DIDACTICAL CLASSIFICATION OF PROBABILITY PROBLEMS LINKED WITH THEIR FORMULATION

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ABSTRACT

Our research aims at the study of the relationships between problems involved in the teaching of mathematics at school and those encountered both during the historical development of mathematics and in educational research.

We study the common elements and features of the formulation of these problems as well as the way they influence each other, aiming at the improvement of mathematical problems used in classroom contexts. This improvement is related to their content and the way they are presented, so that they will have epistemological account and be related as well as be improved by research results. The interrelations that will be presented focus on Probabilistic Problems and their teaching to 5-11 year - old children.

Before coming to these interrelations, we initially collected problems, which were found: 1) in governmental school books different for each level as well as in several published books, 2) in published articles and conference proceedings (problems which have been used in researches), 3) in history, philosophy and epistemology books.

Moreover, after registering the features of their formulation, we created categories and sub-categories, in which each problem was incorporated. Resulting data were statistically analyzed by Factor Analysis methods in order to classify the problems and obtain the appropriate taxonomy. Research results and conclusions will constitute educational material for teacher training because, although Probability Theory is a very important and socially useful branch of mathematics, it has been observed that there are many difficulties in their learning as well as teaching process.

Keywords: Probability problem's formulation, primary school (5-11 year – old), educational material, teacher training.

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1. Introduction

Elements of Probability Theory have recently been included in the curriculum of primary education because this area is considered a very important and socially useful branch of mathematics education. However, it has been observed that there are many controversies and difficulties as far as the comprehension and acquisition of this theory are concerned as well as its teaching process. These controversies mainly derive **a)** from the confusion caused by the parallel and not critical adoption of more than one philosophical theories (classical, frequentist, subjective) and **b)** from the fact that probability acquisition activates a "practical" and "common sense" framework rather than a typical and very abstract approach. Therefore, when a typical and abstract approach is "incumbent" or "implied", the teaching process becomes incomprehensible and creates difficulties in the theory acquisition.

Much research done relate, both to the difficulty of understanding the concept of probability and to the ways these difficulties can be dealt with, but their results do not seem to have influenced education.

Our point of view focuses on the influence the probability problems formulation has on the learning and teaching activities. We have formed the assumption that the mathematical problems used for research purposes differ from the problems we find in schoolbooks. For this reason, we assume that research explanations and interpretations are not sufficient and this is the reason why research results cannot improve the teaching process.

Historically, there is a view claiming that the probability theory has derived from "problem-games". This view has resulted in the study of these "famous" probability problems as well as of new ones. Our first finding was that historical problems could not be found in the context of the teaching of the probability theory in the primary school.

The purpose of this paper is to study we have studied the relationship between probability problems involved in the teaching of mathematics in primary school (5-11 year olds) and those encountered both during the historical development of mathematics and in educational research, focusing on the study and analysis of their formulation. Our aim is to create educational material for the improvement of the teaching process and also for the better understanding of the probability theory.

The formulations of probability problems have some special particularities compared to the formulation of the problems of other mathematics branches.

For example, with regard to formulation **kind**, apart from simple formulations, which we also find in other mathematics branches, we have complex formulations in which many materials are combined with many concepts at the same time. More specifically, we can observe in the same formulation the use of dice, coin and spinner related to the probability of an event, sample space, graphical representation and modeling.

As regards **form**, the formulations can be either verbal or mixed (verbal and iconic), but, in this case, they are diverse in several ways depending on whether they depict the material, a drawing, a table, a probability scale or a graphical representation.

The **material** in probability problems formulation has its own special role. In such kind of formulations we have a large variety of material, often with hidden information presupposing student's "knowledge". For example, if the material is a "dice", most of the times it is taken for granted that the child knows what is a "dice" (cube with numbers 1,2,3,4,5,6). This also happens with lotto as well as with other materials.

The **solving process**, apart from the formulations that request a unique answer, is usually complicated and needs prediction, experimentation, interpretation and comparison of the results.

As regards the suggested **way of solving**, the probabilistic formulations cause the collaboration in groups for problem solving because by nature the probability theory, due to the experimentation involved, requires comparison of results for its better approach.

Due to the fact that there are several "schools" of probability, each with a somewhat different interpretation, we find several approaches within the formulations. One of the interpretations is the classical one, according to which the probability of an event is simply the ratio of the number of alternatives favorable to that event to the total number of equally - likely alternatives. Another one is the frequentist interpretation, which defines probability in terms of the limiting relative frequency of occurrence of an event in an infinite or near infinite, number of trials. This interpretation is applied to events that are composed of non-equally-likely alternatives. Beyond the above mentioned, in many formulations we notice the request for an **experiment** in events with equally - likely alternatives, a fact that creates confusion.

Another particularity is the existence or lack of the sample space, as well as the way it appears (if it is found verbally or iconic, if it does not appear, if it is requested, or if it does not exist). This means that the way it is presented makes problem solving easy or difficult.

What becomes clear from all the above is the different nature of the formulation of probability problems, compared to these of other mathematics branches. In this way, difficulties are caused both to students and teachers: the former need to work in different ways; the latter have to apply a different evaluation method. Using Factor Analysis methods we study the relationships between problems involved in the teaching of mathematics at school and those encountered both during the historical development of mathematics and in educational research.

2. Methodology

We collected 282 probability problem formulations, from Greek and Cypriot schoolbooks, from English and American published books, from research articles and educational activities, which refer to 5-11 year-old children. We also collected historical and philosophical formulations from history books.

Then, we wrote down their basic characteristics and we created variables and categories, which are based on these characteristics. These variables related to:

1) the kind of the text that includes the formulation
2) the book publication date or the original date for historical and philosophical formulations
3) the formulation level
4) the formulation country of origin
5) the kind of formulation
6) how clear the formulation is
7) the formulation form
8) the formulation number data
9) the formulation sample space
10) the existence of a solved example
11) the formulation material
12) the formulation solving process
13) the formulation solution
14) the formulation experimental trials

15) the formulation probability concept

Next, we coded each formulation in a category. The resulting data was statistically analyzed through Factor Analysis methods, in order to classify the problems and obtain the appropriate taxonomy.

3. Results

Factor Analysis pointed to two major taxonomy formulation criteria corresponding to the first two factors.

The first factor mainly represents the diversification between the published for the first time in USA formulations and these published in the other countries with the characteristics that accompany each case.

VARIABLES	CATEGORIES	
	Factor's 1 positive side	**Factor's 1 negative side**
Country	USA	Other Countries
Kind	Complex	Simple
Form	Mixed (Verbal and Iconic)	Verbal
Solving Process	Experimental	Simple Answer
Way of Solving	Teamwork	Individual
Trials	Existent	Not Existent

The second factor mainly represents the diversification between old and more recent formulations. The table below describes this diversification:

VARIABLES	CATEGORIES	
	2nd factor positive side	**2nd factor negative side**
Date	Historical	After 1979
Kind of text	Book, Article	School book
Level	1-6	1,2,3,4,5,6
Country	France, Italy	Cyprus

On the level of factor 1 and factor 2 below (figure 1) we notice the categories-projection configuration, which could lead to four groups. Each group contains different formulation characteristics.

More specifically group I contains formulations from English and Israelian research tests, group II from recent Cypriot schoolbooks, group III French and Italian historical formulations and finally group IV formulations from the USA.

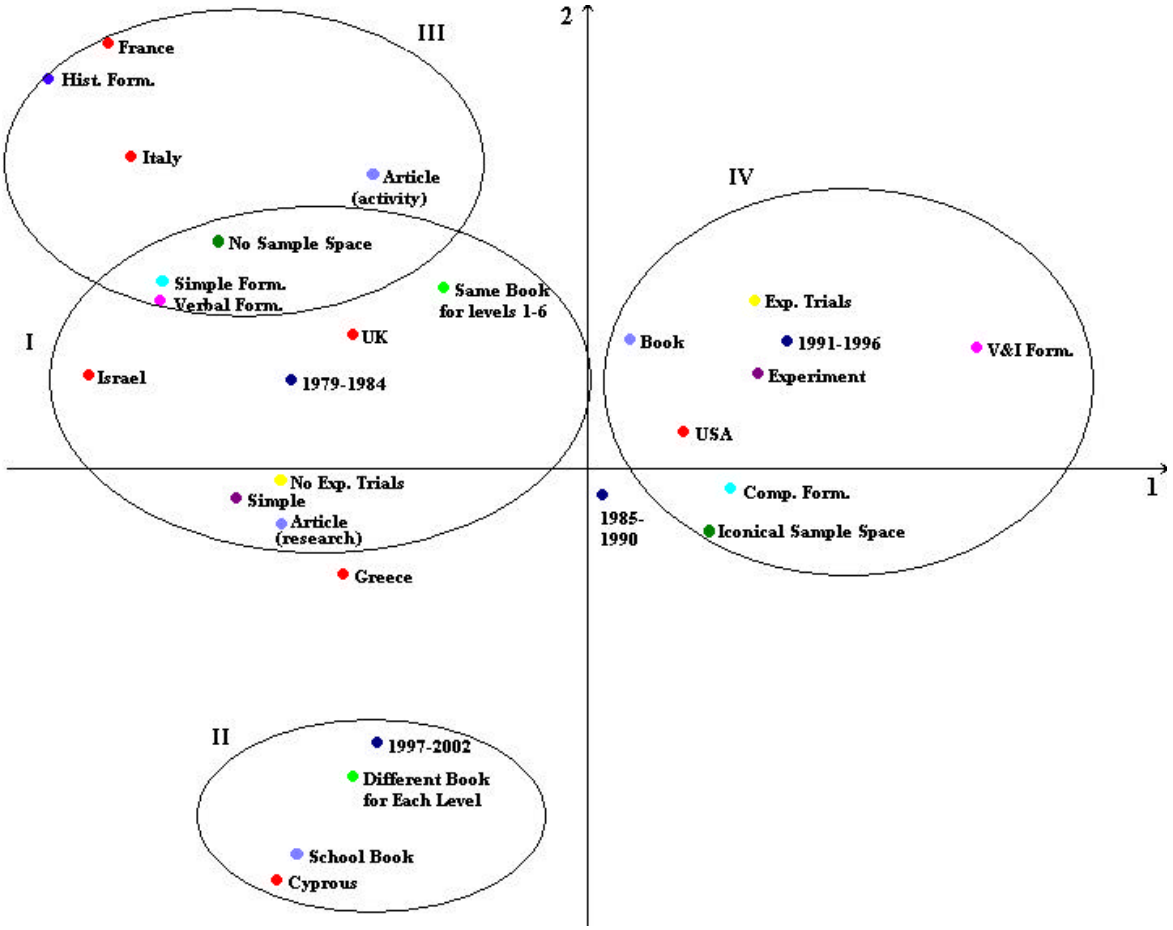

Figure 1: Categories -projection configuration

The hierarchical cluster analysis below (figure 2) confirms the above results. In particular, cluster 4 (group IV in diagram) is clearly separated from the others. Cluster 3 (group III in diagram) follows, which differs from clusters 1 and 2. Finally, there are clusters 2 (group II in diagram) and 1 (group I in diagram).

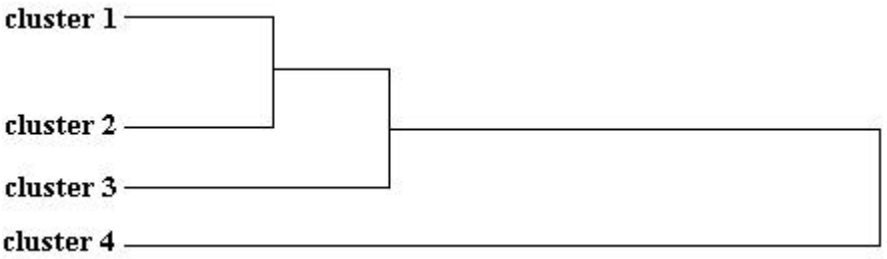

Figure 2: Cluster analysis dentrogram

4. Conclusion

After analyzing the correlations emerging from the characteristics of probability formulation, we noticed that our initial assumption, that problems used for research purpose differ from the

problems we find in schoolbooks, was confirmed. This might be a reason why we assume that research results cannot improve the teaching process.

Also, the "famous" historical problems considered as the foundation of probability theory, which, as a framework, would lead children to reinvent and rediscover the concepts and the real difficulties of this theory, are not presented at all in probability school-formulations.

The simple and verbal school-formulations requesting a unique answer from the student prevail, while, according to the particularities we mentioned in our introduction, the probability problem solving requires the depiction of material, experimentation and teamwork for better results.

The above conclusions (as well as others that will come up from further data analysis) will be utilized for the construction of educational material, especially for problem activities, for the improvement of teaching, for the facilitation of understanding the probability concept by primary school students as well as a research context for teacher training.

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