TACIT KNOWLEDGE IN CURRICULAR GOALS IN MATHEMATICS

Cristina FRADE
Universidade Federal de Minas Gerais – UFMG
Prog. Pós-Graduação em Educação da FAE & Centro Pedagógico
Av. Antonio Carlos 6627 – 31270-901
Belo Horizonte – MG – Brasil
e-mail: cristina@coltec.ufmg.br

Oto BORGES[1]
Universidade Federal de Minas Gerais – UFMG
Prog. Pós-Graduação em Educação da FAE & Centro Pedagógico
e-mail: oto@coltec.ufmg.br

Key words*:* mathematical teaching – curricular guidelines – tacit knowledge – mathematical knowledge

ABSTRACT

In this paper the authors analyze the current curricular goals in mathematics as proposed for school levels K-7 to K-12 (ages 11 at 16) in different countries. Based on Paul Ernest's view of mathematical knowledge, the authors consider school-acquired mathematical knowledge as multidimensional, in the sense that it involves components from different domains: cognitive and social, beliefs and values. Furthermore, most of those components are of a mainly tacit nature. The authors present evidence to support that the goals identified in those curricula foster the learning of a mathematical knowledge that is mainly tacit in nature. On the other hand, they argue that the curricular guidelines for the teaching of mathematics lack the supports to handle the processes involved in the learning of any knowledge of that nature. Part of the current literature on the subject emphasizes that such knowledge can be learned although it cannot be taught in the traditional sense of the word *teach*, that is**,** by the teachers' publicly transmitting or stating their knowledge**.** The same literature, although not dealing specifically with the teaching of mathematics, suggests, for instance, that the act of teaching a knowledge that is mainly tacit is closely linked to the teacher's public actions in face of authentic questions. That is, when he is engaged in a situation which demands the use of his own tacit knowledge. The authors conclude by discussing some curricular implications for the teaching of mathematics, which result from those issues.

[1] Supportted by CNPq – Brazil

Introduction

In the last thirty years we have witnessed a growing movement of changes in the understanding of what mathematical knowledge is about. In order to understand such changes, one has to remember that traditional mathematical epistemology used to assume that mathematical knowledge could be described, on the whole, through a set of explicitly formulated sentences and thus regarded as *essentially explicit* (Ernest 1998a). Such a conception has influenced the teaching of mathematics for years in what concerns primarily the learning of the formal aspects of the systematization of that knowledge.

In contrast, there is a current tendency in the epistemology to regard mathematical knowledge as a social practice in its wide sense. This tendency is clearly seen in the present-time curricula where we find consensus about the need to fill in the gap between school-acquired mathematical knowledge and some of the practices and processes used by mathematicians to produce mathematics (Romberg 1992, Shoenfeld 1992, Winbourne and Watson 1998, Ernest 1998a, 1988b). Mathematical knowledge is then reshaped: besides the relative components to its justification, it includes other equally relevant components, which are, by nature, mainly tacit. That is, knowledge built on experience or action and which cannot be fully described by rules or words.

An analysis of the current curricular goals set for the teaching of mathematics based on Ernest's model of mathematical knowledge (1988b) shows that such goals foster the learning of a knowledge that is more tacit than explicit in nature. This holds true for students at several school levels and in different countries. However, those curricular guidelines lack the support to handle the process involved in the learning of that kind of mathematical knowledge.

With this in mind, the aim of this work is to promote a critical reflection on the implications of the curricula that result from those issues. To this end, this paper is organized in three sections. *In the first*, we digress on Ernest's model of mathematical knowledge (or, as he says, of mathematical learning). *In the second*, we present evidence to support that the current curricular goals set for the teaching of mathematics in different countries foster the learning of a mathematical knowledge that is mainly tacit in nature. This trend was shown in Frade and Borges (2001) in the discussion about a given level of teaching in the Brazilian case. *The conclusion* discusses some curricular implications arising from that trend.

1. Ernest's Model of Mathematical Knowledge

Following and expanding Philip Kitcher's view of mathematical knowledge, Ernest (1998b) regards mathematical knowledge as a social practice and describes such knowledge through a multidimensional model whose components are classified as either mainly explicit or mainly tacit. As we understand it, for Ernest, mainly explicit mathematical knowledge is the knowledge that can be taught through a propositional language, as for instance, the Pythagoras' theorem. Alternatively, mainly tacit mathematical knowledge is that which is built on experience or action and cannot be fully taught explicitly.

To Ernest, mainly explicit mathematical knowledge includes the knowledge of a set of:

1. Accepted propositions and statements **(PS)**
2. Accepted reasoning and proofs, including less formal ones **(RP)**

3. Problems and questions **(PQ)**.

As mainly tacit components he cites:

4. Knowledge of mathematical language and symbolism **(LS)**
5. Meta-mathematical views, that is, views of proof and definition, scope and structure of mathematics as a whole **(MV)**
6. Knowledge of a set of procedures, methods, techniques and strategies **(PMTS)**
7. Mathematical aesthetics and personal values regarding mathematics **(AV)**.

For Ernest, the word *knowledge* covers both theoretical and practical knowledge. In the case of mathematics, the latter corresponds to the use of mathematical knowledge. Secondly, the first two components – accepted propositions and statements and accepted reasoning and proofs – are mainly explicit since they are strictly related with warrants in mathematics. As long as they are kept under discussion within the mathematical community, problems and questions relevant to mathematicians are also mainly explicit.

Based on Wittgenstein's (1995) concept that a word is given meaning through its suitable use in a *language game* or in *forms of life*, on Polanyi's (1962) view that any propositional knowledge rests on the tacit knowledge of language, and others, Ernest (1999a) sets his argument according to which the fourth component – language and symbolism – is mainly tacit. To Ernest, meta-mathematics views constitute a tacit element of mathematical knowledge in the sense that the mathematicians acquired and built them up through the enculturation of the mathematics community. And this experience cannot be fully explicitly taught.

With respect to procedures, methods, techniques, strategies, he argues that although they are often applicable to new problems, they are used differently in different situations. Thus, he states that, "(…) while the applications of these procedures and strategies are explicit, the more general knowledge underpinning them normally is not" (1998b, 13). To Ernest, it is not the procedures, strategies and algorithms that are not explicit but that underlying general knowledge of how and when one uses them, for example.

The last component – aesthetics and values – transcends the meta-mathematics views and is mainly tacit as long as the feelings about the aesthetics and the beauty of mathematics are closely linked to personal beliefs and values, which are only partly articulated.

How we interpret Ernest's model and on what it can help us with relation to the aims of that work

First of all, let us interpret Ernest's model as compared to some aspects of Polanyi's (1983) theory on tacit knowledge. Among the various types of knowledge used to support the task of teaching mathematics are Ernest's mainly explicit and mainly tacit components. This understanding gives us a clear example of how mainly explicit mathematical knowledge, as for instance, the Pythagoras' theorem, may become tacit in Polanyi's sense. As we understand it, when we use certain knowledge as subsidiary to another, the former is mobilized as tacit knowledge. In our case, it means that while the Pythagoras' theorem is being used as a tool to solve a problem, that specific knowledge is not explicitly shown (at that moment we may not even be aware of holding such knowledge) as it is not our focus of attention. Thus, what is taken as tacit knowledge depends on the context of situation.

On the other hand, Ernest's use of the expressions *mainly explicit* or *mainly tacit* implies an attempt to stress these two dimensions as complementary to one and only knowledge. Let us think, for example, about this as represented in a scale where the extremes could be one the totally inarticulate and the other the totally articulate knowledge. In such a scale the components of the mathematical knowledge are either close to one extreme or to the other but never reach any of them. Besides, the position of one component in the scale is directly related to its learning: the closer the component is to an extreme, the easier or the more difficult to reach it through a propositional language, depending on what extreme the component is close to.

It is our understanding that Polanyi's *fragmentary clues* - which allow for the identification of the particulars of a given tacit knowledge someone is trying to communicate - can be more or less meaningful, more easily learned or not, depending on how that someone is handling either mainly explicit or mainly tacit mathematical components. In sum, (1) tacit mathematical knowledge is any type of mathematical knowledge (such as, the mainly explicit components and the mainly tacit components of Ernest's model) used as subsidiary to the performance and control of a mathematical task. (2) If a certain type of mainly tacit mathematical knowledge (in Ernest's sense) is used as subsidiary by a first person, the fragmentary clues that allow for a second person to identify them will demand great effort from the second person to apprehend and integrate them.

Secondly, as long as it does not embrace the cognitive/psychological processes involved in mathematical learning, Ernest's model is more closely related to an ontological than to an epistemological model. However, that model helps us understand the kinds of mathematical knowledge, as for instance, concepts, procedures and attitudes or dispositions, which are currently enhanced in mathematics curricula. In fact, according to Ernest, mathematical knowledge is not a single block of knowledge pertaining to a single domain; it aggregates multiple faces or multiple domains: cognitive and social domains, beliefs and values. Furthermore, most of those components are, by nature, mainly tacit. This means that part of mathematical knowledge can be taught through the transmission of propositional knowledge, but most of it cannot. Only in this sense do we understand Ernest's statement that his model is also able to describe the process of learning mathematics. Finally, we cannot forget that Ernest's model describes a knowledge that is mainly tacit. Thus, it must be considered with all the limitations that result from the attempt to explicit any knowledge of that nature.

One should remember that, according to Polanyi (1983), when one tries to describe a tacit knowledge through the closely scrutiny of its particulars or explicit the relation between them, the meanings of that knowledge are effaced and their original meaning cannot be recovered.

2. Tacit Components of Mathematical Knowledge in Current Curricular Goals

In this section we analyze some current curricular goals for the teaching of mathematics in the light of Ernest's model. The aim of the analysis is to present evidence to support the statement that, in different countries, those goals foster the learning of those components of mathematical knowledge that are mainly tacit in nature.

To this end we analyze the Attainment Target 1 – Using and Applying Mathematics – for Key Stages 3 and 4. The material is suggested by The National Curriculum for Math of the United Kingdom (Appendix). Our choice to analyze Target 1 was based on the belief that it expresses the general goals for the teaching of mathematics in what refers to delimiting the context in which the other targets – Number and Algebra; Shape, Space and Measures; Handling Data – are to be developed.

Although we understand that any sub-target of Target 1 can embrace others, if not all components of Ernest’s model, from the analysis of each sub-target we identify the *dominant* components of the model. These must then be constructed in order to reach or to accomplish those sub-targets. At the end of the analysis of Target 1 we obtained the identification represented in Table 1.

Table 1 – Dominant components of Ernest’s model identified in the curricular goals in United Kingdom

Target 1 (key stages 3 and 4) - Using and applying mathematics		**Components**	**Nature**
1	**Puplis should be given opportunities to:**	PS	ME
a)	use and apply mathematics in practical tasks, in real-life problems and within mathematics itself;	PMTS	MT
b)	work on problems that pose a challenge;	AV	MT
c)	encounter and consider different lines of mathematical argument.	MV	MT
		AV	MT
2	**Making and monitoring decisions to solve problems**	AV	MT
a)	find ways of overcoming difficulties that arise; develop and use their own strategies;	PMTS	MT
b)	select, trial and evaluate a variety of possible approaches; identify what further information may be required in order to pursue a particular line of enquiry; break complex problems into a series of tasks;	PMTS	MT
		MV	MT
c)	select and organize mathematics and resources; extend their view and reflect on alternative approaches of their own;	PMTS	MT
		MV	MT
		AV	MT
d)	review progress whilst engaging in work, and check and evaluate solutions.	AV	MT
		MV	MT
3	**Communicating mathematically**		
a)	understand and use mathematical language and notation;	LS	MT
b)	use mathematical forms of communication, including diagrams, tables, graphs and computer print-outs;	LS	MT
c)	present work clearly, using diagrams, graphs and symbols appropriately, to convey meaning;	LS	MT
d)	interpret mathematics presented in a variety of forms; evaluate forms of presentation;	MV	MT
e)	examine critically, improve and justify their choice of mathematical presentation.	MV	MT
		RP	ME
4	**Developing skills of mathematical reasoning**	LS	MT
a)	explain and justify how they arrived at a conclusion or solution to a problem;	RP	ME
b)	make conjectures and hypotheses, designing methods to test them, and analyzing results to see whether they are valid;	AV	MT
		PMTS	MT
		MV	MT
c)	understand general statements, leading to making and testing generalizations; recognize particular examples, and appreciate the difference between mathematical explanation and experimental evidence;	LS	MT
		PMTS	MT
		MV	MT
d)	appreciate and use ‘if…then…’ lines of argument in number, algebra and geometry, and draw inferences from statistics;	AV	MT
		RP	ME
		PMTS	MT
e)	use mathematical reasoning, initially when explaining, and then when following a line of argument, recognizing inconsistencies.	RP	ME
		MV	MT

Key: **Components** – Dominant components of Ernest’s model; **Nature** – Nature of the components; ME – Mainly Explicit; MT – Mainly Tacit

In the case of sub-target 4 – Developing skills of mathematical reasoning – the identification corresponding to the letter b, for example, results from our interpretation that the action "make conjectures and hypotheses" is closely connected to a favorable disposition to inquire or pose questions. Such disposition originates from personal experience, beliefs and values about mathematics. On the other hand, "designing methods to test them" involves not only the close observation of specific cases to unveil regularities but also the knowledge of accepted mathematical ways to test hypotheses and results. Finally, "analyzing results to see whether they are valid" demands, among other actions, connecting the old and the new and developing a way of thinking based on evidence or argumentation. Such an action demands a type of knowledge that is constructed through a slow process of enculturation and some understanding of how mathematics works in the context that the results are being analyzed. Such identification exemplifies how the process of analyzing the curricular goals was constructed in this work.

We then find not a single and precise identification of the sub-targets and the components of Ernest's model but a combination of the dominant components involved in each sub-target.

On close inspection it was possible to see the prevalence of the mainly tacit components encountered in Target 1, in particular, the more elusive and slower components to acquire: meta-mathematics views and aesthetics and values. Those are the ones that shape our mathematical way of thinking more deeply as they are, to a great extent, stable in time. This prevalence can be found in the curricular goals proposed in other countries, such as Germany (Table 2), Brazil (Table 3) and Portugal (Table 4). (Refer to the Appendix for documentation)

Table 2 - Dominant components of Ernest's model identified in the curricular goals in Germany

General goals of mathematics teaching (general education)	Components	Nature
• mathematics as a theory and as a tool for solving problems in natural and social sciences, including modelling;	MV	MT
	PS	ME
	RP	ME
	LS	MT
	PMTS	MT
• experiences with fundamental ideas in mathematics like the idea of generalization, the need for proving, structural aspects, algorithms, the idea of infinity, and deterministic versus stochastic thinking;	MV	MT
	PMTS	MT
• methods of getting insights like inductive and deductive reasoning, methods for proving, axiomatic, formalization, generalization/specification, heuristic work;	RP	ME
	LS	MT
	PMTS	MT
• variation of argumentation levels and representation levels in all fields and aspects of mathematics teaching;	MV	MT
	LS	MT
• historical aspects of mathematics.	AV	MT

Table 3 - Dominant components of Ernest's model identified in the curricular goals in Brazil

General goals of mathematics teaching (third and fourth cycles of elementary school)	Components	Nature
• identify mathematical knowledge as a means to understand and transform the learner's surrounding world; understand that mathematics is an intellectual game, and as such, a trigger to promote interest, curiosity, investigative mind, and develop the ability to solve problems;	MV AV	MT MT
• use mathematical knowledge (arithmetic, geometric, metric, algebraic, statistic, arrangement, probabilistic) to make systematic observations about the quantitative and qualitative aspects of the real world aiming at establishing relations between those aspects.;	PS LS MV	ME MT MT
• select, organize and produce relevant information to be interpreted and evaluated critically;	LS MV	MT MT
• solve problems, validate strategies and results, develop various forms of reasoning and processes such as intuition, induction, deduction, analogy, valuation. Use mathematical concepts and procedures and every technological tools available;	MV RP PMTS	MT ME MT
• establish mathematical communication, that is, describe, represent and show results accurately, argue in favor of learner's own conjectures, making use of speech and establishing the relations between speech and various mathematical representations;	LS RP MV	MT ME MT
• establish connections between mathematical subjects from distinct fields and between those subjects and the knowledge of other fields of the curricula;	MV	MT
• feel capable to construct mathematical knowledge, develop self-esteem and persist in the search of solutions;	AV	MT
• interact cooperatively with peers, working collectively in search of solutions for the problems posed. Identify common sense about the subjects discussed and be respectful of peer's viewpoints while learning from them.	AV	MT

Table 4 - Dominant components of Ernest's model identified in the curricular goals in Portugal

Mathematical competence at basic education integrates attitudes, skills and knowledge, and includes:	Components	Nature
• the disposition and capacity to think mathematically, this is, to explore problematic situations, search for patterns, formulate and test conjectures, make generalizations, think logically;	AV MV	MT MT
• the pleasure and self-confidence in developing intellectual activities involving mathematical reasoning and the conception that the validity of a statement is related to the consistence of the logical argumentation rather than to some external authority;	AV MV	MT MT
• the capacity to discuss with others and communicate mathematical thoughts through the use of both written and oral language adequate to the situation;	LS AV	MT MT
• the understanding of notions such as conjecture, theorem and proof, as well as the capacity to examine the consequences of the use of different definitions;	MV RP	MT ME
• the disposition to try to understand the structure of a problem and the capacity to develop problem solving process, analyze errors and try alternative strategies;	LS PMTS MV AV	MT MT MT MT
• the capacity to decide about the plausibility of a result and to use, according to the situation, mental computational process, written algorithms or technological devices;	MV PMTS	MT MT
• the tendency to "see" the abstract structure underlying a situation, from daily life, nature or art, involving either numerical or geometrical elements or both.	MV AV	MT MT

A similar pattern can be found in the curricular goals proposed for the teaching of mathematics in the USA, Spain and Canada where the curricula undergone similar changes in the 1990's. (Refer to the Appendix for documentation).

3. Discussion

In face of the changes in the conception of the epistemology of mathematical learning, many scholars (Schoenfeld 1992, Romberg 1992, Winbourne and Watson 1998, among others) have been putting emphasis on the importance of creating learning environments where teachers and students would be involved in actual mathematical experience. On account of that, we can say that Ernest's model re-signifies mathematical learning when it characterizes it as being mainly tacit. In other words, such an approach tells us that a great deal of mathematical knowledge cannot be either taught or learned by means of explicit transmission.

Although schools have incorporated a discourse in favor of the actions advocated by current mathematics curricula, such a discourse has not been given actual support as the practice keeps treating the teaching and learning of mathematical knowledge as mainly explicit (refer to Romberg, 2001). The reason may be that the curricular guidelines for the teaching of mathematics lack the support to handle the processes involved in the learning of a knowledge that is mainly tacit, as the quotes below suggest:

> My third observation is related to the concept of competence and, in the case of mathematics, the definition of mathematical competence. Doubts and criticism on the presented proposal showed that a broad concept is difficult to be widely accepted. Terms like *disposition* (to think mathematically), *pleasure* (in developing intellectual activities) or *tendency* (to look for the abstract structure) have been especially criticized with the argument that is very difficult to make such things "operational". (Abrantes 2001, 35)

> It is said that thinking mathematically and developing mathematical skills through learning mathematical content is important. However, the meaning of mathematical way of viewing and thinking is interpreted in several ways among university mathematics educators. Among schoolteachers there is some confusion about the meaning. (Kunimune and Nagasaki 2001, 2)

Approaching the subject in the light of the literature on tacit knowledge will only help us understand how to teach and how to learn the various types of school-acquired mathematical knowledge that are being valued at present, for example, those commonly labeled *mathematical competencies* in some countries. We understand that Polanyi (1983) and Schön (1987) imply Ernest's sense of mainly tacit knowledge when both stress that tacit knowledge can be learned. However, they claim that it cannot be taught in the traditional sense of *teaching*, that is, by means of stating the knowledge the teacher holds or by making it explicit.

When evaluating the teaching of architectonic design, Schön suggests that the act of teaching tacit knowledge is closely connected to the teacher's public actions in face of authentic questions, that is, as he is involved in a situation that demands the use of his own tacit knowledge. That means, for example, that the teacher's act of doing standard exercises and solving problems on the board, which does not actually challenge him, does not correspond to that type of practice. As for the learning of a disposition to think mathematically, what we interpret Schön suggests is that the students should be exposed to a number of experiences that would allow them to *see* their teacher *think mathematically*.

More generally, according to Polanyi, a person can learn or know about a second person's tacit knowledge through the apprehension of some of its particulars, which are provided by fragmentary clues, and a great effort to understand the meanings of those few apprehended features. On the other hand, for the latter to be able to communicate the features of his tacit knowledge to the former, it is necessary to provide him with suitable means to express it. Thus, Polanyi says that both the communication and the integration of the particulars of a tacit knowledge occur through their meanings. As we see it, mathematical teaching demands, among other things, a great effort from the teacher to develop a sensibility to apprehend the fragmentary clues provided by the students and observe how they become manifest when students mobilize mainly explicit and mainly tacit components of mathematical knowledge.

From all this it is possible to foresee the consequences of a curricular tendency to value the tacit components of mathematical knowledge in the teaching practice. There is however one aspect yet not stressed but of equal relevance and equal consequences for the teaching of mathematics. We refer to the understanding that most of the assessment practices to evaluate mathematical learning in school are based on the assumption that mathematical knowledge is either of a fully explicit nature or possible to be made explicit in its entirety (refer to Romberg, 2001). As such, it is clear that such practices are potentially inadequate as evaluative of a curriculum that stresses the tacit components of mathematical knowledge.

Teachers can use their previous experience with students' evaluation to understand the difficulty the learner finds in apprehending the tacit components of mathematical knowledge. It bears the same nature - and probably similar or higher intensity - of the difficulty that they have when trying to apprehend the tacit knowledge of their students.

Understanding mathematical knowledge as proposed by Ernest's and finding the most adequate way of evaluating students' development demand the teacher's commitment with both the use of new assessment tools and the awakening and tuning of his own sensibility to the new trend. For the curricular trend here discussed to become effective, it is necessary that the nature, the curricula and the teaching and learning processes that characterize the basic qualification required for teachers be fully reviewed. Those changes must place the teacher's formative process in tune with the curricular goals that value the tacit components of mathematical knowledge. All the same, they must aim at the adequate qualification of the reflective teacher.

Appendix: Documents used in section II.

Canada
Mathematics - The Ontario Curriculum
http://www.edu.gov.on.ca/eng/document/curricul/curr97ma/curr97m.html (12/16/01)

Brazil
1998. *Parâmetros Curriculares Nacionais - Terceiro e Quarto Ciclos do Ensino Fundamental - Matemática*. Brasília: MEC/SEF.

Germany
Kaiser, Gabriele 2001. A Description from Germany. *Proceedings of PME25* 1: 164- 169

Weidig, Ingo. 2 Mathematics teaching in Germany.
http://www.mathematik.uni-wuerzburg.de/History/meg/weidiga2.html (12/16/2001)

Portugal
Abrantes, Paulo 2001. Revisiting the Goals and the Nature of Mathematics For All in the Context of a national Curriculum. *Proceedings of PME25* 1: 25- 40

Spain
1989. *Diseño Curricular Base - Educación Secundaria Obligatoria*. Ministerio de Educacion y Ciencia. Vol. 1: Capitulo 2, 478-549

United Kingdom
The National Curriculum For Maths
http://www.dfee.gov.uk/nc/matks34.html (11/16/1998)

United States
2000. Principles and Standards for School Mathematics. Reston (VA): The National Council of Teachers of Mathematics, Inc.

REFERENCES

Abrantes, Paulo 2001. Revisiting the Goals and the Nature of Mathematics For All in the Context of a narional Curriculum. *Proceedings of PME25* 1: 25- 40

Ernest, Paul 1998a. *Social Constructivism as a Philosophy of Mathematics*. Albany: SUNY.

_____ 1998b. Mathematical Knowledge and Context. *Situated Cognition and the Learning of Mathematics.* Edited by Anne Watson. Oxford: Centre for Mathematics Education Research, University of Oxford. Chapter 1, 13-29.

Frade, Cristina and Borges, Oto 2001. Componentes Tácitos e Explícitos do Conhecimento Matemático nas Orientações Curriculares para o Ensino de Matemática. *Anais 2001 Anped.*

Kunimune, Susumu and Nagasaki, Eizo. Curriculum Changes on Lower Secondary School Mathematics of Japan – Focused on Geometry.

http://www.fi.ruu.nl/en/Icme-8/WG13_6.html (12/16/01)

Polanyi, Michael 1962. *Personal Knowledge*. London: Routledge & Kegan Paul.

_____ 1983. *The Tacit Dimension.* Gloucester (Mass): Peter Smith.

Romberg, Thomas A. 1992. Problematic Features of the School Mathematics Curriculum. *Handbook for Research on Curriculum*. Edited by Philip W. Jackson. New York: MacMillan. Chapter 27, 749-788.

_____ 2001. Mathematics Goals and Achievement in the United States. *Proceedings of PME25* 1: 180-185

Schoenfeld, Alan H. 1992. Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics. *Handbook for Research on Mathematics Teaching and Learning*. Edited by D. Grouws. New York: MacMillan. Chapter 15, 334-370.

Schön, Donald A. 1987. *Educating the Reflective Practitioner.* San Francisco: Jossey-Bass.

Winbourne, Peter and Watson, Anne 1998. Participating in Learning Mathematics through Shared Local Practices in Classrooms. *Situated Cognition and the Learning of Mathematics.* Edited by Anne Watson. Oxford: Centre for Mathematics Education Research, University of Oxford. Chapter 7, 93-104.

Wittgenstein, Ludwig 1995. *Tratado Lógico-Filosófico. Investigações Filosóficas*. Tradução de M.S. Loureiro. 2a edição revista. Lisboa: Fundação Calouste Gulbenkian.