USING INTERACTIVE DIGITAL VIDEO AND MOTION ANALYSIS TO BRIDGE ABSTRACT MATHEMATICAL NOTIONS WITH CONCRETE EVERYDAY EXPERIENCES

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ABSTRACT

In an attempt to offer a means for better visualization and conceptualization of abstract mathematical notions, we investigated how the analysis of motion contained in a digital video performed in a special computer software environment, can help students increase their understanding on specific topics. Previous research on Digital Interactive Video Technologies (DIVT) was limited to the domain of kinematics and graph interpretation in particular. It was the conviction and in some cases the conclusion of those researchers that students would benefit more from the study of everyday motion as presented in a video, rather than in simulation software. We believe that this is particularly true in the case of mathematics teaching, where students often have difficulty in perceiving the meaning behind an algebraic or graphical representation. Pre-service teachers need to gain a profound understanding on such abstract concepts, as those are usually the ones they have more difficulty teaching. This pilot study is part of a full-scale research that aims to 1) extend the field of investigation using Digital Video Technologies as a connecting link for the Integration of Mathematics and Science, 2) investigate how different dynamic software environments that offer advanced visualization options affect students' learning of mathematics. This paper is the report of the first part of the pilot-study, where the main aspects of teaching with the aid of DIVT were investigated. Five pre-service teachers participated in this study, which consists of two parts, one without and one with DIVT support. The analysis of data gathered indicates that being able to manipulate the reference frame in the environment of the DIVT software and notice how it affects coordinates, graphs and equations of motion, had the greatest impact on the pre-service teachers' understanding on this subject.

KEYWORDS: Interactive Digital Video, Video-Based Laboratories, Motion Analysis, Coordinate Systems, Graphs

1. Introduction

A major problem in the teaching of Mathematics is finding efficient ways of presenting abstract mathematical notions. Sometimes it is hard enough to introduce and explain the definition of an abstract concept alone. The in-depth discussion of such concepts and the revelation of their properties is a task that usually requires advanced visualization and conceptualization strategies that comply to the way students build their network of concepts. What seems natural in developing these strategies is to create a bridge between the world of the abstract (formal mathematics) and the everyday world (the experiences of a person). With the development of technology and its implementation in education we are constantly discovering new ways to teach abstract ideas and support the above statement. As Kaput (1994) notes in a paper discussing the use of technology in connecting mathematics with authentic experience: "The new availability of interactive and representationally plastic media makes possible a wide variety of operative action representation systems, such as coordinate graphs, that can now be manipulated as if they were physical objects. Thus the move of operative symbolism that led to the scientific revolution becomes newly available to enhance the intellectual power of all manner of representation systems".

The term Interactive Digital Video, as used in this paper, refers to computer software tools that allow viewing a movie in digital video format and analyze the motion presented in that movie. Interactive Digital Video and Motion Analysis have mainly been used in the teaching of kinematics in physics, but researchers have also noted the potential use of this technology and related techniques in an interdisciplinary manner to teach Mathematics with the aid of Physics and Technology.

2. VideoPoint and Motion Analysis

Several computer programs have been developed in order to analyze motion presented in videos for the teaching of mathematics and physics (Boyd and Rubin, 1996). The use of this software in the environment of the classroom is most usually referred to as Video-Based Labs or VBL. The program we have chosen to use is VideoPoint by Lenox Softworks. Using VideoPoint, students are able to view videos of motion events and then analyze that motion.

In a motion analysis students begin by marking with the mouse cursor the position of a moving object(s) in successive frames. There is a primary "Reference Frame" present in all video frames and all coordinates are measured with respect to this frame. The collected (object) coordinate data are automatically stored in a table together with the time value attributed to the corresponding frames (Figure 1 – Appendix A). VideoPoint allows dynamic manipulation of data, in the sense that any changes made by the students to object coordinates are automatically updated in the table. Furthermore, they can move or rotate the Reference Frame at any time and view simultaneously how the coordinate data in the table change. This feature offers students a visualization of the abstract concept of Coordinate Systems.

The second step in the motion analysis is the process of setting a physical scale for the movie. So, the position data, which were initially measured in pixels, can be measured in standard units of distance, such as meters, inches or feet. To scale a movie, it is enough that students inform the software about the real length of an object in the desired units.

The next step is usually the construction of graphs corresponding to the time evolution of several physical quantities, such as distance, velocity, acceleration, force, energy, and momentum. Students need only to determine the quantity they wish to display on the horizontal and vertical axis. Subsequently, VideoPoint creates the graph and students are able to change the way it looks in several manners, including its size, the symbols used, and the region being plotted. They can also manipulate the Reference Frame and examine how the graphs are affected. Furthermore, they can try to guess the function that would produce such a graph and compare it to the real graph, or they can directly fit the best curve that matches the graph. There is also the possibility of displaying several quantities in the same graph, e.g. the horizontal component of velocity of two objects versus time (Figure 2 – Appendix A). Another interesting feature is that students can use several Frames of Reference to analyze motion. These Frames of Reference can be either stationary or moving.

Of particular interest is the case where one of the moving objects is selected as the origin of a Reference Frame. Students have the opportunity of investigating how the coordinate data in the table and graphs change when they are measured with respect to a moving Frame of Reference. This technique can offer a better understanding on how coordinates, the shape of a graph, or the equation describing a trajectory, depend on the position of the origin and the orientation of the Coordinate Systems in which they are measured. Teaching cycloid motion with VideoPoint is a good example of how this technique can be utilized. Students can examine the motion of a marked point on the tire of a bicycle as seen by an observer standing on the street or as it would appear to an observer located at the center of the bicycle wheel.

3. Research with VBL – Review of Literature

Research based on VBL has mainly focused on the field of kinematics in introductory physics courses and kinematics laboratories. Beichner (1990 and 1996) has conducted extensive research using software that he designed for this purpose, which is very similar to VideoPoint. His work has mainly focused on student understanding of kinematics graphs. The results of his research indicate that when VBL are integrated in the curricula to an extensive degree then student understanding of kinematics graphs is improved.

In research on VBL, students' misconceptions have been yet another major subject of inquiry. Zollman and Brungardt (1995) focused on students' misconceptions with kinematics graphs and on the way the simultaneous-time presentation of the graphs and the motion event can help them deal with those misconceptions. Their results however revealed that there was no difference in achievement of students using this method, but there was change in terms of student motivation. However, because of the small size of the sample used these results could not be over-generalized and further investigation is necessary.

The innovation of Andrew Boyd and Andee Rubin (1996) compared to previous research on VBL was the use of Interactive Digital Video clearly as means of bridging motion to mathematics. They focused on making connections on how students perceive and/or experience motion in every day life and motion as presented mathematically in graphs and tables. They investigated how students create their own graphs modeling real situations. Digitized video of these situations helps them to revisit and reflect on an object's motion.

4. Our Research – Research Goals

The aim of our research is to extend previous work by adopting a multidisciplinary approach of motion analysis with interactive digital video, for an integrated teaching of Mathematics and Physics. In particular, we wish to investigate students' interpretation of a numerical table of coordinates as a representation of a real motion event, graph understanding and the role of the

Frame of Reference in analysis of motion. These concepts belong both to the domains of Mathematics and Physics. Our hypothesis is that utilizing the methods of both Mathematics and Physics teaching in an integrated activity will lead to increased student understanding of those concepts. Furthermore, we expect that students would be more motivated than in traditional teaching.

5. Research Design

This research was designed as a pilot project of a forthcoming full-scale research on the subject of Integrating Mathematics, Physics and Interactive Digital Video Technologies. The activities and questions used were designed so that the following topics would be mainly investigated: Tables and Numbers, Graphical Representations, Coordinate Systems and Frames of Reference.

Five case studies were conducted with pre-service teachers, students of the Department of Primary Education of the University of Ioannina, three males and two females. Two of them had no computer skills, two had a few and one was an advanced computer user.

The case studies consisted of three parts. In the first part the students were asked to fill in, within an hour, an initial questionnaire that was designed to investigate their skills prior to interaction with VideoPoint. The second part consisted of one to three meetings (depending on their skills and performance during these meetings) where they performed two activities with VideoPoint. Finally, at the third part of this investigation the students answered a modified version of the initial questionnaire this time using VideoPoint. The need for more than one treatment meeting has been documented (Beichner, 1996), as a single treatment meeting cannot produce the desirable change in student understanding.

The questionnaires were based on a movie that showed three moving objects. A screenshot of that movie is displayed in Figure 3 (Appendix A).

6. Results and Discussion

The most important findings and observations based on the pre- and post-questionnaire are summarized in Table 1 (Appendix B).

The analysis of the results is based primarily on students' answers on the pre- and post-questionnaire and in part at the notes kept during the sessions with the students. The analysis is presented in the following nine sections. The first eight sections correspond on the eight findings presented in Table 1. The last section (number nine) presents the analysis of results concerning student motivation and interaction with technology as a teaching medium.

1) Prior to using VideoPoint, students were asked to read a table which consisted of 20 measurements of the x and y coordinate (taken every 0.1 seconds) of three objects moving simultaneously (Figure 4 – Appendix A). In four out of five cases the students tried to estimate the rate of change for x and y, even though that was not requested. Their estimations were either based on mental or written calculations. In the case of written calculations no more than two pairs of numbers were used.

This effort to perform mental or "rough" calculations has been abandoned after they had interacted with VideoPoint in three out of four of the cases. It is our hypothesis that VideoPoint helped them realize that the data contained in the table were not just a collection of numbers but were representing quantities with physical meaning. Using VideoPoint a link was made, between the data in the table and the real motion event. Thus, they realized that it was not possible to make "generalized" or "rough" comments about those quantities, as they concerned the actual motion of the three objects. So, the chance of making the wrong assumptions about their rate of change would now be of significance and not unimportant as in the case of 20 numbers of no meaning.

2) Prior to using VideoPoint, students were asked to identify the points of intersection of three curves displayed in one graph. Only one out of the five was successful in identifying the two intersection points of these three curves. Two more were partially successful as they identified one of the two intersection points. The remaining two were completely unsuccessful. It is very interesting to observe that these two had successfully completed this task after having interacted with VideoPoint!

One possible explanation for this observation is that the "mental" interpolation of the three curves is successfully performed after using VideoPoint, because students had a more concrete and uniform image concept regarding the motion of the three objects.

3) Extending observation (2), we see that only two out of the three students that were able to identify one or two intersection points of the three graphs, named those points, using either their coordinates on the given graphical representation or by associating them with the corresponding video frame. Being able to name a point on a coordinate system is an important task that students should master after being taught coordinate systems. The remaining three students that were not able to name the intersection points prior to using VideoPoint are successful after having used it. This was an expected difference in performance. The obvious justification is that of VideoPoint's dynamic feature of displaying the coordinates (in relation to the given graphical representation) of the user's mouse index when it is within the boundaries of a graphical representation.

4) In the case of three moving objects a graph of the x-coordinate of velocity versus time was given to the students. As two of the objects were moving to the opposite of the positive direction of the x-axis of the Frame of Reference their algebraic value was a negative number.

Students were asked to describe the motion of the three objects by interpreting the meaning of the negative values for velocity. We did not receive satisfactory answers to this question. Though, an interesting reaction was that some of the students noticed that in fact one of the three objects did not have a negative velocity.

In particular, prior to interacting with VideoPoint only two out of the five students noticed that one of the objects did not have negative velocity and marked this on their questionnaire. After interaction with VideoPoint, the ratio has gone up to four out of five. The remaining one student showed no difference prior to and after using VideoPoint.

As the graphs presented to the students prior to using VideoPoint were identical to those provided by VideoPoint in the post-examination, we cannot attribute this change to any of VideoPoint's features. Rather, we can assume that it was the whole activity design that made a difference. Students participating in this activity have a more active role than the one they have when answering a questionnaire on paper. Because of this, we believe, that they were more motivated and more concentrated in their work. The result of placing students in a more active role-control is that they behave as if they are working on a project of their own and not taking some sort of examination. This made students more cautious and suspicious regarding the information given to them.

5) When students were asked to answer questions based on graphs some of them consulted the table in order to provide an answer. This unexpected behavior could indicate that perhaps VideoPoint did not help students improve their understanding of graphs and they resort to the table in order to answer. Another possible explanation, though, is that VideoPoint operates as a link between the different representations of the table and the graphs. Students realize that the table and the graphs both represent a mathematical expression of the motion they observed, so they decide to consult any of the two representations when providing an answer as they are now convinced that they are two versions of the same thing: In both cases they see a mathematical expression of a motion event. In any case, this observation needs to be further investigated.

6, 7, 8) We will explain the change in achievement regarding observations 6, 7 and 8 together, as we believe that it is due to the same reasons.

It is obvious from these three observations that VideoPoint has the potential to make a big difference in students' understanding of the concept of the Frame of Reference. Students were much more successful when answering questions regarding the role of the system of reference after using VideoPoint than before.

Based on the results of the pre- and post-questionnaires, but mostly on the interviews, we could claim that the reason for this change is the dynamic nature of the Frame of Reference in VideoPoint. The Frame of Reference as presented mathematically, is an abstract concept that cannot be conceptualized unless it has been visualized in a drawing representing a motion event. Thus, comprehension derived from this visualization is not enough to provide students with the ability to make predictions of how equations of motion, graphs and coordinates would change if the Frame of Reference were to rotate or/and change position. VideoPoint may serve as a means for an advanced conceptualization.

In VideoPoint the Frame of Reference is a notion of dynamic nature. Students can manipulate it at will and whenever they want and observe how coordinates, graphs, and equations of motion related to it are updated.

It is our hypothesis that being able to "experiment" with the Frame of Reference and observe the change it causes to coordinates, graphs and equations of motion, enhances students' conceptual knowledge on this subject. Students realize that there is a dynamic link between the Frame of Reference's position and orientation and the way that graphs and tables of coordinates look. Furthermore, by bringing the Frame of Reference to particular positions of "special" interest, such as positioning one of the axes to be parallel to an inclined level, or bringing the x-axis vertically and the y-axis horizontally, they can deal with misconceptions and gain a better understanding and insight to the role of a Frame of Reference.

The students that took part in this research seemed to particularly enjoy the part of moving and rotating the system of reference and noticing the change it causes to the graphs and tables. Most of them made remarks on this that indicate some sort of "insight" regarding this topic when it was demonstrated to them for the first time.

9) At the beginning of this research there was some concern regarding students' familiarity with computers. Only one out of five students had advanced computer skills. Two had a few and the remaining two had none. It is very encouraging to see that at the end of the activities all five had almost mastered the skills required to use VideoPoint. They could all run VideoPoint, open a movie, collect data, scale the movie, read tables and create graphs. As two of the students struggled with the use of mouse at the first activity it is amazing that after a maximum of five hours they were able to successfully perform the above tasks on their own. Students were themselves surprised by how well they performed on the computer, which increased their self-esteem. They confessed that they had never thought they could do work on the computer so easily. We consider that this feeling of success was a major factor for the increased motivation that they displayed throughout the activities.

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Appendix A – Figures

Figure 1

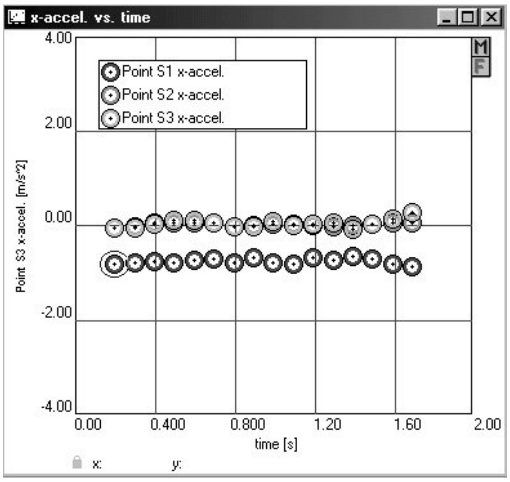

Figure 2

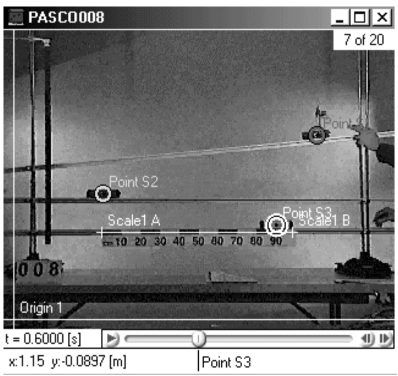

Figure 3

Table							
		Point S3		Point S2		Point S1	
	time [s]	x-pos [m]	y-pos [m]	x-pos [m]	y-pos [m]	x-pos [m]	y-pos [m]
1	0.000	1.678	0.4904	0.1720	0.6433	1.599	0.9490
2	0.2000	1.592	0.4873	0.3312	0.6401	1.564	0.9427
3	0.4000	1.510	0.4873	0.4841	0.6433	1.513	0.9395
4	0.6000	1.420	0.4904	0.6338	0.6433	1.459	0.9331
5	0.8000	1.338	0.4873	0.7803	0.6433	1.398	0.9268
6	1.000	1.255	0.4873	0.9299	0.6465	1.331	0.9236
7	1.200	1.169	0.4873	1.070	0.6433	1.258	0.9140
8	1.400	1.086	0.4841	1.220	0.6465	1.182	0.9076
9	1.600	1.006	0.4841	1.360	0.6433	1.099	0.8981
10	1.800	0.9236	0.4841	1.506	0.6433	1.013	0.8917
11	2.000	0.8408	0.4841	1.646	0.6433	0.9236	0.8854
12	2.200	0.7643	0.4809	1.796	0.6401	0.8312	0.8758
13	2.400	0.6783	0.4841	1.939	0.6433	0.7325	0.8662
14	2.600	0.5987	0.4809	1.971	0.6401	0.6306	0.8567
15	2.800	0.5255	0.4809	1.981	0.6369	0.5255	0.8503
16	3.000	0.4459	0.4809	1.975	0.6401	0.4140	0.8376
17	3.200	0.3662	0.4809	1.978	0.6369	0.3057	0.8280

Figure 4

Appendix B – Tables

Table 1

			Subject 1	Subject 2	Subject 3	Subject 4	Subject 5
1	Using only 0-2 random pairs of numbers to find the rate of change of the data for a list of 20 numbers, even though it was not requested	Pre		●	●	○	○
		Post			●		
2	Ability to identify points of intersection of two or more curves on a graph	Pre	○		●		○
		Post	○	●	●	●	○
3	Ability to name points of intersection of two or more curves on a graph	Pre	●				●
		Post	●	●	●	○	
4	Comparing – crosschecking the truth of information given at the questionnaire with the graphs displayed	Pre	●				●
		Post	●		○	●	●
5	Using the corresponding data table to answer questions about graphs	Pre					○
		Post		●		●	○
6	Prediction of the consequences of a parallel transportation and a rotation of the Frame of Reference regarding position	Pre	●		●		
		Post	●	○	●	○	○
7	Prediction of the consequences of a parallel transportation of the Frame of Reference regarding velocity	Pre					
		Post	●		○	○	
8	Prediction of the consequences of a rotation of the Frame of Reference regarding velocity	Pre				○	
		Post	○	○	○	○	●

● Represents success **○ Represents partial success**