

SUSPENSION OF SENSE-MAKING IN MATHEMATICAL WORD PROBLEM SOLVING: A POSSIBLE REMEDY

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ABSTRACT

In common teaching practice the habit of connecting mathematics classroom activities with reality is still substantially delegated to word problems. But besides representing the interplay between mathematics and reality, word problems often are the sole example of realistic mathematical modeling and problem solving. During the past decades, a growing body of empirical research (e.g. Freudenthal, Schoenfeld, Verschaffel, De Corte) has documented that the practice of word problem solving in school mathematics promote in the students an exclusion of realistic considerations and a “suspension” of sense-making and hardly matches the idea of mathematical modeling and mathematization. If we wish situations of realistic mathematical modeling, that is both real-world based and quantitatively constrained sense-making, we have to make changes: i) we have to replace the word problem solving with classroom activities that are more relatable to the experiential worlds of the pupils and consistent with a sense-making disposition; ii) we will ask for a change in the teacher conceptions, beliefs and attitude towards mathematics; iii) a directed effort to change the classroom socio-math norms will be needed. In this paper we discuss how these changes can be realized through classroom activities based on the use of suitable cultural artifacts and interactive teaching methods.

1. Introduction

In normal teaching practice, establishing connections between classroom mathematics activities and everyday-life experiences still regards mainly word problems. But besides representing the interplay between formal mathematics and reality, word problems are often the only means of providing students with a basic sense experience in mathematization, especially mathematical modeling (Reusser & Stebler, 1997). Recent research has documented that the practice of word-problem solving in school mathematics actually promotes in students a “*suspension of sense-making*” (Schoenfeld, 1991), and the exclusion of realistic considerations. Primary - and secondary - school students tend to ignore relevant and plausible familiar aspects of reality and exclude real-world knowledge from their mathematical problem solving.

Several studies point to two reasons for this lack of use of everyday-life knowledge: textual factors relating to the stereotypical nature of the most frequently used textbook problems (“*When problem solving is routinised in stereotypical patterns, it will in many cases be easier for the student to solve the problem than to understand the solution and why it fits the problem*”, Wyndhamn and Säljö, 1997, p.364) and presentational or contextual factors associated with practices, environments and expectations related to the classroom culture of mathematical problem solving (“*In general the classroom climate is one that endorses separation between school mathematics and every-day life reality*”, Gravemeijer, 1997, p.389). Furthermore, it has been noted that the use of stereotyped problems and the accompanying classroom climate relate to teachers’ beliefs about the goals of mathematics education (Verschaffel, De Corte, and Borghart, 1997).

This indicates a difference in views on the function of word problems in mathematics education. The researchers, and probably the drafters of new curricula such as the Italian one, relate word problems to problem solving and applications. The student-teachers (and probably teachers in general) see another role for word problems. That is as nothing more, and nothing less, than exercises in the four basic operations which also have a justification and suitable place within the teaching of mathematics, though certainly not that of favoring “realistic mathematical modeling”, which is “*both real-world based and quantitatively constrained sense-making*”, Reusser (1995).

If we wish to establish situations of realistic mathematical modeling in problem-solving activities, changes must be made.

1. The type of activity aimed at creating interplay between reality and mathematics must be replaced with more realistic and less stereotyped problem situations, founded on the use of concrete materials.

2. We must endeavor to change students’ conceptions of, beliefs about and attitudes towards mathematics; this means changing teachers’ conceptions, beliefs and attitudes as well.

3. A sustained effort to change classroom culture is needed. This change cannot be achieved without paying particular attention to classroom socio-mathematical norms, in the sense of Yackel and Cobb (1996).

In this paper we discuss how these changes can be realized through suitable classroom activities. These activities are related more easily to the experiential world of the student and which are consistent with a sense-making disposition must be designed. They make extensive use of cultural artifacts that could prove to be useful instruments in creating a new link between school mathematics

and everyday-life, which incorporates mathematics. We will show how suitable cultural artifacts and interactive teaching methods can play a fundamental role in this process.

2. Connections between classroom activities and everyday-life experience

The connection between students' everyday and classroom mathematics is not easy because the two contexts differ significantly. Just as mathematics practice in and out of school differs (Lave, 1988; Nunes, 1993) so does mathematics learning (Resnick, 1987). Masingila, Davidenko, and Prus-Wisniowska (1996) outlined three key differences between in- and out-of-school practices (goals of the activity, conceptual understanding, and flexibility in dealing with constraints). In out-of-school mathematics practice in particular, people may generalize procedures within one context but may not be able to generalize to another since problems tend to be context specific. Generalization, which is an important goal in school mathematics, is not usually a goal in out-of-school mathematics. On the other hand, many studies have pointed out that local strategies developed in practice are more effective than algorithms which are usually taught in school to give students powerful general procedures, but which are, in fact, often useless in out-of-school contexts (Schliemann, 1995).

Although the specificity of both contexts is recognized, we think that the conditions that often make out of school learning more effective can and must be re-created, at least partially, in classroom activities. Indeed, while there may be some inherent differences between the two contexts, these can be reduced by creating classroom situations that promote learning processes closer to those arising from out-of-school mathematics practices.

Through our studies, and the paradigmatic example that we will present, we wish to make a contribution towards resolving the problem of 'permeability' between school and life experiences (Freudenthal, 1991). As in the Realistic Mathematics Education (RME) perspective of the Dutch school of thought, we think that progressive mathematization should lead to algorithms, concepts and notations that are rooted in a learning history which starts with students' informal experientially real knowledge. In our approach everyday-life experience and formal mathematics, despite their specific differences, are not seen as two disjunctive and independent entities. Instead, a process of gradual growth is aimed for, in which formal mathematics comes to the fore as a natural extension of the student's experiential reality. The idea is not only to motivate students with everyday-life contexts but also "*to look for contexts that are experientially real for the students and can be used as starting points for progressive mathematization*", Gravemeijer (1999, p.158).

Furthermore we stress that the process of bringing "reality into mathematics" by starting from student's everyday-life experience, is fundamental in school practice for the development of new mathematical knowledge. However it turns out to be necessary, but not sufficient, to foster for example "*a positive attitude towards mathematics, intended both as an effective device to know and critically interpret reality, and as a fascinating thinking activity*", as is stressed for example in the Italian primary school program. We contend that these educational objectives can only be completely fulfilled if students and teachers can bring mathematics into reality. In other words, besides *mathematizing everyday experience* it is necessary to "*everyday*" mathematics. This can be implemented in a classroom by encouraging students to analyze '*mathematical facts*' embedded in

appropriate ‘*cultural artifacts*’, and which for brevity we might call “*cultural mathfacts*” or “*social mathfacts*”. There is indeed a great deal of mathematics embedded in everyday life.

Cultural artifacts embody theories that users accept, even when they are unaware of them (Saxe, Dawson, Fall, & Howard, 1996). Their use mediates intellectual activities and, at the same time, enables and constrains human thinking. Through these subtle processes social history is brought into any individual act of cognition (Cole, 1985).

The cultural artifacts we introduced into classroom activities (e.g. supermarket bills, bottle and can labels, railway schedules, a cover of a ring binder), or those to be constructed by students, e.g. calendars, are concrete materials which children typically meet in real-life situations. We have therefore offered the opportunity of making connections between the mathematics incorporated in real-life situations and school mathematics, which although closely related, are governed by different laws and principles. These artifacts are relevant to children; they are meaningful because they are part of their real life experience, offering significant references to concrete situations. This enables children to keep their reasoning processes meaningful and to monitor their inferences. As a consequence, they can off-load their cognitive space and free cognitive resources to develop more knowledge.

We believe that immersing students in situations which can be related to their own direct experience and are more consistent with a sense-making disposition, allows them to deepen and broaden their understanding of the scope and usefulness of mathematics as well as learning ways of thinking mathematically that are supported by mathematizing situations. This allows students to become involved in mathematics and to break down their conceptions of a remote body of knowledge. Only in this way can we encourage a positive attitude towards school mathematics.

Obviously, usefulness and its pervasive character are just two of the many facets of mathematics that do not entirely capture its special character, relevance and cultural value; nonetheless these two elements could be usefully exploited from the teaching point of view.

3. Cultural artifacts in classroom activities

The use of cultural artifacts in our classroom activities has been articulated in various stages, with different educational and content objectives.

First, the dual nature of the artifacts, that is belonging to the world of everyday life and to the world of symbols, to use Freudenthal’s expression, allows movement from situations of normal use to the underlying mathematical structure and vice versa, from mathematical concepts to real world situations, in agreement with ‘*horizontal mathematization*’ (Treffers, 1987). Using a receipt, which is poor in words but rich in implicit meanings, overturns the usual buying and selling problem situation, which is often rich in words but poor in meaningful references (Basso & Bonotto, 1996).

As we will see, these artifacts may also become real “*mathematizing tools*” with some modification, e.g. removing some data. On the one hand they create new mathematical goals, on the other they provide students with a basic experience in mathematization. In this new role, the cultural artifact can be used to introduce new mathematical knowledge through the particular learning processes that Freudenthal (1991) defines ‘*prospective learning*’ or ‘*anticipatory learning*’. We think that this type of learning is better enhanced by a ‘rich context’ as outlined by Freudenthal, that is a context, which is not only the application area but also a source for learning mathematics. The cultural artifacts and classroom activities we introduced are part of this type of context. These experiences

have also favored the type of learning “*retrospective*” that occurs when old notions are recalled in order to be considered at a higher level and within a broader context, a process typical of adult mathematicians.¹ This different use of the artifacts also made it possible to carry out ‘*vertical mathematization*’, from concept to concept, compatible with grade level. Vertical mathematization may be described as the process of reorganization within the mathematical system itself, for instance discovering connections between concepts and strategies and then applying these discoveries.

The use of some artifacts, receipts, bottles, labels, the weather forecast from a newspaper, a cover of a ring binder (see for example Bonotto, 2001, Bonotto & Basso, 2001, and Bonotto 2003), allow the teacher to propose many questions, remarks, and culturally and scientifically interesting inquiries. The activities and connections that can be made depend, of course, on the students’ scholastic level. These artifacts may contain different codes, percentages, numerical expressions, and different quantities with their related units of measure, and hence are connected with other mathematical concepts and also other disciplines (chemistry, biology, geography, astronomy, etc.). It could be said that the artifacts are related to mathematics (and other disciplines) as far as one is able to make these relationships.

To summarize, the artifacts can be used

- as tools to apply ‘old’ knowledge to ‘new’ contexts, thus becoming good material for ‘meaningful exercises’;

- to reinforce mathematical knowledge already possessed, or to review it at a higher level;
- as motivating stepping-stones to launch new mathematical knowledge.

Furthermore we ask children

- to select other cultural artifacts from their everyday life,
- to identify the embedded mathematical facts,
- to look for analogies and differences (e.g. different number representations),
- to generate problems (e.g. discover relationships between quantities).

In other words children should be encouraged to recognize a great variety of situations as mathematical situations, or more precisely “mathematizable” situations. In this way children are offered numerous opportunities to become acquainted with mathematics and to change their attitude towards mathematics, in contrast with the traditional classroom curriculum.

From our experience, children confronted with this kind of activity also show flexibility in their reasoning processes by exploring, comparing and selecting among different strategies. These strategies are sensitive to the context and number quantities involved, and are better mastered and controlled from the meta-cognitive point of view. They are therefore closer to the procedures that emerge from out-of-school mathematics practice.

4. The basic characteristics of the teaching/learning environment

Besides the use of suitable cultural artifacts discussed above the teaching/learning environment designed and implemented in our classroom activities is characterized by:

¹ Freudenthal (1991, p.118) states that “*prospective learning should not only be allowed but also stimulated, just as the retrospective learning should not only be organized by teaching but also activated as a learning habit*”.

- the application of a variety of complementary, integrated and interactive instructional techniques (involving children's own written descriptions of the methods they use, individual and class discussions, and the drafting of a text by the whole class);

- an attempt to establish a new classroom culture also through new socio-mathematical norms.

Regarding the first point, most of the lessons follow an instructional model consisting in the following sequence of classroom activities: a) a short introduction to the class as a whole; b) an individual written assignment where students explain the reasoning followed and strategy applied; c) a final whole-class discussion. We consider that the interactivity of these instructional techniques is essential because of the opportunities to induce reflection as well as cognitive and metacognitive changes in students. This process may be very important for teachers also, since it enables them to recognize and analyze individual reasoning processes that are not always explicit (corresponding to the individual written report). In the collective discussion, comparing the different answers and strategies, children's first attempts at generalizing, and further remarks made during the discussion, lead to collectively drawing up a text aimed at socialization of the knowledge acquired, which completes the activity.

As far as the second point is concerned, we expect students to approach an unfamiliar problem as a situation to be mathematized, not primarily to apply ready-made solution procedures. This does not mean that knowledge of solution procedures plays no part, but the primary objective is to make sense of the problem. In practice, it is often a matter of shuttling back and forth between interpreting the problem and reviewing possible procedures or results. At the same time, the teacher is expected to encourage students to use their own methods, exploring their usefulness and soundness with regard to the problem. The teacher should stimulate students to articulate and reflect on their personal beliefs, misconceptions and problem-solving strategies. Other possible strategies for solving the same problem when it appears next are emphasized and students are encouraged to make comparisons between strategies.

According to the socio-constructivist perspective, these norms are not predetermined criteria introduced into the classroom from outside. Instead, the understandings are constructed and continually modified by students and teacher through their ongoing activities and interactions. The development of mathematical reasoning and sense-making processes is seen as inseparably interwoven with their participation in the interactive constitution of taken-as-shared mathematical meanings and norms (Yackel and Cobb, 1996).

5. Conclusions and open problems

In this paper we discuss some classroom activities based on the use of suitable cultural artifacts, interactive teaching methods and on the introduction of new socio-mathematical norms was combined in an attempt to create a substantially modified teaching/learning environment. This environment focused on fostering a mindful approach toward realistic mathematical modeling, that is both real-world based and quantitatively constrained sense making (Reusser & Stebler, 1997).

We do not suggest that the activities described here are a prototype for all classroom activities related to mathematics, although in agreement with Verschaffel, L., De Corte, E, et al. (1999, p.226), we think that *“the development of mathematical problem-solving, skills, beliefs, and attitudes should not emanate from a specific part of the curriculum but should permeate the entire curriculum”*.

We do believe however that by enacting some of these experiences, children are offered an opportunity to change their beliefs about, and attitudes towards school mathematics. Immersing students in situations more relatable to their direct experience and more consistent with sense-making, provides a means to deepen and broaden their understanding of the scope and usefulness of mathematics as well as learning ways of thinking mathematically that are supported by mathematizing situations. Using appropriate cultural artifacts, which students can understand, analyze and interpret, we can present mathematics as a means of interpreting and understanding reality and increasing the opportunities of observing mathematics outside the school context. Teaching students to interpret critically the reality they live in, to understand its codes and messages so as not to be excluded or misled should be an important goal for compulsory education. The computer, as well as other more recent multimedia instruments, has a remarkable social and cultural impact and huge educational potential that perhaps has not yet been fully explored.

For a real possibility to implement this kind of activity, there also needs to be a radical change on the part of teachers. They have to try i) to modify their attitude to mathematics; ii) to revise their beliefs about the role of everyday knowledge in mathematical problem solving; iii) to see mathematics incorporated into the real world as a starting point for mathematical activities in the classroom, thus revising their current classroom practice. Only in this way can a different classroom culture be attained. On the basis of the experience of this and our other studies, we entirely agree with Freudenthal (1991), that the main problem regarding rich contexts is implementation requiring a fundamental change in teaching attitudes. As in other studies (Verschaffel, De Corte et al., 1999), the effective establishment of a learning environment like the one described here makes very high demands on the teacher, and therefore requires revision and change in teacher training, both initially and through in-service programs.

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