56(5)*, 21-25.

Kuzler, B. (2000). The Algebraic Calculator as a Pedagogical Tool for Teaching Mathematics. http: // www. kutzler.com

Tinker, R. (2001) Information Technologies in Science and Math Education, carson.enc.org/ reform/journals / 104633/ nf_4633.htm

Zehavi, N. (1996). "Challenging teachers to create mathematical project with DERIVE®", International DERIVE® Journal, **3** (2)1-16.