PREPARING TEACHERS FOR A NEW CHALLENGE: TEACHING CALCULUS CONCEPTS IN MIDDLE GRADES

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ABSTRACT:

The main focus of this paper is to discuss possibilities of teaching and developing students' conceptual understanding of advanced Calculus principles in middle grades. Authors explore conditions in teacher preparation for the successful teaching of a "Visual Calculus" course (integrated 3-D Geometry and multi-variable Calculus concepts of differentiation, integration and optimization) in a middle school with a culturally diverse student population. Basic assumption is that conceptual learning leads procedural development (L. Vygotsky, V. Davydov, R. Skemp, etc.). The main distinction of the "Visual Calculus" course is its orientation toward method of ascending from general to specific, multiple connections with science and technology, as well as multiple representations with focus on the power of cognitive visualization in the development of students' conceptual understanding of advanced Calculus ideas. Final research destination of the project is the measurement of an impact that early conceptual development of students' advanced mathematics principles has on students' progress in Calculus at the high school and college level. Current stage of the project is focused on the middle grades teachers' perception of early development of Calculus concepts, relationship between teachers' content and pedagogy knowledge as well as their readiness and confidence to teach Calculus concepts in middle grades.

Key Assumptions

The project is based on the following key assumptions about learning and teaching:

- Conceptual learning leads development of cognitive acquisition of formal procedural operations. Vygotsky claims that development of advanced concepts might start much earlier and it depends on learning, on how you can create a successful learning environment to develop this concepts.
- The development of students' procedural Calculus skills is a derivative of students' conceptual understanding of big Calculus ideas and principles. Thus, development of students' conceptual understanding of Calculus principles should start earlier in the middle school and should be achieved by ascending from general to specific, from big idea to specific procedure (ascending from multivariable Calculus concepts to single-variable principles). We also believe that cognitive-visual conceptualization through the use of modeling and technology will play a powerful role in early learning of Calculus principles.
- Importance of learning through teaching approach in teacher preparation: students learn what they have to teach. Traditional teacher education programs and student teaching experiences do not provide enough time for pre-service teachers to teach mathematics in actual classrooms. This limited experience in mathematics teaching reinforces the low confidence level of most teacher education students in their ability to understand and teach these subjects. Calculus is a subject that few teachers have studied and the very word evokes massive mathematics anxiety from most teacher education candidates. The field-based experience based on learning through teaching approach provides university students an opportunity for immediate application of their knowledge and skills in actual classroom settings in a real public school environment with feedback from university teams and public school teachers. At the same time, the team teaching of mathematics content, methods, and pedagogy classes between faculty in Colleges of Education and Science helps pre-service teachers to integrate Calculus concepts with its active application to teaching and learning.

Current Research in Calculus Teaching and Learning

Last two decades Calculus is at the forefront of research and curriculum reforms in mathematics education. Majority of research in Calculus learning have been done at the level of undergraduate education and some at the high school level. Researchers observed that students enter calculus courses with a primitive understanding of concepts of function, change, continuity, etc. (Tall, D., 1996, Ferrini-Mundy, J., & Lauten, D., 1993). They also noted that students have cognitive difficulties in coordinating function concept in algebraic and graphical representations which is critical in constructing a foundation for fundamental calculus ideas (Schnepp, M., & Nemirovsky, R., 2001). Other research concentrates on different approaches to teaching calculus principles: comparison study on technique-oriented approach vs. conceptual and infinitesimal approaches of learning calculus shows that different approaches have different impact on students' language use and sources of conviction (Frid, S., 1994).

Researches also determined that cognitive obstacles to the learning of calculus arise in at least two different ways – one related to linguistic/representational aspects and the other related to intuitions. Given that so many of our algebra and calculus courses are immersed in symbolic manipulation, often at the expense of understanding, it is not surprising that linguistic/ representational factors give rise to cognitive obstacles. Also, since learners basically want to understand and make sense of what they are being asked to learn, the intuitions that students bring to bear on the concept of calculus often play a crucial role in the appropriate construction of those concepts. Researches “propose that a potentially useful framework in which to embed considerations of cognitive obstacles lies in the framework of Krutetskian cognitive processes of reversibility, flexibility, and generalization” (Norman, A., & Prichard, M., 1994, p. 76).

There is an emerging importance of making connections between different representations (concrete, visual-spatial, numeric, graphical, algebraic, etc.) in helping students’ to learn calculus concepts. One of the guiding principles of Harvard Consortium Calculus text is the “Rule of Three”, “which says that wherever possible topics should be taught graphically and numerically, as well as analytically. The aim is to produce a course where the three points of view are balanced, and where students see each major idea from several angles” (Hughes-Hallett, D., 1990, p. 121).

One of the most significant points which come from the analyses of research in Calculus learning is that there should be more emphasis placed on conceptual learning using multiple representations and connections before students immerse into symbolic manipulations. In order to build a rich conceptual foundation for successful learning of Calculus at the high school and college level there should be a lot of preparatory work done at the early years of schooling. “Calculus needs to be studied across many years of school, from early grades onward, much as a subject like geometry should be studied” (Kaput, J., 1994, p. 132).

“Visual Calculus” Course Content Design

In contrast to previous remarkable attempts in early development of advanced Calculus concepts (e.g., SimCalc project, CoVis (Scientific Visualization) project) which basically considered development of single-variable Calculus concepts, we start teaching Calculus principles from general multi-variable to single-variable concepts: from generic 3-D surface to arbitrary 2-D curves and then to specific elementary curves (linear, quadratic, exponential, etc.), from tangent plane to tangent line (including concept of gradient), from general infinitesimal methods to procedural calculations of derivative and integral, etc.

V. Davydov (1990) first examined the effectiveness of the method of ascending from general to concrete by teaching algebra concepts to elementary school students in early 1970’s in Russia. We consider an application of pedagogy ascending from big, general idea to specific procedure as a methodological tool for designing a middle school “Visual Calculus” intervention course and a supplementary teacher education course. The main purposes of this course are development of middle school students’ conceptual understanding of Calculus principles and a supplemental Calculus module for teacher education students.

In teaching multi-variable Calculus concepts we use one of the advantages of local Greater El Paso landscape – mountains (a natural model of generic arbitrary 3-D surface). In parallel with this we introduce basic 3-D Geometry concepts (3-D coordinate system, projections of 3-D objects, sections of arbitrary 3-D surface, etc.) to middle school students. We consider multi-variable Calculus as mathematically natural way to introduce 3-D Geometry concepts.

One possible extension of the project is a development of an inquiry-based “Mathematics of a Mountain” initiative for elementary school students. Hiking on the local Franklin mountains, El Paso, TX or skiing on the mountains of Ruidoso, NM will help students to understand the meaning behind the general 3-D concepts of slope (steepness of a mountain), tangent plane, tangent line, gradient (vector of maximum steepness), points of relative maximum and minimum, saddle points, etc. We consider field trips to mountains as a part of developing students’ learning experiences in understanding basic multi-variable Calculus principles. Afterwards students visualize multi-variable Calculus concepts using 3-D arbitrary mountain models (made from play-dough or other materials), constructing contour diagrams, cross-sections, etc.

We provide a thorough visual hands-on introduction to three-dimensional geometry including two-dimensional surfaces in three dimensions. Students go back and forth between the three-dimensional models and surfaces and the two-dimensional representations. Planes, and their slopes, are studied as a special case, and as a transition to studying directional derivatives and gradients. We also introduce multivariable integration by finding volumes of actual three-dimensional objects, by repeated slicing. We model instruction of concepts for the middle grades teachers as well as teacher education students and they in turn teach all strategies in the actual classrooms.

Research Methodology and Professional Development Activities

Research is taking place in conjunction with ongoing NSF funded PETE (Partnership for Excellence in Teacher Education) program at UTEP, with its emphasis on field-based intervention for improvement of pre-service math and science teachers preparation. Clinical quasi-experimental design is focused on the relationship of pre-service teachers’ content and method knowledge in math and upper elementary and/or middle school students’ achievement in “Visual Calculus” and regular mathematics classes.

During the summer and fall-2001 we were piloting “Visual Calculus: Early development of students’ advanced mathematics concepts” experimental class at UTEP in the form of professional development seminar/workshop. In summer we had 15 pre-and-in-service upper elementary and secondary teachers involved into the workshop. Some of the teachers have taken Calculus courses (up to Calculus-III), and some of them – have no Calculus experience at all. We have formed heterogeneous groups in order to involve them into discussions at multi-level Calculus learning experiences and help them to understand basic multivariable Calculus concepts. Each group worked on particular concept of Multivariable and Single variable Calculus: differentiation, optimization, and integration. In this workshop we use Harvard Consortium Calculus text (Hughes-Hallett, D., Gleason, A., et al.) and supplementary materials. During the summer session each group came up with a set of conceptual tools (activities, hands-on manipulations with physical models, technology based illustrations, etc.) which from UTEP students’ perspective would be appropriate to teach to the middle school students.

The distinctive feature of the fall-2001 session of the workshop is that it reflects multi-tiered teaching experiment design (Lesh, R. & Kelly, A., 2000). We have a group of 3-4 researchers (with background and expertise in mathematics, mathematics education, cognition, engineering), the group of 15 pre- and in-service teachers with different level of Calculus experiences, and a group of 4-5 multiage students (from upper elementary, middle and junior high schools without any experience in learning Calculus) in one classroom during each seminar sessions (table 1). Each group of teachers have a chance to teach the activities, developed in summer session, to the multiage group of students with main emphasis on conceptual understanding of particular Calculus principle ascending from general multivariable idea to specific single variable case. After the teaching episode researchers, teachers, and students participate in discussion on how the teaching impacted the students' understanding of the concept.

Table 1. Multi-tiered teaching experiment

Tier 3. **The Researcher Level**	Researchers develop Visual Calculus conceptual model to make sense of pre-and in-service teachers' and middle school students Calculus learning activities. Researchers reveal their interpretations as they create conceptual tools and learning situations for teachers and students, and also as they describe and predict teachers' and students' behavior in increasingly complex mathematics teaching and learning environment.
Tier 2. **The Teacher Level**	Teachers through the study, summer workshops, and professional development seminars learn and design shared conceptual tools (activities, hands-on manipulations with physical models, technology based illustrations, assessment instruments, etc.) in order to help middle school students to develop advanced Calculus concepts. As teachers describe and predict students' behaviors, they construct and refine models to make sense of students' learning activities.
Tier 1. **The Student Level**	Students work on a series of conceptual model-eliciting activities/projects in which the major goals include further refining of models that reveal how students are interpreting and learning advanced Calculus concepts.

We plan to start first pilot experiment of teaching "Visual Calculus" supplementary course at middle school in fall 2002. The university content, method, and pedagogy classes for 4-8 concentration pre-service mathematics teachers will be team taught in a local middle school. The university students will participate in visual calculus projects and then be responsible for teaching the same projects to the middle school students. Lesson study method and video analysis are going to be major tools for qualitative assessment of teaching behaviors. It provides formative evaluative feedback, which guides teachers in their conceptualization of effective teaching practices. A sample of teachers participating in the project will be videotaped throughout their progress in teaching "Visual Calculus" course. In addition, paired problem solving interviews will supplement the documentation and assessment of the teacher's understanding of content, methodology, and pedagogy (different patterns of interaction).

Middle Grades Teachers' Perceptions of Learning and Teaching Calculus

Preliminary outcomes of the pilot multi-tiered teaching experiment show positive changes in teachers' perception of the early development of advanced math concepts as well as their readiness to teach Calculus concepts in middle grades. After completing summer and fall 2001 "Visual Calculus" professional development seminar, we asked teachers to evaluate each given statement below (table 2) based on the following scale:

1 - "Strongly Disagree",
2 - "Disagree",
3 - "Neither Agree nor Disagree",
4 - "Agree",
5 - "Strongly Agree".

Table 2. Middle grades teachers' perceptions of early development of Calculus concepts

##	Statement	1	2	3	4	5
1.	I had no Calculus experience before the "Visual Calculus" seminar	31%	23%	0	38%	8%
2.	Before the "Visual Calculus" seminar my overall attitude toward learning and teaching of Calculus was negative	15%	31%	23%	23%	8%
3.	Emphasis on procedures helps students to understand advanced Calculus ideas	15%	31%	15%	23%	15%
4.	Visualization is an effective approach in learning Calculus concepts	0	0	0	46%	46%
5.	Learning Multivariable Calculus concepts first helps me to better understand Single variable concepts	0	0	46%	38%	15%
6.	It is possible to develop students' Calculus concepts early in the middle school	0	0	0	54%	46%
7.	Principle "Conceptual leads procedural" underlines the main distinction between traditional and innovative way of teaching and learning Calculus	0	0	0	62%	38%
8.	Graphing skills play an important role in conceptual learning of Calculus	0	0	8%	38%	46%
9.	Discussion and reflection on micro-teaching of Calculus activities help me to understand how kids learn Calculus concepts	0	0	0	46%	46%
10.	Local landscape (mountains) and real life applications are good sources to introduce Calculus concepts to middle school students	0	0	0	38%	62%

11.	My confidence in *learning* of Calculus after the seminar is low	69%	23%	8%	0	0
12.	My confidence in *teaching* of Calculus concepts after the seminar is low	38%	46%	0	15%	0
13.	My overall attitude to learning and teaching of Calculus concepts after the seminar is positive	0	0	0	46%	54%

The main indicator of teachers readiness to teach Calculus concepts to middle school students is the answer to the question #6 (table 2): all the participants of "Visual Calculus" professional development seminar believe that it is possible to develop students' Calculus concepts early in the middle school. Another promising indicator is that if at the beginning of the seminar 31% of participants had a negative attitude toward teaching and learning of Calculus, by the end of the seminar all of them had positive attitude.

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