LEARNING WITH TECHNOLOGY: SIMILARITIES IN MATHEMATICS & WRITING

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ABSTRACT

In this paper, we will identify similarities in the learning and the creative process using technology in mathematics and writing at the college level and draw a parallel. Specifically, we will examine the parallel on learning symbolic representations at different levels with special attention to how controversial technologies--such as numeric, graphic, and symbolic calculators in math or word processors, spell checkers, grammar checkers, and graphic organizers in writing--help learning. We identify comparable learning variables in writing and mathematics using a theoretical model. Finally, we present specific parallel examples in solving problems in mathematics and in the writing process using technology.

Keywords: symbolic representation, learning variables, writing

Introduction

Learning mathematics and English composition is, on the surface, two very different enterprises. In this study we examined whether the use of technology makes learning these two subjects more similar in some fundamental ways. Do we teach the students how to use the existing technology to further their learning and creativity? We will identify similarities in the learning and the creative process using technology in mathematics and writing at the college level and draw a parallel.

Basic steps in the learning process such as constructing, exploring, and experimenting can be personalized and accelerated using technology. We will oversimplify and omit many obvious technical and discipline-specific differences for the sake of the comparison. Many other simpler parallels with examples from the natural sciences and engineering can be made. We compare writing to mathematics to show learning commonalities across the wide range of subjects and across developmental ages.

This inquiry draws on three sources of information. Reflecting the experiences of authors in both disciplines, they include a review of the literature on learning and technology, an ongoing study of undergraduate students in honors mathematics classes, and clinical casework with children and adults with learning disabilities. O'Donnell headed a federally funded, two-year, writing team with a jury of national learning specialists who reviewed the research on learning and technology, including content areas of writing and math. Gavosto has used the symbolic capabilities of a graphing calculator in teaching multivariable calculus, linear algebra, and ordinary differential equations in honors courses at the University of Kansas. Her observations of the students in her classes motivated part of this work.

This paper is organized as follows. We start by discussing the common features of the learning process between mathematics and writing, followed by examples. Finally we draw some conclusions and propose areas of future study.

General Similarities in the Learning Process using Technology

To talk about the similarities, we start by describing the technology considered. In writing, the technology refers to a word processor like Microsoft Word with a built-in spelling and grammar checker, a dictionary, and the capability of inserting graphics from a file in the text. It also includes cognitive mapping options or graphic semantic organizers, such as *Innovation* software commonly used in the brainstorming and conceptual structuring phases of writing. In mathematics, the technology considered is a TI-89, that is, a graphing calculator with symbolic capabilities. Similarly, a computer algebra system like *Mathematica* or *Maple* could be used.

During their first two years of college students typically demonstrate basic knowledge of the technology used in classes and have covered material with equivalent content in lower level courses. While we acknowledge significant individual differences in both content and technology skills, our comments here refer to students with basic knowledge of the technology and the content taught.

Learning, as we use it in this discussion, refers to the evolving science of human learning as described by the National Academies of Science in three recent works. The first, *How People Learn: Brain, Mind, Experiences, and School* (National Research Council, 1998/2000), reviews the current empirical research base. The second, *How People Learn: Bridging Research and*

Practice (National Research Council, 1999a), addresses the relevance of that research base to classroom practice and teaching. The third report, *Improving Student Learning: A Strategic Plan for Education Research and Its Utilization* (National Research Council, 1999b), proposes a 15-year plan of changes, now underway, to advance the learning of students and teachers. In brief, learning involves the change from not knowing to knowing. This essential "change," which manifests learning itself, results from experiences with technology merging with content of mathematics or writing.

What does the technology offer to the students? First, the technology gives multiple representations of the concepts. For example, many mathematical concepts consist of three different representations: numerical, symbolical, and graphical. The calculator or computer software allows the student to generate these representations easily. The corresponding paradigm in writing could be the semantic, syntactic, and graphical representation with words, picture, or graphic organizers in the form of concept mapping. Areas in which we note similarities between learning to write and learning mathematics are represented in the learning variables research grid (see Table 1).

Table 1. Learning Variables Research Grid

	Association of Elements	**Rule Systems**	**Reasoning & Comprehension**	**Cognitive Levels**
Math & Science	Symbol-Referent (& properties)	Numerical Rules Geometric Rules Formulate Rules	Problem Solving & Scientific Comprehension	**Features Analysis** **Discriminate vs. Generalize**
Reading	Grapheme- Phoneme (& properties)	Phonic Rules Syllabic Rules	Reading Comprehension	**Recognize vs. Recall**
Language	Phoneme-Referent (& properties)	Features of Lexicon Transformation Grammar Rules Phase-Structure Rules	Language Comprehension (written & oral)	**Attain Concept vs. Form Concept** *Among many others*

The cells in Table 1 describe different components of mathematics and written language within the categories of association elements, rule systems, underlying comprehension components, and multiple cognitive levels. For example, in the first column, associative elements, the various symbol systems link to content-specific referents. Technology may help with the number symbol associations in mathematics and the letter symbol associations in writing or the musical notation associations in musical composition by providing the learner with a way to manage the rule systems. We mention the musical notation to emphasize that, in addition to the three rows on mathematics, reading, and language (written and oral), one could add other rows whenever a new symbol system would fit in the first column. The musical notation symbol system (association elements in column one) is genuinely different from these symbol systems listed here. As a result, music would merit its own row; the row would link across for music with its own rule systems, reasoning and comprehension, and cognitive levels.

In the second column, rule systems, technology offers a functional perimeter within which allowable responses can be permitted. Disallowed responses can be prevented within the rule systems in math and writing using technology. This extraordinary active learning feature assists the mechanistic aspects of written language by means of spelling and grammar checkers for easier editing, and the mechanistic computational aspects of mathematics.

In the third column, reasoning and comprehension, a fundamental commonality crosses subject content areas. For example, in both mathematics and language, the reasoning and comprehension demands must link to stored prior knowledge in memory. When no existing connections are possible, the learner may cognitively seek this connection to *assimilate* incoming information or to *accommodate* a restructuring of the internal information. Building on prior learning means a vast collection of errors and correct information continues to be gathered by the learner. Comprehension efforts tap into to this cognitive storehouse. Understanding of the content in math may require problem solving with this knowledge, for example, while understanding of written language may involve composing complex written ideas with this knowledge.

All subject content areas (in rows 1, 2, and 3) share the cognitive levels (in column four). They represent intelligence, learning, memory, and cognition, which change rapidly across the developmental years from birth to adulthood. The leading explanations of the revolution in behavioral and cognitive psychology in the 1960s and the new science of learning in the 1990s include structural/behavioral models of development (Horowitz, 1994; Sameroff, 1983), cognitive/information processing models (Anderson, 1983; Anderson, Reder, & Simon, 2001; Newell & Simon, 1972); and the connectionist/ neurocognitive models (McLeod, Plunkett, Rolls, 1998). Composing text through the writing process demands linking the symbols and ideas through behavioral, cognitive, and connectionist processes.

Several analyses have been published describing the relationships between cognition and mathematics (relevant to the fourth column of Table 1). A more extensive explanation includes, for example, the work by Tall (1992) about the mathematical processes and symbols underlying undergraduate mathematics education. In addition, Anderson et al. (2001) analyze the "applications and misapplications of cognitive psychology to mathematic education (p. 1)."

An extensive research literature on learning and technology suggests that technology can facilitate the learning experience thorough diverse symbolic representations, though not always in the ways people expected. For example, achievement gains did not show up as expected. Perhaps better clinical trials in longitudinal studies will be needed to demonstrate expected gains. However, results are clear that students' motivation increases in mathematics and writing with technology tools. Students spend more time on the task. Universal design of instruction and universal design of classrooms, as well as accommodations with technology, provide access to students with disabilities who might otherwise be unable to participate in math and writing. Collectively, these results benefit learning (Anderson & Horney, 1997; Applebee, 1984; Bangert-Drowns, 1993; Cochran-Smith, 1991; O'Donnell, 2001; O'Donnell, Alexander, Jensen, 1999; Okolo, Hinsey, Yousefian, 1990; Woodward & Rieth, 1997). A National Academies of Science study concluded that technology impacts learning by: "(1) bringing exciting curricula based on real-world problems into the classroom; (2) providing scaffolds and tools to enhance learning; (3) giving students and teachers more opportunities for feedback, reflection, and revision; (4) building local and global communities that include teachers, administrators, students' parents, practicing scientists, and other interested people; and (5) expanding opportunities for teacher learning" (National Research Council, 1998/2000, p. 207).

At the college level the technology may help students either write a better text or give a better solution to a problem. Some of the crucial elements of the learning process include experimentation, exploration leading to discovery, and construction. By using technology, students may accelerate these steps at their own pace in the following way.

Experimentation with technology allows students to try many different investigational approaches, as text and formulas can be freely manipulated without the worry of correctness. Different versions may be "cut and pasted" repeatedly in writing and mathematics. Word processors give the text great mobility and so do calculators with mathematical expressions. Speller checkers, grammar checker, and symbolic capabilities of the calculator reassure students and help them persevere on difficult tasks.

Exploration with technology opens up many possible capabilities. A complicated problem or challenging essay can be simplified to a manageable problem. In the same way a finished product may be generalized. The length of the final product can be easily changed, shortening or lengthening the solution.

Construction with technology, particularly the input-output nature of the technology, provides an ideal tool to construct an essay or the solution of a problem. Building blocks (text and mathematical expressions) can be saved and combined in many different ways, yielding multiple levels of text and multiple levels of computations.

Our observations lead us to conclude that the technologies of writing and mathematics are especially well suited for learning that involves experimentation, exploration, and construction. The learner benefits from the multiple forms of numeric, graphic, and symbolic representation with more fully articulated understanding of concepts. Three core attributes of learning selected by the National Research Council (1998/2000), with which we think the technology helps, are (a) learning with understanding is essential, (b) learning builds on pre-existing knowledge, and (c) learning is an active not a passive process. The multiple representations created with calculators and computerized word processing and cognitive mapping graphics reveal concepts that enrich understanding, build on prior learning, and necessitate active participation in learning. Through the technology's reiterative editing features, through symbolic representations, and through successive approximations of the correct final work, the word processor and calculator generate learning episodes across the K-16 experience.

Examples

In this section, we give examples of the similarities between learning mathematics and learning writing described above. We will draw a parallel using the classical approach of Polya (1957) for problem solving with the addition of technology. The concrete example in mathematics to illustrate the use of the technology is a word problem from an ordinary differential equations course. The problem is an application of Newton's Law of Cooling. The data are the constant ambient temperature and the temperature of a body at two different instants of time. The question is to determine the amount of time needed by the body to reach a certain given temperature. We describe how this problem could be solved using graphing calculator with symbolic capabilities, like TI-89, by students working in small groups.

1. *Understanding the problem*: defining the unknowns, data, and conditions. In our problem, the wish to use the technology forces the definition of the variables. For instance, the two known points of the graph of the temperature function can be plotted after identifying the corresponding variables. Conjectures about the possible solutions and the model can be made.
2. *Devising a plan*: identifying what type of differential equation the model gives and how to solve the equation. The calculator can help in setting up the solution since the functions that solve the equations symbolically will only accept input in a certain order. This feature will not set up the equation, but it may help detect very rough conceptual errors like the dependence of the variables.
3. *Carrying out the plan*: finding the solution. The calculator can be used to solve the equation and can help find the two constants involved. It can also be used to check algebraic and numerical computations.
4. *Looking back*: examining the solution. After obtaining the solution, plotting its graph with the calculator translates the symbolic solution to a graphical one. The graph provides ample qualitative information of the temperature. In particular, this information verifies the validity of the model and the answer to the problem.

Gavosto observed that her students were able to solve problems like this using a symbolic calculator without any previous experiences with comparable problems. With the calculator, the students demonstrated more confidence in handling all the variables and constants involved in attacking the problem. The calculator also helped communication among the students about the steps involved in the computations. The best students were able to be analytical and critical of the feasibility of the solution obtained.

Examples in writing parallel those in mathematics. The steps of the writing process have been delineated through a compelling body of empirical research, ushered in by Janet Emig (1971) and Donald Graves (1983), and expanded in the work of many others such as MacArthur and Graham (1987) and Cochran-Smith (1991). *Process writing* features the steps of planning, prewriting, writing or composing, and revising or editing. Students share their work with peers and seek editorial response to their work in each prepublication step of the writing process. The number of parts in the writing process and their names have changed over time. Now, experts in the field have settled on five: prewriting, drafting, revising, editing, and publishing. The arrows in the process model represent how writing steps operate in a recursive, not sequential linear process (see Figure 1).

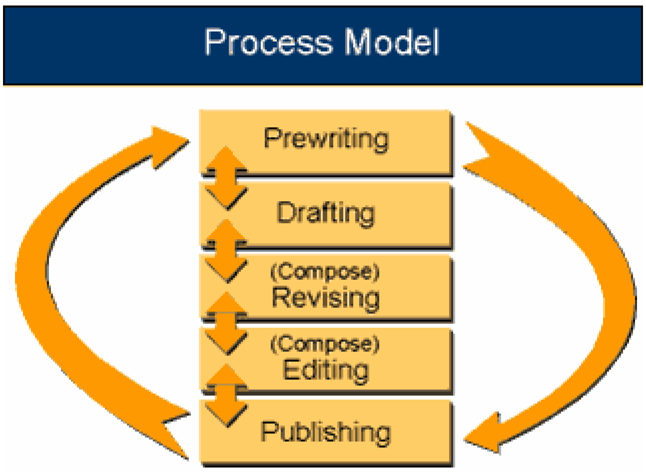

Figure 1. Writing Process Model (O'Donnell et al., 1999).

1. *Prewriting* involves brainstorming discussions and divergent thinking about alternative approaches to the writing problem or content to be written. This is analogous to Polya's first step of understanding the problem. The technology designed for this step includes computerized cognitive mapping and graphic organizing tools (such as *Innovations* software).

2. *Drafting* refers to the initial step of generating and structuring the ideas in written words. The outlining tools on the word processor sometimes help in this stage, but more often drafting is the initial attempt to put the ideas into rough connected written text. Several versions of drafts may show significant change during this stage. Since writing is a recursive process, the writer(s) may return to the previous prewriting step and back to drafting interchangeably several times as the text takes shape. This step may be most nearly analogous to the Polya's "devising a plan."

3. A. *Revising* is the first part of composing text. This step grooms the ideas and structure of the work. It fleshes out the text to full length, taking different shapes depending on the type of writing. For example, in an essay it involves the introduction to the topic, the analysis supporting the focus, delineating the arguments, providing evidence, and drawing conclusion from the discussion. Writers employ the full range of word processing options. Some authors use connectivity tools through the Internet to gather information, or e-mail to engage in shared writing projects with advanced track changes (such as *Microsoft Word Track Changes* for multiple authors). Revising processes resemble Polya's "carrying out the plan."

B. *Editing*, the second part of composing, consists of bringing the digital text into finished form. At this point spell checkers and grammar checkers, plus language tools like Thesaurus and Dictionary, are the features of word processing most commonly employed. This component may still be part of Polya's "carrying out the plan." It corresponds to checking the solution in mathematics, and is a major step in the writing process.

4. *Publishing* refers to the final step of making the work available in its finished form and receiving feedback from the intended audience. This takes many forms, in addition to the formal publishing well known to academics. It is the end-stage for all writing. For young children it can include sharing work with classmates and family. It may involve students writing in newsletters, yearbooks, journals, and multimedia digital alternatives with web sites, PowerPoint presentations, and Access Grid (supercomputing or Internet2) presentations. Publishing means entering into a dialogue about the written work with the intended authentic audience as a way of examining the validity of the approach taken. In this way the last step in the writing process may be similar to the last step in Polya's problem solving process of "looking back" after obtaining the solution to verify the validity of the answer to the problem.

Conclusions

We have not attempted a full comparison of both writing and math technology applications. Rather, we selectively described writing technology, such as the word processing and concept mapping of ideas in written work, to illuminate the potential advantages of improved learning. The learning of mathematics using technology such as numeric, graphic, symbolic calculators, visualization software, and computerized modelling tools suggests many areas yet to be researched.

Empirical evidence demonstrating which approaches work best with which aspects of writing and math learning, for which students, should lead to changes in the approach of faculty who still prefer traditional lecture format without technology. Researchers initially mistakenly presumed that the mechanistic aspects of technology would make the word processor a good teacher of rote and simple learning, helping the poorest writers do better. However, results consistently show that the best writers benefit most from using the technology, while the poorest writers still need significant help to compose text. This lends credence to the observation that technology advances learning of complex and abstract ideas in situations involving problem solving and experimentation, especially for very bright students. Better understanding the learning process will help widen the benefits of the technology to a larger number of students. If longitudinal clinical trials research supports the observations noted by Gavosto and O'Donnell, then applying these ideas in university classrooms would change the way faculty members help their students learn about writing and mathematics.

The similarities described here in the learning process across subjects should be studied further. Faculty across disciplines can learn from each other how students learn using technology. Pointing out the parallel with their preferred discipline can motivate students who like one of the disciplines but not the other. A research challenge ahead will be to analyze the developmental learning variables of symbolic representations in mathematics and in writing using technology.

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