CASE STUDIES IN THE SHORTCOMINGS OF MAPLE IN TEACHING UNDERGRADUATE MATHEMATICS

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ABSTRACT

In this paper, we will investigate some of the algorithmic inadequacies and limitations of Maple as well as the common misuses of the software when used as a tool in teaching undergraduate mathematics. We will present examples for which Maple produces misleading or inaccurate results. We will also refer to situations where Maple gives accurate, but incomplete, results which are misused or misinterpreted by novice users of the software, specifically the undergraduate students. The authors have over ten years of experience in using Maple as a teaching tool and some examples presented here are based on those classroom experiences. Other cases have been reported by our students, by our colleagues and in various newsgroups devoted to discussions on Computer Algebra Systems (CASs). Many of the previously reported software bugs, observed in the earlier versions of Maple, are now corrected in the most recent release of the software. So, although we have occasionally referred to the older versions, we have presented the actual output only from the latest version of Maple, namely Maple7, in this paper. For the sake of brevity, we have limited our discussions to the topics which are ordinarily covered in the first two years of a typical undergraduate mathematics curriculum such as limits, single and multivariable integration, series, and floating point arithmetic. We have also tried to limit our case studies to the most common features of Maple, specifically those features that are widely used by the undergraduate students who are new to Maple.

Keywords: Maple; technology; shortcomings; bugs; undergraduate mathematics

Introduction

Computer Algebra Systems (CASs) have become increasingly popular tools in teaching mathematics in the past decade. The use of CAS has caused drastic changes in teaching undergraduate mathematics courses, particularly pre-calculus and calculus courses. According to the CBMS survey [1], 18% of Calculus I and II courses involved computer assignments in 1995, up from 9% in 1990. Assuming that the same trend has continued throughout the 90's, one could speculate that CAS has now become a major component of teaching in approximately 50% of Calculus I and II classes. The extensive mathematical assistance, symbolic manipulations, computational power and graphical abilities of CAS can greatly help students to explore mathematical topics and experiment with ideas without labouring through cumbersome calculations. The educators in mathematics community have hoped that CAS would enable students to develop an investigative attitude toward mathematics. A multitude of textbooks, workbooks and project manuals have been published to encourage and help the students toward this goal. Unfortunately, most of the literature is focused on the power of CAS, use of the commands, and to a lesser extent the programming aspects of CAS. Few of these books discuss the limitations and inadequacies of the software and the potential for misuse of CAS. As a result, the novice users such as beginning undergraduate students, who lack mathematical maturity, often mistakenly, assume that the "black box" software can solve any mathematics problem completely and accurately. This paper is written to demonstrate some of the shortcomings of one of the most popular CASs, namely Maple. We'll present examples from a typical pre-calculus and calculus course where Maple produces incomplete, inaccurate, or misleading results. We'll start each section with an example or two where the earlier versions of Maple produced inaccurate results and later these algorithmic bugs were corrected in the most recent version of Maple (version 7), and conclude the section with examples and actual output of Maple7 where the software still has difficulty to produce an accurate result. The examples are taken from a variety of topics. Although we have many examples in our disposal, we have limited our presentation to those examples which best demonstrate the shortcomings of the software. In section 1, we'll discuss solving scalar equations, section 2 is devoted to limits, and section 3 deals with sums and series. Single and multivariable integrals are discussed in section 4. Some of the examples presented here are based on the authors' classroom experience and our students and colleagues have reported some examples to us. However, most of our information is based on the *Maple User Group archives* and the internet discussion groups devoted to CASs, specifically: *sci.math.symbolic*, and *comp.soft-sys.math.Maple.*It is important to note that the authors have no intention of downplaying or downgrading the importance of CASs in general, and Maple in particular. CASs have revolutionalized the teaching of mathematics and we wholeheartedly endorse the CAS-based mathematics instruction. The pitfalls of earlier versions of Maple (which many have been corrected in Maple7) have not diminished our interest in the use of the software in our classes. We have used Maple in our classrooms for over a decade and we'll continue to do so enthusiastically in the future.

1. Solving scalar equations:

It was reported in [2] that *fsolve(x^5-5^x,x,x=3..5)*, using MapleV3 gives an output of *x=4* which is clearly incorrect. The solutions are *x=1.76 and x=5*. Apparently there was a bug in the Newton's algorithm. The algorithm is corrected in Maple7. In [3], it is reported that MapleV5 command of *solve(x^2=Pi*tan(1)+sin(1),x)* which should have two obvious and trivial solutions produces no answer. The following two examples are the actual output of Maple7 which demonstrates some of the inadequacies of the software. The first example is using the command *allvalues*, which should return all of the solutions of the polynomials. It appears that for the first problem *x^3*(x-1)*, Maple produces the expected four roots (three roots of 0 and one equal to 1), but in the second example we get only one root of 5 (instead of 4 equal roots of 5):

```
> #Example1.1- find all roots of x^3(x-1)
> allvalues(RootOf(x^3*(x-1)));
```

$$1, 0, 0, 0$$

```
> #Example1.2- find all roots of (x-5)^4.
> allvalues(RootOf((x-5)^4));
```

$$5$$

The *solve* command of Maple sometimes has difficulty with equations that contain floating-point numbers, particularly when the expression involves exponents. The following example and solution taken from [4], demonstrates such a case and offers a remedy. Specifically, it suggests that we replace the value of the exponent by a symbolic parameter, then solve the equation in terms of the parameter and substitute the value of the parameter at the end of the procedure.

```
> #Example1.3-solve the given equation using the floating-point values.
> solve(1.03*x^0.67=67,x);
Warning, computation interrupted

> # solve appears unable to get the solution.Abort the computations and
use rational representations.
> evalf(solve(103/100*x^(67/100)=67,x));
```

$$508.5395605, 506.3050286 + 47.62040174I, 499.6210698 + 94.82231377I,$$
$$488.5464231 + 141.1909244I, 473.1784128 + 186.3187436I,$$
$$453.6520937 + 229.8091870I, 430.1390637 + 271.2800590I,$$
$$402.8459560 + 310.3669124I, 372.0126237 + 346.7262499I,$$
$$337.9100311 + 380.0385447I, 300.8378729 + 410.0110472I,$$

We get the solution we want (508.5395605) and a lot of complex solutions, which are omitted for the sake of brevity, so we'll try another approach [4]:

```
> # solve by replacing the exponent with a symbolic parameter.
> solve(1.03*x^p=67,x);
```

$$e^{\left(4.175133817 \frac{1}{p}\right)}$$

```
> eval(subs(p=0.67,%));
```

$$508.5395595$$

2. Limits

It appears that if the command *limit* is used to determine limit of unassigned functions *f* and *g*, all versions of Maple, including Maple7, return *f(0)g(0)* which is clearly incorrect. Example 2.2, taken from [5], demonstrates another strange behaviour of the command *limit*. The limit in both cases should return unevaluated. Consider the Maple 7 output:

```
> #Example2.1- find limit of f(x)*g(x) as x approaches 0
> limit(f(x)*g(x),x=0);
```

$$f(0)\, g(0)$$

```
> #Example2.2- find limit of f(x)*exp(-x) as x approaches infinity
> limit(f(x)*exp(-x),x=infinity);
```

$$0$$

3. Sum and Series

The earlier versions of Maple (V3 and V5) had an algorithmic bug in summing infinite terms of divergent series. Apparently, Maple did not check for convergence first, rather it used various sum formulas, which are only valid outside the range of convergence of the series. Here are a few examples: it is reported in [6] that the command *sum((-1)^(n+1),n=1..infinity)* produces a sum of 1/2, which is clearly incorrect, since the series is a well- known divergent series. The command *sum(n!,n=0..infinity)* produces a surprising (complex) result of *0.69717488 –1.1557273i* [7]. Most of these bugs have been corrected in Maple7. However there are still a few left. Following is an actual output of Maple7 for a limit/series problem. Note that generally Maple looks at the leading term of a series for finding limits. In the following example [8], Maple clearly fails to see that the sum of the two trigonometric terms is zero and mistakenly returns zero (instead of *x*) as the limit of the expression.

```
> #Example3.1- find limit of the given expression
> g:=x+(-cos(9/50*Pi)+sin(8/25*Pi))/h;
```

$$g := x + \frac{-\cos\left(\frac{9}{50}\pi\right) + \sin\left(\frac{8}{25}\pi\right)}{h}$$

```
> limit(g,h=0);
```

$$0$$

Here is another example from Maple7 that perhaps has more to do with the floating-point arithmetic [9] than series. Note that a simple change of exponent from an integer “1”to a floating-point representation “1.”creates a totally different and incorrect result.

```
> #Example3.2-Comapre series expansion of 1/(1-x)^1 and 1/(1-x)^1.
> series(1/(1-x)^1,x);
```

$$1 + x + x^2 + x^3 + x^4 + x^5 + O(x^6)$$

```
> series(1/(1-x)^1.,x);
```

$$1$$

4. Integration

There is a multitude of problems in single-variable integration that Maple, specifically the earlier versions of Maple, fail to produce correct results. In fact, the majority of reported software bugs to Maple-related Internet sites were (and continue to be) about antiderivatives and definite integrals. The primary reason behind many of the inaccurate or incomplete results appears to be the issue of multivalued functions in the complex plane. If the path of integration crosses the branch cut then the definite integral often returns an inaccurate result. We suspect that there are also problems with the implementation of Risch’s algorithm. Here are a few examples from MapleV which since have been corrected in the most recent versions of Maple (versions 6 and 7): it is reported in [2] that both MapleV3 and V4 fail to produce an accurate result for the simple antiderivative problem of *int(sqrt(x)*sqrt(1+1/x),x)* .In another example [10], *int(log(sin(t)),t=0..Pi)* returns 0 which is incorrect, while *int(log(sin(x),x=0..Pi)* returns *–Pi*ln(2)* which is correct. Following is the actual output produced by Maple7, which demonstrates some of the persisting bugs in the software. In the first example [11], Maple gives a complex result to a definite integral, which clearly has a real value. However, If we use the inert command for integration (*Int* instead of *int*) we'll get the correct result. The reason appears to be that by using Int, Maple avoids finding antidrivatives and employs a numerical approach to find the result of the integral. Whereas, if we use the int command, Maple first finds the antiderivative, and then uses the Fundamental Theorem of Calculus to calculate the integral and somewhere in that process Maple7 commits an error. In the second example, the *int* command again produces a complex result to a positive integrand evaluated over a real interval. Although the integral is not a trivial one, but one expects that it be either returned unevaluated or some kind of message is given about the non-existence of an elementary antiderivative. The numerical integration using *Int* produces the correct result.

```
> #Example4.1-evaluate the integral using int and Int
> evalf(int(log(5+cos(x)),x=0..1));
```

$$1.764697796 + .88\ 10^{-9}\ I$$

```
> evalf(Int(log(5+cos(x)),x=0..1));
```

$$1.764697791$$

```
> # Let's increase the digits to see if int does better
> Digits:=15;
```

$$Digits := 15$$

```
> evalf(int(log(5+cos(x)),x=0..1));
```

$$1.76469779083464 + .136\ 10^{-13}\ I$$

```
> #Example4.2-evaluate the given integral using int and Int
> evalf(int(1/sqrt(2+x^4),x=0..1));
```

$$.4790759386 - .4790759386 I$$

```
> evalf(Int(1/sqrt(2+x^4),x=0..1));
```

$$.6775156893$$

We close this section with an example on double integrals. The example is taken from [12] and involves a trivial double integral over a rectangular region. It appears that Maple7 produces different results when the order of integration changes. The correct answer is 3.066667. The error first reported in 1996 and it appears that it has not yet been corrected. Here is the actual Maple7 output:

```
> #Example4.3-evaluate the double integral over the rectangular region.
> evalf(int(int(abs(y-x^2),x=-1..1),y=0..2));
```

$$3.216988933$$

```
> evalf(int(int(abs(y-x^2),y=0..2),x=-1..1));
```

$$3.066666667$$

As a final note, it is worth mentioning that the users of Maple or any other CAS sometimes use the words "pitfall", "bug" or "error" improperly. The user of the software, occasionally, makes an assumption (presumption?) about a command, which simply is not shared by Maple. In the following example [13], the user is surprised at the fact that Maple7 can not simplify ln(exp(f)) which is expected to be f. However, as it is explained in [13], Maple does not know that the parameters, t, C and R represent time, capacitance and resistance which are real numbers. Therefore, one has to inform Maple7 explicitly that all the parameters are real. Maple7, then returns the simplified expression. The actual output of Maple7 is presented here.

```
> # simplify the given expression
> result:=ln(exp(t/R*C));
```

$$result := \ln\left(e^{\left(\frac{t\,C}{R}\right)}\right)$$

```
> simplify(result);
```

$$\ln\left(e^{\left(\frac{t\,C}{R}\right)}\right)$$

```
> # simplify does not work since Maple needs more information about t, C and R
> simplify(result,assume=real);
```

$$\frac{t\,C}{R}$$

Concluding remarks and acknowledgements

In this paper, we examined some of the shortcomings of Maple through examples. We presented examples from older versions of the software, which are now corrected in the latest version of Maple. We also presented examples to demonstrate some of the bugs, which still exit, even in the newest version of Maple. Some of the examples presented in this paper are taken from the posted problems and solutions in various newsgroups, most notably *sci.math.symbolic*, *comp.soft-sys.math.Maple* and *the Maple user Group archives*. We are very thankful to all of our colleagues in the mathematics community, who were, and continue to be, involved in these discussions, particularly those who utilize CASs in teaching of undergraduate mathematics. Our main objective in writing this paper is to encourage a conversation among the mathematics educators on the practical aspects of using CASs. We hope that our work will be of some use to the educators in the mathematics community who are involved in CAS-based mathematics instruction. We also would like to thank the Texas Lutheran University, which partially supported this research through the TLU Research and Development fund.

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