"INTERDISCIPLINARY PROJECTS IN THE ARTS AND SCIENCE PROGRAMME AT MCMASTER UNIVERSITY"

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ABSTRACT

Interdisciplinary component of the Mathematics course taught in the Arts and Science Programme at McMaster University is implemented in several ways, varying in depth, width and level of involvement of other courses. Of a number of issues related to the course, this paper uses instructors' experience and examples of students' writing to discuss the features of the narrative in mathematics. Used as a vehicle to enhance understanding of mathematics and to build and improve research and communication skills, good writing is a key to a successful and productive interdisciplinary mathematics course.

1. Introduction

Arts and Science Programme at McMaster University[1] is an interdisciplinary program that offers students an opportunity to use their university years to further their intellectual growth through a study of significant achievements in both arts and sciences. The main goal of the program is to give students an understanding of sciences, arts, and technology, to help them develop skills in communication, in qualitative and quantitative reasoning, and to help them become critical and independent thinkers.

Acquiring valuable skills as undergraduate students, Arts and Science graduates have always been in demand; even today, in what is perceived as a 'digital and high-technology economy.' The following is an excerpt from the statement, signed by leaders of Canadian high-technology corporations, underscoring the importance of liberal arts education[2]:

> "A liberal arts and science education nurtures skills and talents increasingly valued by modern corporations. Our companies function in a state of constant flux. To prosper we need creative thinkers at all levels of the enterprise who are comfortable dealing with decisions in the bigger context. They must be able to communicate - to reason, create, write and speak - for shared purposes: for hiring, training, managing, marketing, and policy-making. In short, they provide leadership."

Mathematics has always played and important role in the Arts and Science Programme curriculum. The mission of the Programme (as outlined in the opening paragraph) creates an ideal environment for learning mathematics the way it should be learnt. The Arts & Science Mathematics course (Mathematics course, for short) exposes students to all aspects of mathematics, from its 'rigid' and 'abstract' sides (axioms, theorems and definitions) to its applied (modeling) and 'non-mathematical' sides (history, ethnomathematics). This two-semester course, taught in the first year, reveals mathematics at its foundations, presents its theoretic aspects and investigates its meaning and purpose in social and cultural contexts. Although its 'backbone' is differential and integral calculus, the scope of topics discussed in the course is much broader. The skills that the students develop in the course (formation of precise mathematical and logical arguments, written and oral communication, research, problem-solving and critical thinking skills) are the skills that are not needed just in the 'digital and high-technology economy,' but rather in any area of human endeavour.

In this paper, I plan to describe aspects of the Mathematics course that are related to the interdisciplinary mission of the Programme. The second part is devoted to a discussion of the use of writing in the course, in the context of knowledge construction and acquisition, criticality and depth in approach, and originality and creativity in thought and presentation.

2. Arts and Science Mathematics as an Interdisciplinary Course

Appropriate examples, problems, and ideas selected from other disciplines motivate students and stimulate their interest in mathematics content. The main purposes of an interdisciplinary approach are

[1] McMaster University is a medium-sized, full service university located in Hamilton, Ontario, Canada
[2] from: 'Hi-tech CEOs Say Value of Liberal Arts is Increasing,' http://www.trentu.ca/news/ceo.html

to deepen students' knowledge and understanding of major ideas and concepts in mathematics, and to develop their research, communication and critical thinking skills. Well designed interdisciplinary projects will enable students to place mathematics into historic, cultural and societal contexts.

Interdisciplinary approach needs to elevate learning to a new level, by providing something new in all disciplines involved - something that otherwise would not be present.

In the Arts and Science Mathematics course interdisciplinary approach is implemented in several ways - varying in depth, width and level of involvement of other courses:

- First and foremost, the Mathematics course itself is interdisciplinary in nature.
- "Cultural Meaning of Mathematics or Science" is a project that links Mathematics with the course on formal logic and writing (via team-teaching).
- "Standard" links with the statistics and physics courses in the Programme have been established.
- "Science Inquiry" course - under construction at the moment - will use the 'powers' of mathematics, physics and chemistry to investigate questions in biology.
- Interdisciplinary themes (such as "Symmetry," "Knowledge and Popular Culture," "Infinity," or "Construction of Reality") link courses across several disciplines and across all levels (years one to four) in the Programme.

I will describe the first two models in some detail, and then say a few words about the remaining ones.

The Mathematics course itself is interdisciplinary in nature.

Besides introducing new material and establishing connections with the previously taught mathematics material, lectures in the course are used to broaden students' viewpoint and understanding - by presenting historic and cultural aspects of the development of mathematics and by discussing related topics. An example: construction and definition of the definite integral is motivated by a real-life, 'applied' problem - how to compute the area of a plot of land (Ancient Egyptians paid taxes based on the amount of land they owned). The amount of material needed to construct a temple, or a pyramid, was based on calculations of volume. In lectures, students are shown how ancient (and very often intuitive) methods of computation of areas and volumes got formalized in the framework of 19th century calculus.

Number theory is probably one of the most fascinating fields of mathematics - and, quite possibly, one that is among the easiest to discuss on an elementary level (to a certain degree, of course). Yet unproven conjecture stating that every even number greater than two can be expressed as a sum of two prime numbers - so-called Goldbach conjecture - uses mathematics concepts that are understood by an average high school student. In my Mathematics class, Goldbach conjecture is used in a two-fold way: on the one hand, it illustrates the difference between a theorem and a conjecture. Students are asked to articulate what would be needed to prove Goldbach conjecture, and also what would be needed to disprove it. Creation of the conjecture itself mimics a process of creating mathematics. By 'playing' with numbers, we are actually performing an investigation - conducting an equivalent of an experiment in chemistry or physics. Sooner or later - hopefully - we start noticing a pattern (e.g., even and odd numbers behave differently when we try to express them as a sum of two prime numbers). Based on the pattern, we try to formulate a conjecture (that is not a theorem unless we prove it).

Writing about mathematics is one of the best ways of learning mathematics. Only when we are able to clearly and unambiguously communicate an idea, or a result of a computation to somebody else (and answer their questions about it), we can claim that we have learnt and understood. An example: it is a matter of technical expertise to compute a horizontal asymptote. But how does one explain the ide a to somebody who has not heard of limits? In their written answer to "what is a horizontal asymptote," my students are not allowed to say "as x approaches infinity;" instead, they are expected to explain in words how "values of a given function f(x) can be made arbitrarily close to some number by taking x large enough." Then, they must further elaborate on statements "can be made arbitrarily close" and "large enough." Finally (now thinking of talking to a mathematically sophisticated audience) they are asked to return to mathematics, and to translate their English statements into mathematics symbols and formulas. Early in their narrative students are encouraged to identify examples of horizontal asymptotes in 'real life' (or argue why they cannot find any) but that is by no means the only goal of the exercise.

"Cultural Meaning of Mathematics or Science"

In order to investigate and discuss mathematics in contexts of society, history and culture, Arts and Science Mathematics course requires that students complete a project, tentatively called "Cultural Meaning of Mathematics or Science." The aim of the project is to investigate one mathematical (or scientific) issue and to explore the cultural significance of it. To start, students are asked to formulate a question within the given categories. The categories are quite broad: assess popular myths about mathematics (science) or competing histories of the origins and/or models of the development of mathematics (science); assess mathematics (science) as an authoritative and powerful institution controlling knowledge production; is mathematics (science) value-free; consider gender, class, race, non-Western approaches and contributions, etc. This project is done jointly (team-teaching) with the course on writing and formal logic, and the final essay and oral presentation are the parts of the requirements for both courses.

After receiving a feedback from instructors and teaching assistants, students revisit their question, reformulate it or narrow it down if necessary. They must identify a reference that they will use (could be several pages, or a chapter from a book, or a newspaper article)[3], and then write a critique of it. Their work should not be merely a summary, or an apology, or celebration of science or mathematics. Rather, it should interrogate and assess the role of mathematics or science in relationship to society.

[3] Several references are listed here, to show the variety of students' interests and the topics they investigated:
* Ascher, Marcia, Code of the quipu: a study in media, mathematics, and culture. Ann Arbor: University of Michigan Press, 1981 (cultural history and sociological aspects of scientific discovery)
* Golinski, Jan. Making Natural Knowledge: Constructivism and the History of Science. 1998 (study of the recent histories of science and their connections to culture)
* J.A Paulos, A Mathematician Reads the Newspaper. New York: Anchor Books, 1995 (use and abuse of mathematics and mathematical reasoning in media)
* LaTour, B. and Wo olgar, S., Laboratory Life: The Social Construction of Scientific Facts. 1979 (classic in sociology of science)
* G.H. Hardy, A Mathematician's Apology. Cambridge: University Press, 1940 (why mathematics - by one of the most famous 20th century mathematic ians)
* Menninger, K. Number Worlds and Number Systems. New York: Dover, 1969 (cultural history of numbers)
* E. Rothstein, Emblems of Mind. New York: Evon Books, 1995 (among other topics, explores the relation between music and mathematics)
* Henrion, Claudia. Women in Mathematics: the Addition of Difference. 1997 (profiles of professional mathematicians)

The question that a student formulated helps her/him focus on one issue. The final part of the project consists of oral presentations, followed by a question-and-answer period and a discussion.

Other Interdisciplinary Models in the Arts and Science Programme

It is very easy to identify topics that are common to mathematics, probability and statistics, and physics (these three courses form a major part of the core of the science curriculum in the Programme). For example, concept of the area introduced in calculus is revisited in the sections on continuous probability distributions in the statistics course; data from physics experiments is analyzed using statistical methods, etc.

Instructors for the three courses hold regular meetings. Organized initially to adjust and synchronize the syllabi of the courses, the meetings provide a forum for discussions on a variety of topics related to teaching science.

The "Science Inquiry" course - presently under construction - will use the 'powers' of mathematics, physics and chemistry to investigate questions in biology. Students will be assigned to work (in small groups) on a project in biology that will use at least one of mathematics, physics or chemistry in a significant way. The final product - depending on the level of involvement and depth of investigation - will be an essay, a final course report or an undergraduate thesis. In any case, it will be a narrative piece.

Interdisciplinary themes (such as "Symmetry," "Knowledge and Popular Culture," "Infinity, " or "Construction of Reality") link courses across several disciplines and across all levels (years one to four). Last year's theme, called "Bodies of Knowledge," involved students, faculty and guest speakers from several departments within the University. Unlike other interdisciplinary projects in the Programme, this one is not a part of a specific course, and students do not get a credit for participating in it.

3. Writing in Mathematics and Writing About Mathematics

The fact that writing in mathematics - and writing about mathematics - are good for learning mathematics can be taken as an axiom; or, in the least, it is an easily provable theorem (as shown by a significant body of literature in mathematics education). Writing helps students learn mathematics better and teaches them how to communicate effectively their ideas to others. Students' writing assignments represent a valuable resource for the teachers: among the many benefits, they could reveal a nature of students' conceptual misunderstandings and problems.

If we expect our students to write about mathematics, we need to teach them how to do it first. Moreover, if the project they are involved with is interdisciplinary, we should clearly state the expectations in terms of each discipline involved.

Stephen King said that " … the only way to learn how to write is to read a lot and to write a lot."[4] The same is true if we replace 'write' by 'write mathematics' and 'read' by 'read mathematics.' My experience tells me that one of the most efficient ways of teaching how to write mathematics is to analyze samples of good and bad mathematics writing - both of which are easy to find, especially the latter. I usually use books on popular mathematics and my students' old essays.

[4] 'On Writing: a Memoir of the Craft.' Scribner, 2000.

The most important aspects of mathematics writing include knowledge construction and acquisition, criticality and depth in approach, and originality and creativity in thought and presentation.

What does one write about in mathematics? Most common approach for an elementary interdisciplinary topic is to try to address a 'real-world' situation (such as building the most optimal box, or using exponential growth to model a population, etc. - mathematics textbooks are full of those). However, one must be a bit critical about it. Whose 'real-world' is it that is being investigated? Almost every calculus textbook has a story problem about a person on a ladder. The bottom end of the ladder is sliding away, so the unlucky person on the ladder is falling down. The problem usually asks to compute how fast is the top of the ladder falling. Is that a 'real-world' problem? To whom is it really relevant? Does it present a good opportunity for a (short) narrative in math?

Linked to political goals of 'accountability' in universities, investigation of 'real-world' problems is promoted as a tool that will motivate students, and provide them with better understanding of mathematics. This is true, but only in some cases. Working on 'real-life' problems requires an appropriate level of mathematical sophistication - it cannot be done too early, when students are still struggling with technical intricacies and basics of mathematics concepts (can one appreciate reading a poem without being able to recognize all letters?).

The calculus textbook that we have been using in Arts & Science Programme[5] contains a primitive model of a blood flow, but does not provide sufficient clues as to how the formula has been arrived at. About all a student can do is to answer the questions from the book - which are mostly of technical nature. Actual models of (blood) flow are far too sophisticated for a first-year calculus student.

Without understanding background mathematics, investigation of a 'real-life' problem - unfortunately, in many cases - reduces to repetition of material presented in the text and memorization without much understanding or sophistication.

Let us consider an example, taken from a student's essay:

> "... take for example Edward Lorenz's discovery of the butterfly effect ... an assumption was widely held [in science] that the rounding of numbers would have little effect on the final answer of a calculation, because the rounded values would cancel each other out. Lorenz proved that, by rounding, a discrepancy in value would compound itself until the final value was completely incorrect ... this discovery went against a basic scientific concept, but still proved to be valuable, as it underlies the unpredictability and consistency of weather."

On top of obvious problems - such as not explaining why is the phenomenon under investigation is called the butterfly effect, and what is meant by 'unpredictability and consistency of weather' - the student missed to mention a crucial fact: the described type of behaviour characterizes non-linear systems, and does not occur in linear systems.

A common misconception among students (and not only among students) is that any use of mathematics objects is mathematics; e.g., an essay on the appearance of number seven in the Bible is mathematics; or, the existence of half-tones and quarter-tones in jazz shows that jazz is somehow linked to mathematics. Likewise, talking about three-dimensional objects (say, in architecture) is not geometry – it will become geometry, if one proceeds by asking good and 'provocative' questions … how do the shapes of the buildings fit together, how do they relate to each other? What mathematics

[5] James Stewart, 'Calculus: Concepts and Contexts.' Brooks/Cole, 2001.

functions would best describe their shape? Finally, when the investigation is finished, is it possible to use the experience and knowledge in a different context - say, to visualize regions used in double or triple integration?

How do I explain to my students what it means to be critical? There are no convenient definitions or recipes - I start by discussing examples. A student once wrote that

> " … it has recently been proved that the number of prime numbers is infinite."

What is the meaning of the word 'recently' in this sentence? How long ago was it - hundred, thousand, two thousand years ago? Another student wrote:

> "... parabola is another form that appears repeatedly in nature; the curve is created by gravity, and can be observed in the pathway of a flying stone, spear or arrow, water drops of a fountain or cascade […] a rainbow takes a similar shape."

What does it mean that 'a rainbow takes a similar shape?' Actually this is an excellent opportunity for an investigation - exactly what is the shape of the rainbow? Flying stone is affected by gravity - does the same principle work for the rainbow? Consider one more example:

> "… Greeks saw the structure of math as beautiful ... Greek math avoided the irrational number because they did not believe that such a thing existed. The concepts of numbers and theories were described as being good. Evil presided with the unknown, the irrational numbers and theories. The irrational number is ugly and frightening ..."

Good start, but a math essay should not end here. Investigating why was an irrational number 'ugly and frightening' should lead a student towards a concept of a rational number - that, in turn, can lead to more sophisticated math topics, such as Diophantine equations.

Mathematics writing requires a good degree of originality and creativity. How does one explain a difficult mathematics concept (or a formula, or a computation) to a layperson? One has to simplify the content, without placing the integrity of mathematics in jeopardy. A creative approach will look at experiences of the expected audience, and try to incorporate it into the presentation. A good story is one of the elements I am looking for in my students' essays.

For example, an interdisciplinary project could start by exploring symmetry, say, in the work of M.C. Escher. Then, it proceeds towards investigating mathematical foundations of symmetry, such as concepts of rigid motions and groups. The works of Escher can now be revisited and described in the newly acquired mathematical framework. A final step could aim at identifying symmetric objects in calculus, algebra, or differential equations.

4. Conclusion

Interdisciplinary component of the Mathematics course taught in the Arts and Science Programme at McMaster University is implemented in several ways, varying in depth, width and level of involvement of other courses. Of a number of issues related to the course, this paper uses instructors' experience and examples of students' writing to discuss the features of the narrative in mathematics. Used as a vehicle to enhance understanding of mathematics and to build and improve research and communication skills, good writing is a key to a successful and productive interdisciplinary mathematics course.

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