TRANSPOSITION OF DIDACTICAL KNOWLEDGE : THE CASE OF MATHEMATICS TEACHERS' EDUCATION

Isabelle BLOCH
IUFM d'Aquitaine
160 avenue de Verdun
33700 Merignac Cedex - FRANCE
isabelle.bloch@aquitaine.iufm.fr

ABSTRACT

The objects that the didactics of mathematics want to study are not exactly the same as a naïve or only vocational approach could identify as pertinent; didactic tools are an efficient way to analyse teaching situations, and anticipate new ways of learning, but are not always easy to communicate to future teachers, knowing that these students often get a very formal conception of mathematics during their university courses.
The aim of this work is to analyse situations that can be given to novice secondary teachers to help them understand the articulation between advanced mathematical notions and the contents of what they will be teaching themselves. Beyond that, it is to describe some principles of a didactical study of the instruction of trainee-teachers.

It leads to the use of a complex theoretical framework, which:
1) Identifies the didactical contract of the novice teacher;
2) Determines what kind of mathematical and didactical work can ensure the transition from the student's place to the teacher's place, and how this transition will become evident;
3) Uses transposition of the theory of situations due to Guy Brousseau, to build specific situations for young teachers, and to lay out the aims, the criteria and the constraints of such situations;
4) Questions the didactical knowledge on what is useful to drive complex situations in a classroom.
The organization of teachers' education at the IUFM d'Aquitaine is evoked, and examples of situations on the vectors and algebra are expounded.

Keywords: teachers' training, theory of situations, didactical contract of novice teachers, vectors, algebra.

This paper presents an outline of training methods for mathematics teachers, and some examples of the work we offer future teachers. This work has been initialised by the following questions that we are required to meet as educator in a training Institute with young teachers.

What are the conceptions of novice teachers on the mathematics to be taught? On teaching practice? How can we bring these conceptions to light ? What means are at our disposal to make them evolve?

Is theoretical didactical knowledge effective to make future teachers broaden their conceptions of mathematics? Which complex learning situations can be introduced in the preparation of young teachers? Are these situations the same ones as those useful to understand mathematics for oneself?

Which pedagogic knowledge is necessary to help teachers drive complex learning situations ? Does the preparation take care of this knowledge, or is it left to the teachers' own initiative?

This paper consists of four parts:

1) A presentation of the organization of the academic year;
2) The theoretical background we use to analyse the needs and the means of training;
3) Examples of situations (vectors and algebra);
4) Conclusion.

1. Alternate training and conceptions of young teachers

1.1. *Organization of the training*

Once they succeeded at the theoretical examination for teaching, whose content concerns only mathematical knowledge, French students are made responsible for teaching mathematics to a secondary school class, even if they have no experience in teaching, which is frequent. An older and more “expert” teacher has the responsibility for helping the trainee-teacher, and this one must learn from experience their elder's, by attending lessons in the tutor’s class, and performing a few lessons under his/her direction. During nine months, novice teachers must also follow some fifteen days of training in one of the twenty-four national training Institutes (IUFMs). During these days of training, trainers of the Institute try to bring young teachers’ conceptions of teaching to light and to make them evolve when desirable. The role of the training in the Institute is first to help new teachers to do their job: conceive and perform lessons of mathematics in front of pupils in a secondary school. But beyond that, the role of the trainers is to let the novice teacher build reflexive tools to analyse his/her practice and to improve it, assuming that "improving one's practice" has a clear meaning.

At the beginning of their careers, novice teachers can become aware of dysfunctions only by analysing the pupils' reactions: these can manifest in the form of inappropriate or quite unpleasant behaviour: noise, even the refusal to do the required work. So we see young teachers saying that pupils are lazy, because they do not want to do anything; but the teacher does not wonder whether the given work is interesting or not, or even if the pupils have any real possibility of doing it.

A lever to make the young teachers evolve is the degree of success they meet in their class, but it is difficult because it questions their practice in a very personal way especially if they don’t succeed; and because they see no reasons for changing, if they believe they will succeed in a way that the training institution does not consider very pertinent!

A second part of the training has to supply the teachers with mathematical knowledge to help them understand the articulation between advanced mathematical notions and the contents of what they are to teach. This second component leads to revisit some mathematical notions, but differently from the way it was taught at the University: the aim is to make teachers see what a notion means, that is, which problems it allows to solve. A large part of this component of the training is to study teaching organizations, as Thales' Theorem, and try to analyse why it is difficult (for example, that it supposes continuity of real numbers, or that an homothetic path has a length of k the length of the first one, implies to know something about rectification of paths…)

A third component of the training will be the structuring of didactical knowledge, as far as it is possible and useful to future teachers, that in itself is a question of research.

1. 2. *Conceptions of young teachers about mathematics to be taught and ways of teaching*

a) Mathematics

Students often get a very formal conception of mathematics during their university courses, and they are not at all accustomed to solve problems with the mathematics they know. For them, a theorem has to get a proof, but no justification in terms of problem solving; it is seen as a part of a mathematical theory, which is its own justification. They have a very poor culture of problems to be solved with the mathematical tools they have studied at University; and, as many authors have pointed, their own mathematical knowledge is often inefficient (Robert, 2001).

b) Ways of teaching

What are the novice teachers' conceptions of the mathematics to be taught and on teaching practice ? They still keep the illusion that "a good course" of mathematics is done by a teacher in front of the students, and that the teacher "tells the law", that is, the mathematical law. They have no idea that this law could be contested, and no idea that the mathematical law could not be understood, overall, considering that only elementary mathematics are in question at that level. The mathematical formalism seems transparent to them, it is as if it was self-explaining. This is to say that they themselves hardly ever question mathematics, they are accustomed to take what the mathematics teacher said at University for granted and cannot imagine any other behaviour from the students in their own classes.

When these conceptions are brought to light, how can we work with them? The teachers' expertise includes two components (at least) : one of them involves education skills, and is related to vocational habits ; and the other gets an epistemological dimension.

It leads to make the hypothesis that it is necessary to offer novice teachers, both analysis of teaching practice (theirs and the experienced teachers') ***and*** new situations for their students, to make them question the way they are accustomed to be taught themselves. And it is also necessary to provide them with vocational knowledge to drive the situations we propose.

2. Theoretical background

2.1. Didactical constraction of students versus young teachers

At University, the students are not responsible for the mathematics they are taught, either in their dimension of proof or in the global organization of the course; when they become teachers, they have succeeded, so they think that their mathematical studies are achieved, but they know very little about how the mathematics they have learned can be applied at the secondary school. It is therefore difficult for them to get a critical and reflexive point of view on this mathematics. So they receive a real subjection to the didactical transposition of the mathematical knowledge at the secondary level[1].

[1] And young teachers want to become member of the institution "Secondary School", so they tend to be much more conformist than they are expected to, it is a well-known phenomenon of integration.

We saw that young teachers often think that mathematics reduce to the formal point of view; but the didactical transposition in the secondary school emphasizes the pragmatic point of view: mathematics are reduced to manipulation of semiotic tools[2]. The dimension of problem solving is not considered neither by one nor the other institution (the secondary school or the University). It is then difficult for a young teacher to imagine problems relative to a mathematical concept, and all the more problems that can be attainable by students at the secondary level.

Then the didactical contract of young teachers is characterized by :

- an illusion that manipulation of ostensive objects carries sense of mathematical notions ;
- a lack of knowledge about pertinent problems related to concepts taught at secondary level;
- an absence of means to take the responsibility for the mathematical organization of a long course.

How does this contract appear ? We can observe it through the tasks that the students and the teachers consider as theirs. When they are at University, students try to solve roughly the problems at the exams; they do not consider themselves responsible for the exact solution and it is more "profitable" for them to solve more or less numerous questions, than to solve only a little question in detail. It derives from the University's habit: University makes students frequent the mathematics more as large components of theories than exactitude of a closely defined research.

At the opposite, in his/her class the teacher is responsible for the mathematical exactitude, in terms of what is right and what is false. When novice teachers arrive in a training Institute, they very often refuse to write a complete solution of the exercise they give their students: they do not see the use of this work, neither do they see why they should correct an exercise's text.

Similarly, it is difficult for them to anticipate the planning of a week, a month, a term of mathematics with their class. It is also problematic to provide a series of exercises at a given level, because this work does not concern mathematical concept in a usual way (define a new concept, fit it in a well known theory …), but it is a technical or technological work (Chevallard 1999), a new work : how to express *this* concept for *these* students ?

The didactical contract of the novice teachers must evolve to enable them to :

- organize the didactical time, on a short or long term, and define the objectives they want to reach ; control the schedule of the teaching/learning organization ;
- define the corpus of learning situations and exercises to offer to students, in order to study a mathematical notion, and to reach a given objective ;
- link how to teach and how pupils can learn, and give themselves means for assessment.

2.2. Theory of situations

The theory of situations has proposed some situations for primary school, but not so many for secondary school and college. At this level, the question is not to build ONE good situation (as for multiplication or proportionality - see Brousseau 1997) but to find collections of problems, activities for students that permit to explore the fundamental meanings of a mathematical concept. What we could expect from university knowledge was to enable young teachers to understand these fundamental meanings, but as we already said, young teachers do not know how to converse their formal knowledge into a problematic one.

[2] On ostensive objects and semiotic tools, see Chevallard 1999.

Another question is the adidactical component of a situation, or, in other terms, the place for the work of pupils or students in the mathematics class (Bloch 1999). One reason for building complex situations is to try and broaden the role of pupils in mathematical research, and their confrontation to mathematical truth through debate.

With young teachers we make the hypothesis that it is possible to play with such situations during the training time, and the objective is :

- to provide them with robust situations to apply in their classes (that is, situations that permit a real mathematical work for their pupils, even if the teacher is novice) ;
- to enable them to make their own mathematical knowledge evolve, thanks to the interactions with situations.

It is also necessary to make the teachers analyse class practice: tools of the theory of situations are also used to analyse the teachers' practice, but this work is not presented here.

So :

a) We have to build situations that can be submitted to the novice teachers to help them understand the articulation between advanced mathematical notions and the contents of what they will be teaching themselves; and research shows that the mathematical knowledge of teachers evolves when they have left University: it broadens in a way, but it is used in restrictive domains, considering the field of university knowledge. What we want to do is to guide this evolution.

b) We try to introduce complex learning situations in the instruction of young teachers in order to enable them to teach in these situations; and we make the hypothesis that these situations are useful to make their mathematical knowledge evolve.

Then we can see that situations for teachers' education are built under a double constraint:

- first, allow the young teachers to question and broaden their mathematical knowledge, by a confrontation with the situation ;
- and, retain components that could be transferable in a class situation, and could be managed by a novice teacher. That is why it is also necessary to anticipate teachers' regulations.

To build situations that are relative to a notion we apply principles from the theory of situations :

- identify a game where the concept is pertinent ;
- make the main didactical variables appear and choose their value ;
- organize the game in two phases : a direct one and an inverse one, the last one being the only one that leads pupils to confront their action to the "milieu".

These conditions will be explained below.

2.3. Didactical knowledge to drive complex situations

What teaching knowledge is necessary to help teachers drive complex learning situations? And if the training takes care of this necessity, how to do it?

It is of course impossible, and would not be efficient, to try and provide future teachers with theoretical didactical knowledge out of a pertinent teaching context: didactical knowledge is always the synthesis of the observation and the analysis of precise situations.

When the teaching device is an adidactical situation, or a partially adidactical one, we analyse difficulties in driving the class, as related to the different phases of the situation: first, devolution of the problem, then, activity of the students, during which the teacher must adjust the work and collect the procedures, and finally, assessment of the best ways of success and institutionalisation of the aimed knowledge.

Organising the phases of a situation: what vocational knowledge is it and how can it be the object of the training work? We make the hypothesis that in this case, a theoretical knowledge is useful, even if not sufficient; and a "theoretical" course is planned, but it is based on realized sessions about well known situations (Bloch 1999, Bloch 2001). It is not presented here.

In the case of "ordinary" practice, the work is based on the analysis of practice, and regulations take place when the trainers of the Institute go in the novice's class to make a visit[3]. The emphasis is laid on the regulations that the teacher can anticipate, and on the place and ways of pupils' work.

3. Examples of situations: vectors and algebra

3.1. Product of vectors by real numbers

A situation to introduce the product of vectors by real numbers has been tested with both novice teachers and pupils. The aim is to build lessons on the vectors that permit to make the functionality of this notion appear in different kinds of problems. It consists of a game, whose support is a grid (see annex 1).

The game that is presented here is the inverse one. The direct game would be to design sums of vectors, and associate them the good points; it must be known before the inverse game, but it is much more common. The inverse game permits a validation, and above all, the pupils cannot succeed if they do not "put the good knowledge" in the game.

The game must be played with trainee-teachers themselves. Afterwards they can build another grid with numbers of points to define with vector equalities; the teacher can identify didactical variables and fix them (supports of the vectors being parallel to the edges of the paper or not; nature of the numbers – natural, rational, irrational ; number of vectors of the system : one – in which case some points cannot be reached, two – in which case all the points can be reached in one way, or three – in which case the points can be reached by different ways).

This situation has got two objectives for teachers:

- doing to understand that the concept of vector is the notion that permits generally to reach every point of the affine plan ;
- show how to build a situation on the link point-vector in such an environment (the grid) ;

and two objectives directed at working with pupils :

- working about the technique : calculate sums of vectors and find the corresponding points ;
- use the grid as a tool for validation in the class, and link this tool with others that will be met afterwards, as coefficient of a straight line.

This situation is particularly interesting to play with young teachers because they generally have a especially formal view of the linear algebra, and they tend to see the situation as quite a gap between what they are accustomed to do on vectors and what they are invited to work on with pupils in this situation. And moreover, they immediately get difficulties to see why the situation is adequate and to find the didactical variables. The idea that it is possible to "put on stage" the concept of basis of a vector space in such a way is very amazing for them.

---

[3] There are two kinds of visits : formative ones, and assessment's ones.

3.2. Algebra

The second situation is due to Boris Veron: its aim is to work on equations, and more generally literal expressions, considering the difficulties that pupils meet with the notions of variables and unknown quantities. The support is Fibonacci's sequences: the problem is to find one of Fibonacci's sequences, knowing only one of its terms, and its rank (e.g. the tenth term). It allows the introduction of parameters, as it is logical to "do the calculation once and for all". The good didactical variables are available, as to see that elementary algebra is not only a collection of rules, but that it permits to solve problems. Pupils must try to build Fibonacci’s sequences, and find one sequence knowing some of its terms (see annex 2). The situation makes the main functionalities of algebra appear as Gascon (1994) describes it.

4. Conclusion

It is difficult to make an assessment of this training: it is not easy to evaluate the situations the trainee-teachers drive in their classes, when we visit them. There are numerous factors which can create conflicts when teachers plan a classroom-situation: even if the situation is interesting, their know-how can be too uncertain to permit the success.

Trainee-teachers get a questionnaire at the end of the training, and they mention that this work allows them to consider mathematics from another viewpoint: they see dimensions they ignored in their mathematical studies.

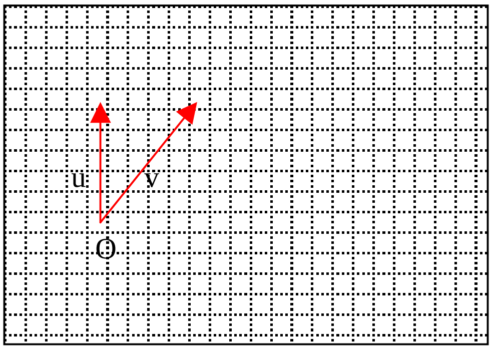
Anyway, we notice that almost all the trainee-teachers become able to try one of the studied situations in their classes, and this is an important result for us.

REFERENCES

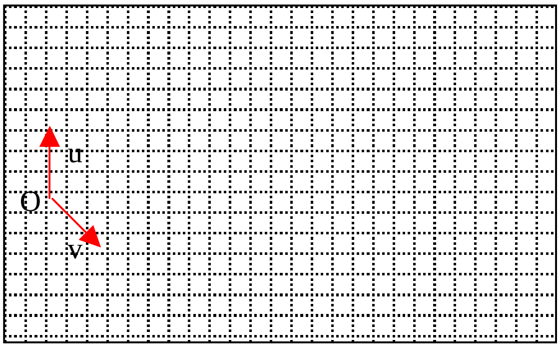
- Bloch I. (1999) L'articulation du travail mathematique du professeur et de l'eleve dans l'enseignement de l'analyse en Premiere Scientifique. *Recherches en Didactique des Mathematiques*, **19/2**, 135-194. Grenoble : La Pensee Sauvage.
- Bloch I. (2000) Le role du professeur dans la gestion des situations : consigne et dévolution, mises en commun, cloture des seances du point de vue cognitif. Actes du XXVIIeme Colloque Inter-Irem des formateurs et professeurs de mathematiques. Grenoble : Universite Joseph Fourier.
- Brousseau G. (1997) *Theory of Didactical Situations in Mathematics*. Mathematics Education Library, Kluwer Academic Publishers, Nederlands.
- Castela C. Eberhard M. (1999) Quels types de modifications du rapport aux mathematiques en vue de la possibilite de quels gestes professionnels ? *Actes de la Xeme Ecole d'Ete de didactique des mathematiques*, 164-172. Houlgate : ARDM.
- Chevallard Y. (1999) L'analyse des pratiques enseignantes en theorie anthropologique du didactique. *Recherches en Didactique des Mathematiques*, **19/2**, 221-265. Grenoble : La Pensee Sauvage.
- Gascon J. (1994) Un nouveau modèle de l'algebre elementaire comme alternative a "l'arithmétique generalisee". *Petit x* **37**, 43-63. Grenoble : Universite Joseph Fourier.
- Robert A. (1997) Outils d'analyse des contenus mathematiques a enseigner au lycee et a l'Universite. *Recherches en Didactique des Mathematiques*, **18/2**, 139-190. Grenoble : La Pensee Sauvage.
- Robert A. (2001) Les recherches sur les pratiques des enseignants et les contraintes du metier d'enseignant. *Recherches en Didactique des Mathematiques*, **21-1/2**, 57-80. Grenoble : La Pensee Sauvage.
- Veron B. (2001) Calcul litteral, equations, inequations. Bulletin de l'Association des Professeurs de Mathematiques, **435**, 440-444. Paris : APMEP.

Annex 1 : The grid game

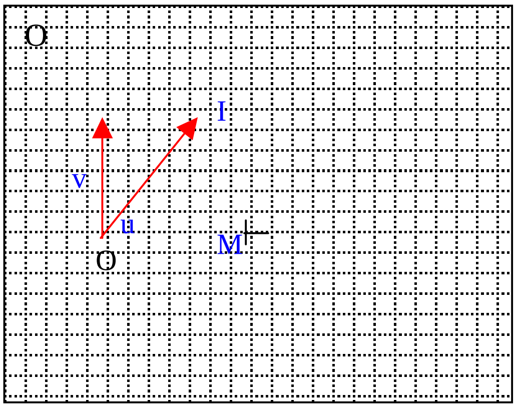
Game n°1
Grid for the receivers



Game n°2
Grid for the receivers

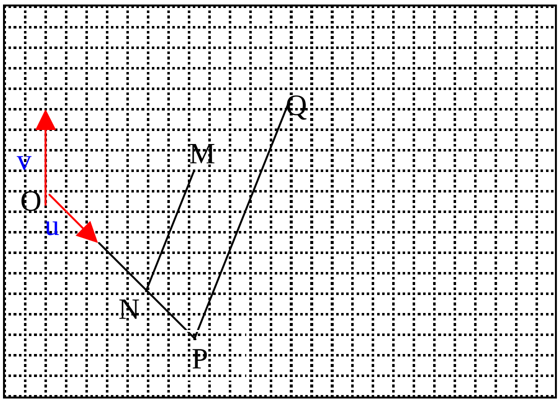


Game n°1
Grid for the transmitters



The other team has got the same grid as you, but with only the point O and the vectors **u** and **v**. Send them a message to place the point M. It is on the circle (O, OI) , and on a straight line orthogonal to v.
But you're not allowed to tell it in your message that must contain only O, **u**, **v** and numbers.

Game n°2
Grid for the transmitters



The other team has got the same grid as you, but with only the point O and the vectors **u** and **v**. Send them a message to place the point M. It is at a place so that (MN) // (PQ) and the points N, P, Q are exactly at crosses of the grid.
But you're not allowed to tell it in your message that must contain only O, **u**, **v** and numbers.

Annex 2 : The Fibonacci's sequences

The first phase (for the pupils) is to explain by examples, what a Fibonacci's sequence is. Then it is to induce them to find a Fibonacci's sequence, knowing its tenth term, or the fifth one.... It appears that there are many solutions. Then the question is : find one Fibonacci's sequence, knowing the first *and* the tenth term. The pupils must put the problem into an equation. This leads them to name *x* the second term of the sequence and resolve equations, and this work shows the functionality of algebra to solve such problems. Moreover, the calculation of numbers of such Fibonacci's sequences leads to the question: could one do the calculation once and for all? Then the calculation changes its signification, the aim is to show that such a sequence depends only on the first two terms, *a* and *b*, seen as parameters (and the underlying structure of vector space is present of course, even if it is not the object of the pupils' work).

But it is possible to go on with this algebraic work, since some questions lead to inequalities: how can one be sure that the terms of the sequence are positive integers ? And this permits further work with systems of equations.

First phase:

Find a Fibonacci's sequence such that:

2	5	7	12						212

Second phase:

Find a Fibonacci's sequence such that:

									178

						51			

									301

Third phase:

Find a Fibonacci's sequence such that:

7									45

9							241

8					77

This leads to name *x* the second term, and the equations with *x* have positive or negative coefficients, which is interesting for the algebraic work.

Calculation "once and for all":

a	*b*	*a + b*	*A + 2b*	*2a + 3b*	*3a + 5b*	*5a + 8b*	…

Possible questions: is there a formula for the nth term? How many Fibonacci's sequences are there, with positive integer terms, and when the tenth term is given?