TEACHERS ESTIMATE THE ARITHMETIC SKILLS OF THEIR STUDENTS WHEN THEY ENTER THE FIRST GRADE OF PRIMARY SCHOOL

Charalambos LEMONIDIS
Aristotle University of Thessaloniki
Pedagogical School of Florina
Florina 53100, Greece.
e-mail: lemonidi@eled-fl.auth.gr

Athanasios DIAMANTIS
Aristotle University of Thessaloniki
Pedagogical School of Florina
Florina 53100, Greece.
e-mail: trianev@otenet .gr

Evanthia TRIANTAFILLIDOU
Aristotle University of Thessaloniki
Pedagogical School of Florina
Florina 53100, Greece.
e-mail: trianev@otenet .gr

ABSTRACT

Nowadays, in the area of contemporary teaching, the role of the teacher in the class has changed. Teaching is based on students and a great emphasis is given to the communication between teachers and students; the preliminary knowledge of children is taken under serious consideration for the establishment of a new knowledge. Teachers have to know this pre- established knowledge and estimate the abilities of their students, so that they will be able to organize their teaching according to this knowledge.

In the present paper we investigated the predictions and estimations of Greek teachers about the arithmetic skills of their students when they enter the first grade of Primary School. In the first stage of the research, teachers were interviewed and asked to estimate their students' abilities in enumeration, addition and subtraction, writing numbers and solving problems. In the second stage, teachers themselves tested their students, one after the other, in the above - mentioned processes. After completing this test and gathering all the answers, teachers were interviewed again and this time they were asked to evaluate their assumptions about children's knowledge.

The final results of our research show that the teachers' predictions about their students' mathematic abilities, in some cases are away from reality. For example, teachers underestimate their students' abilities in writing numbers, solving simple problems of addition and subtraction, etc. It seems that this perception is enforced by the instructions of the Greek analytical program, which ignores what students already know before they enter school.

Introduction.

During the last years many researches have taken place about teachers' theories and how these theories affect their teaching. One of the most important representations of teachers is relevant to the way by which students elaborate the information they get and learn. However the majority of teaching theories are not specific and they usually refer to educational systems and groups of students. Although it is important for teachers to know these learning theories, it is not obvious to the most of them that they need them in their every day teaching. Late researches on learning theories have focused on the cognitive dimension of learning and have shown new ways of examining students' knowledge and using this knowledge in the teaching process.

Researchers have deliberated on students' mathematical abilities and their knowledge about addition and subtraction (Carpenter, Fennema, Franke, Levi, Empson, 1999). For the purposes of the CGI program (Cognitively Guided Instruction) a number of researches have taken place in order to be examined whether the knowledge of the research findings in addition and subtraction can affect teaching decisions. The results of the research taken place in 1998 (by Carpenter, Fennema, Peterson & Carey) show that teachers are not aware of the strategies that students use when they solve a problem and they do not distinguish the different kind of problems. Their knowledge is organized in a way that does not allow them to understand their students' way of thinking.

In another research taken place in 1989 (by Carpenter, Fennema, Peterson, Chiang & Loef.) the researchers compared two groups of teachers; at first students were asked to solve problems and teachers were asked to predict how the students would solve these problems. In the end it was proved that teachers from the experimental group who knew more about their students' solving strategies had used this knowledge in every day teaching and their students were more capable in solving problems than others. Furthermore, the students from the experimental class had developed their metacognitive abilities the relevant to the understanding of solving strategies.

In our research in 1995 (Lemonidis), we examined teachers' theories about Mathematics and asked their opinions about the best way of teaching Mathematics. According to what they said it seems that during their studies teachers do not take enough special courses in teaching Mathematics. In a later research (Lemonidis 2001) we tested arithmetic skills of students at the first grade of Elementary school. Taking under consideration the findings of this research we decided to proceed a new one.

Firstly, we interviewed teachers asking them to predict their students' arithmetic abilities (for example, in enumeration, addition and subtraction). After that, teachers themselves examined their students' mathematical knowledge. In the end, teachers were interviewed once more and this time were asked to evaluate their predictions comparing them to the answers given by students.

Methodology of research.

Our research, in which 72 students from five classes of the first grade of three schools took part, was taken place in Karditsa. The first school was placed in the center of the city, the second one in the suburbs and the third one in a small town of the prefecture of Karditsa.

1ˢᵗ stage: Prediction interview

As we have mentioned before, first we asked teachers to appraise their students' abilities in mathematical tests. We chose the method of *half-structured interview* that allowed us to use a small general questionnaire adjusting questions according to teachers' answers.

2nd stage: Students examination.

After having completed interviews of the first stage we asked teachers to start examining their students. Throughout the examination we were writing down all the answers concentrating on students' solving strategies.

Students were examined one after the other: 1) in enumeration of objects, 2) in enumeration of dots and in writing the correct numbers, 3) in solving problems, 4) in addition and subtraction. In enumeration and in problems students could use small cubes; in the second test students were asked to find and write down the correct number of dots drawn on separate cards.

3rd stage: Evaluation interview.

Finally teachers were asked to comment on students' performance and evaluate their predictions.

Findings of research.

I. Enumeration.

Every teacher asked her students to enumerate three different collections: one of 5 cubes, a second one of 12 cubes and a third one of 20 cubes. The next table shows the percentages of success:

Table 1. Success in enumeration.

	Enumeration of 5 objects	Enumeration of 12 objects	Enumeration of 20 objects
Total Success N= 72	69 (95,8%)	42 (58,3%)	28 (38,9%)

The results of the first test showed that all teachers had predicted correctly; the majority of their students succeeded in the enumeration of the five objects. Although one of the teachers had underestimated her students' abilities, a percentage of 85,7 percent of her students succeeded to enumerate the collection of 12 and a percentage of 78,6 managed to enumerate the collection of 20 objects. On the other hand, one of the teachers had overestimated her students' abilities; she had stated that only two children could not count the collection of 12 objects, while a percentage of 50 percent failed to give correct answers; she had also mentioned in the first interview that the most of the children would succeed in counting the collection of 20 objects, but only two of her students gave correct answers to this test (12%).

II. Counting dots and writing numbers.

In this examination teachers gave children cards with big black dots asking them to count the dots and write down the correct number of each card. Students were given:
  a. One card of three dots (in a diagonal order)
  b. One card of four dots (in line in a horizontal order)
  c. One card of five dots (dots were in line in a horizontal order)
  d. One card of six dots (pairs of dots in a horizontal order).
The following table shows the results of this test.

Table 2*. Success in counting dots and writing the correct number.*

	3 dots	4 dots	5 dots	6 dots	
Success in Counting.	72 (100%)	72 (100%)	71 (98,6%)	53 (73,6%)	
Success in Writing numbers.	72 (100%)	72 (100%)	71 (98,6%)	70 (97,2%)	53 (73,6%)

On this subject teachers had predicted that most of the children would be able to give correct answers. The main procedure that students used in order to find out the correct number of dots in each card was enumeration. It is worth mentioned that for the card of three dots almost 50 percent of students gave their answers subitizing while all the rest followed the procedure of enumeration. In the first interview teachers had reported that students would follow the procedure of enumeration. Only two of them had mentioned the possibility that some students could give correct answers without counting (subitizing).

Writing the correct numbers was the second part of this examination. As we can see in the category of writing number ''6'' there are two columns: the first one refers to children who didn't find the correct number (6), but they wrote correctly the number they found. The second column refers to the percentage of children who not only found, but also wrote the correct number. As we can also see at table 2, all children wrote correctly numbers ''4'' and ''5''. Regarding to the writing of number ''3'' two children identified and tried to write the number, but they made the known *mirror mistake* (they wrote ''ε'' instead of ''3''), something that teachers had mentioned that it might happen. However these answers were listed as correct. It is quotable to cite here an extract of an interview, when a teacher talks about the ability of children to write numbers correctly:

Teacher: *"...Children count, but they can not write before they go to school. There are exceptions of course...for example, in a class of 14 children only two or three children can write".*

Generally teachers agreed that children would face difficulties in writing numbers and most of them stated that students wouldn't write correctly all the numbers. However, table 2 shows that children achieved better results than those that teachers expected.

III. Solving problems

Every teacher was reading aloud the problems to her students, who had objects in front of them (small cubes) in case they wanted to use them to solve the problems. The first problem was of the kind " part - part - all" and the final whole was asked. In the second problem - where subtraction was needed - children had to find the result of the transformation:

- Mary has four balloons. Kiki has two balloons. How many balloons do they have together?
- Helen had five candies. She made her sister a present of three candies. How many candies have left for her?

In problems teachers could use other names familiar to students (for example, names of schoolmates), but they had to maintain the structure of each problem. The percentage of success in solving problems is presented at table 3. The same table shows the procedures that most of the students followed in this test. Percentages of the chosen procedures include both correct and wrong answers of students.

	Success	Known fact	sequence counting	Counting all with fingers	Counting all with objects
Problem of Addition.	52 (72,2%)	3 (4,2%)	1 (1,4%)	8 (11,1%)	51 (70,8%)
Problem of Subtraction.	33 (45,8%)	2 (2,8%)	–	9 (12,5%)	32 (44,4%)

In their first interview four of five teachers declared that students could not solve problems. They strongly believed that they had to direct children step by step to the solution of each problem. Only one of the teachers supported that students could easily handle problems of addition and subtraction. The same teacher reported that young children could answer almost immediately in these problems, because it is easier for them to find the solution of a problem (with realistic information) than it is to find the answer to the question: "How much is 4+2?"

Teacher: *"Yes, immediately. The problem is easier. If you ask children to find the solution of the addition 2+3 they will probably use their fingers. I noticed that when they want to solve a problem they calculate more easily".*

IV. Addition and subtraction.

In addition and subtraction every teacher was reading aloud the exercises to children. For example: "I want you to tell me how much is two plus two (2+2), two plus one (2+1)…etc". Students were asked to solve five additions (2+2, 2+1, 3+2, 3+3 and 4+4) and three subtractions (4-2, 5-3 and 6-4). In this particular test children didn't have any objects in front of them. Their teachers encouraged some children to think carefully and use their fingers to find the solution. Table 4 shows the results of this test. Again, the percentages of the chosen procedures refer both to the correct and wrong answers:

Table 4. Percentages of success in additions and subtractions and percentages of procedures.

	2+2	2+1	3+2	3+3	4+4	4-2	5-3	6-4
Success	64 (88,9%)	65 (90,3%)	46 (63,9%)	33 (45,8%)	25 (34,7%)	32 (44,4%)	23 (31,9%)	17 (23,6%)
Known fact	47 (65,3%)	44 (61,1%)	9 (12,5%)	21 (29,2%)	8 (11,1%)	11 (15,3%)	5 (6,9%)	5 (6,9%)
Counting with fingers	2 (2,8%)	2 (2,8%)	5 (6,9%)	5 (6,9%)	3 (4,2%)	3 (4,2%)	9 (12,5%)	5 (6,9%)
Counting all with fingers	17 (23,6%)	20 (27,8%)	41 (56,9%)	25 (34,7%)	36 (50%)	24 (33,3%)	21 (29,2%)	23 (31,9%)

As it was proved teachers couldn't estimate their students' abilities in addition and subtraction:

Researcher*: "What about addition and subtraction? Do you think that they can find how much is 2 plus 2?"*

Teacher*: "No way, they do not even know the numbers 2 and 4."*

Researcher*: "What about 2 plus 1?"*

Teacher: *"No, no…perhaps they know the number (1), if they have seen it somewhere, but they can not calculate with addition or subtraction I do not think so."*

One of the other three teachers said that the majority of children would face difficulties in calculations with addition and subtraction. She mentioned that children would use their fingers to find the solution, because they couldn't calculate without using their fingers. One teacher predicted almost correctly the success in calculations with small numbers and the failure in bigger numbers.

A teacher who said that children would not have a specific difficulty in calculations gave the more optimistic prediction. She reported that 80% of the students would solve correctly the additions and the subtractions, something that was confirmed by the results of the test. The same teacher also reported that students usually remember or learn easily the sums 2+2, 3+3, 4+4, (or else the "double sums").

Second interview: commenting the results

In the second interview, although teachers found out - comparing their predictions to the results - that in main points their predictions were wrong, they did not look surprised; they behaved as if they had predicted correctly. However, in this interview the attitudes of teachers were quite different. For example, one teacher who failed to the most of her predictions persisted in her opinions about her students' abilities, even after the opposite results of the test. When she was interviewed for a second time, among other things, she also said:

Teacher: *"According to the results I predicted correctly. When children come to the first grade, they are able to read something or write their name, but they are not able to use numbers".*

Generally, teachers who had underestimated their students' knowledge discussed the results with us, but they did not look surprised. In a very few cases they admitted that they did not expect these results:

Teacher: *"I did not expect that some children would go so well in the addition…"*
Teacher: *"…Most of them wrote number 6, something that I didn't expect."*

Finally, two teachers who had predicted correctly the performance of their students in some tests simply commented on the performance they did not expect. For example, one of them mentioned:

Teacher: *"From what we can see after the examination, children didn't face any difficulty at all in enumeration; almost 98% of children are able to count from 1 to 20 and some students are able to count over 20."*

This teacher in the first interview had estimated that only a small percentage would actually manage to enumerate 12 and 20 objects. The second teacher commented:

Teacher: *"I was sure that children couldn't find number 6 and most of them would not answer; I did not believe that they could count the dots and find the answer. To my surprise I realized that most of them did very well. I was also sure that that they would count in order to find the number 3; on the contrary most of them did not. They just saw the card of the three dots and answered automatically and correctly."*

From the second interview we can conclude that teachers who had predicted more accurately the performance of their students were more careful during the examination and as a result they evaluated better the abilities of their students.

Conclusions

As we have already seen, teachers' predictions were in many subjects far away from students' real arithmetic skills. Regarding to the enumeration of 5 objects teacher's predictions were verified. One teacher estimated that children were not capable of enumerating collections of 12 and 20 objects and another one overestimated her students' abilities.

Three of the five teachers did not expect that their students would answer immediately without counting the three dots (subitizing). In addition, most of the teachers had underestimated the ability of children to write digits. In the first interview they explained that their students could not be capable to write numbers, because they had never been taught how to do this.

On the subject of the problems four of the five teachers declared that their students could not solve simple problems of addition and subtraction. They had expressed their doubts about how students could ever solve problems since they had not even learned the numbers.

Generally, it seems that most of the teachers underestimate their students' arithmetic abilities and they are not familiar with all the procedures that students use to solve an exercise. They support that students not only need objects in order to calculate correctly, but they also need to be guided to the solution step by step.

From the second interview where teachers were asked to make comments on the performance of their students we concluded the following:

a) The teachers who failed to their predictions did not seem to understand the real abilities of their students and they insisted on their opinions about students' weakness in Mathematics.

b) On the other side, two teachers who predicted more accurately were more perceptive; they were interested in the performance of their students and were willing to compare their first predictions to the results; they commented on their students' performance and admitted that some of their predictions proved by the results wrong.

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