THE NEW MATHEMATICS MATRICULATION/ENTRANCE EXAM SYSTEM IN HUNGARY

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ABSTRACT

We are witnessing a long-term educational reform after the political changes in Hungary. The main elements of this reform - beside the question of educational management and finance - are the changes of the curriculum and the matriculation examination. Matriculation examination will have double function in the future that is, on the one hand, a final exam for secondary education and, on the other hand, an entrance examination for the tertiary level of education.

Within the frame of this examination reform we analysed the advantages and disadvantages of the present examination in the mirror of the expected social, educational policy and curriculum changes.

During our research and developmental work we considered the international trends and the applicable Hungarian traditions. We concentrated on the development of a new examination model and new types of tasks and items.

We had the opportunity to field test the new tasks and items and also to collect teacher’s opinions and suggestions. After a careful analysis the experiences has been built in the new examination model.

In our presentation we would like to demonstrate the new crystallized examination model and some of the new examination tasks.

The main characteristics of this model:

- Two levels, the upper level has the selective function for the tertiary education.
- The examination has a centrally developed written and oral part.
- Among the tasks there are short answer questions and some complex mathematical problems with multiple questions.
- The evaluation of the written part is based on a detailed evaluation guide.

The model will be illustrated with concrete examination tasks and their solutions.

Key words: Mathematics, curriculum reforms, matriculation and entrance examination, examination model, requirements for the matriculation and entrance examination, Hungary.
1. The reform of the matriculation/entrance exam in view of the education reform

In Hungary, education had a centralized system for nearly 40 years. This was apparent from the unified, central curriculum, which was compulsory for all, and the lack of choice in textbooks. In the teacher training university and college research workshops research and innovation has been done from as far back as the beginning of the 80s, whose aim was to create new textbooks and textbook families that will better serve methodological directions. Other important steps in the direction of decentralization were the following: the running of schools was decentralized, and the role and responsibility of local councils and communities increased. At the same time, new schools appeared that were run by foundations or churches. These changes have, of course, brought along a renewal of the context and regulation of the education system. As a result of a long innovation process a new curriculum was born in 1995, which instead of the old, strictly specified subjects was based on "cultural domains". It made a summary of its requirements for two-year periods, and it did not fill the whole number of lessons but gave way to - and in fact expected - additions to the curriculum on the basis of local needs. (NAT, 1995) This meant more freedom but also more responsibility for teachers – and it was welcomed by many, but was too fast a change for the majority. The preparation of local curricula meant such new tasks that teachers found it difficult to cope with them - and they made their changes with very mixed quality levels. Therefore, after the change of government in 1998 the new education ministry overruled the introduction of the core curriculum and created a new type of frame curriculum, one that gives more freedom to teachers than the old centralized one but which also has stricter regulations than the core curriculum. (Kerettantervek, 2000a; Kerettantervek, 2000b) For example, this new frame curriculum went back to the old subject system and to a yearly definition of requirements.

In the process of the education reform the main change from the point of view of the graduation/entrance exam system is that whereas in the past the core of the exam was determined by the contextual elements of the curriculum, there has now appeared - as new elements of the exam reform, based on the curriculum changes - a detailed description of requirements and a more strictly structured exam description. These changes will also serve the new needs that society creates, which will in turn increase the reliability of the exam results and ensure equity and comparability. This is not only a Hungarian but an international trend too. (Galbraith, 1993; Niss, 1993; Wain, 1994; Gipps and Murphy, 1996, Mátrai, 2001)

2. Description of the Mathematics matriculation/entrance exam

The current Maths graduation exam can be, in short, summarized as follows. Students can choose between two ways of taking their exam according to their plans for further studies.

1. A school exam can be taken by students who do not want to continue their studies or would like to apply to a higher education institute that does not require them to take a Maths entrance exam. These exams are based on the material covered by the minimum compulsory number of lessons. Such an exam has two versions, which are linked to the two types of secondary schools.
2. A **joint matriculation/entrance exam** must be taken by students who would like to continue their studies in a higher education institute that requires a entrance exam in Maths. With regard to their material context, these exams are not different from the school exam, though the questions are more complicated and require a higher level of Maths problem-solving skill.

The most important characteristics of the current Maths exams are summarized in Table 1:

**Table 1. Summarizing the possibilities of the current Maths Graduate exam**

<table>
<thead>
<tr>
<th>Exam attributes</th>
<th>Centrally designed, but locally taken exam- school exam</th>
<th>Centrally designed, externally taken exam – joint matriculation/entrance exam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Version</td>
<td>For secondary grammar schools</td>
<td>For vocational secondary schools</td>
</tr>
<tr>
<td>The set up of the exam</td>
<td>6 open-ended problems and the verification of 1 known theorem.</td>
<td>5 open-ended problems, 1 definition and the verification of 1 known theorem.</td>
</tr>
<tr>
<td>Duration</td>
<td>180 minutes</td>
<td>8 open-ended problems</td>
</tr>
<tr>
<td>Scores</td>
<td>Maximum 80</td>
<td>240 minutes</td>
</tr>
<tr>
<td>Evaluator</td>
<td>Secondary school teacher</td>
<td>External (+) secondary school teacher</td>
</tr>
<tr>
<td>The source of the assigned problems</td>
<td>Chosen from known problems (Gimes, 1992)</td>
<td>Chosen from known problems (Gimes, 1992)</td>
</tr>
</tbody>
</table>

The chart refers to the way of evaluation and the method of problem assignment. The fundamental difference between the two ways of evaluation is that while the school exam paper is checked by the secondary school teacher of the student, the joint exam is evaluated by two independent teachers - for two different reasons. On one hand, the secondary school teacher will decide the grade that a student will obtain as his/her graduate exam result; on the other hand, the external evaluator assigned by the given higher education institute gives the result that the success of an entrance exam will depend upon. The assignment of the problems for the school exam is based on a collection of problems that has a 20-year history and which contains over 4000 problems that have remained basically unchanged during this time and which are announced on the day of the exam via the media. (Gimes, 1992) The set-up of the test has also stayed unchanged over the years. The design of the joint exam is undertaken by a professional board and contains problems that are especially designed for the exam every year. (The taking of this type of exam is helped by the publishing of test papers from previous years.) We examined the advantages and disadvantages of the current matriculation exam as part of the research/innovation process pertaining to the new matriculation exam. On one hand, our research covered the analysis/evaluation of the design method and evaluation instructions going with matriculation exam test papers coming from previous years. (Tompa, 1999.) On the other hand, we analysed and
re-evaluated randomly chosen actual written test papers and their corrections and evaluations as
done by teachers.

From an analysis of the documents and a comparison of the results achieved by students in
concurrent years it was evident that the exams of each year came with a different level of
difficulty. (Tompa, 2001) As a result of this, the Maths grades of the various years are unable to
serve as a reliable basis for an evaluation of students’ actual knowledge. This research also showed
that exams set up according to these principles do not fulfil the criteria of objectivity and equality
and comparability; in other words, owing to a lack of sufficient evaluation instructions there is
room for subjective evaluations. As a result of a teacher’s strictness or leniency the results going
with individual classes can easily become up- or downgraded – so that an equal value being given
to different results cannot be guaranteed. (Frisbie, 1988; Gipps and Murphy, 1996)

Teachers’ opinions given during the course of the creation of the new requirements and the
testing of the new type of graduate exam show that Hungarian Maths teachers in general rejected
the type of exam containing closed-ended test questions i.e. which would be the best way to ensure
objective evaluations. When analysing exam models coming from other countries, such elements
are more common in exams that serve as higher education entrance exams. (Mátrai, 2001) Thus,
we concluded that, basically, our new exam model also favours open-ended test questions. We
simply cannot ignore the great amount of rejection involved here and choose closed-ended (e.g.
multiple-choice) test questions to out-rule the possibilities of subjectivity (Osterlind, 1998). This
view - which most teachers share - is also in line with the Maths exam philosophy of the exam-
designing workgroups.

3. The development of the joint Maths matriculation-entrance exam

Before we give further details about the new elements of the Maths matriculation-entrance
exam, we would briefly like to summarize those educational policy decisions that have an effect
on the whole of the graduate-entrance exams.

The new matriculation exam is unified - which means that it measures students’ knowledge
under the same regulations, with the same test papers and evaluation mechanisms both in the
framework of regular and adult education, and both in secondary grammar and vocational schools.

The other important difference is the introduction of two levels relating to all subjects, i.e.
students can choose between a lower and a higher level of graduate exam; this latter will also serve
as an entrance exam. (This in the past was only possible in the case of a few subjects.)

The Maths exam design process is similar to that of the other subjects. The development was
preceded by a research period that made an analysis and comparison of Hungarian traditions and
international trends. (Lukács, 1997; Mátrai, 2001) The development has been carried out by a
diversely selected workgroup (among its members one can find experienced grammar and
vocational school teachers, higher education experts, curriculum and evaluation experts, and
textbook writers). Every document created by the workgroup (exam requirements, exam model,
exam descriptions, sample test papers, evaluation guidelines etc.) has to succeed in a multiple
professional evaluation, which means (among other things) professional proofreading, tutorial and
higher educational opinion polling, and the collection and use of the points of view of professional
pedagogical organizations. There are, of course, in these design-groups people whose main task is
to make up these new types of test questions and the detailed answers. The creation of the new
exam model was also preceded by testing some of its versions in schools.
The legal document for the exam contains the requirements in detail and a description of the exam for both levels.

4. The introduction of the joint Maths matriculation-entrance exam under development

In our present study we only have the chance to introduce the high level exam.

The requirement system consists of the following content elements, which is given further detail in the exam document, thus describing the contextual and underlying differences between the two levels.

4.1 Mathematical content of the requirements
1. Methods of Mathematical thinking, sets, logics, combinatorics, graphs
   1.1 Sets
      1.1.1 Operations on sets
      1.1.2 Cardinality, sub-sets
   1.2 Mathematical logic
      1.2.1 Concepts, theorems, proof and verification in Math
   1.3 Combinatorics
   1.4 Graphs
2. Arithmetic, algebra, number theory
   2.1 Basic operations
   2.2 Set of natural numbers, basic knowledge of number theory
      2.2.1 Divisibility
      2.2.2 Number Systems
   2.3 Rational and irrational numbers
   2.4 Real numbers
   2.5 Powers, roots, logarithm
   2.6 Formulas (“letter equations”)
      2.6.1 Notable identities
   2.7 Proportionality
      2.7.1 Percentages
2.8 Equations, equation systems, inequalities, inequality systems
   2.8.1 Algebraic equations, equation systems (linear, quadratic and higher order, square-root)
   2.8.2 Non-algebraic equations (absolute values, exponential, logarithmic, trigonometric)
   2.8.3 Inequalities, inequality systems
2.9 Means, inequalities
3. Relations, functions, the elements of calculus
   3.1 The concept of functions
   3.2 One-variable real functions
      3.2.1 Graphs of functions, transformation of functions
      3.2.2 Characteristics of functions
   3.3 Series
      3.3.1 Number series, geometrical series
      3.3.2 Infinite geometrical series
      3.3.3 Compound interest, allowances
   3.4 The elements of calculus – One-variable real functions
      3.4.1 Limit, continuity
      3.4.2 Differential calculus
      3.4.3 Integration
4. Geometry, coordinate geometry, trigonometry
4.1 Elementary geometry
   4.1.1 Elements of solid geometry
   4.1.2 Sets of points defined by the concept of distances
4.2 Geometric transformations
   4.2.1 Congruency (on the plane, in the space)
   4.2.2 Similarity transformation
   4.2.3 Other transformation (orthogonal projection)
4.3 Geometrical shapes (Plane shapes – solid figures)
   4.3.1 Plane shapes (triangles, quadrilaterals, polygons, circle)
   4.3.2 Solid figures
4.4 Vectors (two dimensional, three dimensional)
4.5 Trigonometry
4.6 Coordinate geometry
   4.6.1 Points, vectors
   4.6.2 Line
   4.6.3 Circle
   4.6.4 Parabola
4.7 Circumference, Area,
4.8 Surface, volume
5. Probability, statistics
   5.1 Descriptive statistics
      5.1.1 Data collection, systematisation of the data, data representation, visualization, diagrams
      5.1.2 The characteristics of the mass of data, measures of central tendency and dispersion statistical indicators
   5.2 Probability and the elements of inductive statistics (point-estimation)
      5.2.1 Characteristics of stochastic phenomena, probability
      5.2.2 Estimate of the relative frequency of a sample by the parameters of a population (Lukács, 2001a)

4.2. The structure of the joint matriculation-entrance exam

The high level Maths exam consists of a 240-minute written test and a 20-minute oral exam. Students can use a calculator and a Collection of Formulas and Functions both for the written and the oral parts. The parameters of these will have to be redefined every year.

Written exam

Content structure

The test thematically covers the 5 main topic groups of the requirement system.
When designing the exam paper, the following proportions will have to serve as guidelines:

<table>
<thead>
<tr>
<th>Topic Group</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methods of Mathematical thinking, sets, logics, combinatorics, graphs</td>
<td>25%</td>
</tr>
<tr>
<td>Arithmetic, algebra, number theory</td>
<td>20%</td>
</tr>
<tr>
<td>Relations, functions, the elements of calculus</td>
<td>20%</td>
</tr>
<tr>
<td>Geometry, coordinate geometry, trigonometry</td>
<td>20%</td>
</tr>
<tr>
<td>Probability, statistics</td>
<td>15%</td>
</tr>
</tbody>
</table>

These proportions, of course, are only guidelines, as a considerable number of test questions could belong to more than one thematic group, being built on a complex circle of knowledge; also due to the arbitrary parts of the exam these proportions could vary with each and every student depending on their choice of test questions. The first thematic group includes the parts of all those problems that require a translation of the text into the language of Mathematics or the creation of mathematical models.
40% of the test problems are situation-based, problems in connection with the everyday life, which will require the application of simple mathematical modelling.

*The attributes of the test paper*

The exam paper consists of 3 different parts that need to be attended to continually. Students have a maximum of 240 minutes to complete it, which time can be freely used. The maximum number of points that can be achieved is 115.

*Part I* consists of 4 questions. These can be regarded as easier problems based on the requirements of this high level exam; in general, they can be solved with the knowledge of the lower level requirements. (There is no free choice of questions in this section.) The questions might contain more than one sub-question. The maximum score is 50 points.

*Part II* consists of 4 questions, all worth 15 points. The candidate has to solve three of the four and only these three can be taken into account. The questions are, in general, based on the knowledge of one or two thematic groups. The maximum score for part II is 45 points.

*Part III* contains one complex question that combines several sub-questions, ones that are based on several thematic groups and which require practical problem solving and mathematical modelling. The correct solution to this problem is worth 20 points.

*Evaluation*

The guidelines for the evaluation contains a detailed solution to the test questions and its possible versions as well as the different sub-points that can be given in the various steps of the solution process.

*Oral exam*

*Content structure*

The oral exam is an external exam. The proportion of contents in the central list of series of questions reflects the proportion in the description of the written exam.

*Attributes*

Each series of questions is chosen from a specific thematic group. Every series of questions requires a student:

- to give a definition,
- to verify a theorem,
- to solve a problem,
- and to give an example for the application of the given thematic group within or outside Mathematics.

As the difficulty of the various theorems can vary, the equal level of the oral exam can be granted by a balance being given to the complexity and difficulty of the chosen questions.

*Evaluation*

The maximum score in the oral exam is 35 points.

The elements of evaluation:

1. Theoretical question and the problem-solving \( 25 \) points
2. The example demonstrating the application \( 5 \) points
5. Some results gained from the development of the tests and the evaluation of the documents

The earlier-mentioned trail exams had about 250-300 participants on every occasion on both levels. The students represented the two school types in equal proportions. From these trail exams, we gained information partly regarding the difficulty of the test questions and partly about how well the different tasks are capable of measuring the mathematical knowledge, skills and abilities as laid out in the requirements. During this experiment both teachers and students were asked to give their opinions about the types of tasks and the whole structure of the exam; and teachers were also questioned about how useful they found the evaluation guidelines given to them.

Students liked the new types of practical questions; however, some tasks, especially the ones that required a higher level of theoretical knowledge, were not carried out to an acceptable standard. (Lukács-Vancsó, 2001) The majority of teachers did not like the idea of free choice among the questions, fearing that their students would be put under even more stress when having to make such a decision in an exam situation. Yet the actual results show a different picture, in that students welcomed this new opportunity and used it well. Nevertheless, the experiment proved that this decision situation requires sufficient time to be allocated to it - and this will have to be taken into account when setting up the exam model.

Teachers had their reservations about the new type of practical questions, which could be summarized as follows:

- The new contents that appeared in the exam requirements and the actual questions are at the moment quite frustrating for some teachers. This is especially true with the theory of probability and statistics, which they will have to teach without actually studying or will have to do differently from the way they were taught. (This problem, of course can be solved by the further training of teachers.)
- Hungarian Maths teaching in general was always more theory-centred, and many teachers would not like to change that for reasons of conviction.
- Some teachers experienced that some of the new types of questions are more favourable to students who are less hardworking but have the necessary intelligence and creativity.
- In this modelling, several of the situational questions require the sort of communicational (comprehension) skills that have so far not been emphasized in Hungarian Maths teaching; thus, some teachers would find it a little problematic to test these types of question in an exam.
- Some teachers feel that Mathematics will suffer if this new exam drops the reproductive verification of mathematical theorems.

On the whole, however, the majority of teachers understand and accept the need for a change. This is shown in the following data. With regard to the higher level of exam, teachers gave the following responses:

- 92% of them agree with the set up of the detailed requirements
- for 90% of them the requirements of the framework curriculum are in line with exam requirements
- 74% of them agree with the introduction of free choice in the written exam
74% of them think that this type of written exam is suitable for the reliable measuring of a student’s performance

67% of them think that this type of oral exam is suitable for the reliable measuring of a student’s performance (Lukács, 2001b)

School will receive the new exam document at the beginning of the 2002/2003 school year in order to enable teachers to prepare their students for the new exam first taken in 2005.

REFERENCES


Annex 1: An example of the test papers

Part I

1. In a 70-membered sports delegation the average age of men is 37, of the women it is 23, and of the whole group it is 28. How many men and women were in the group?

2. The radius of the Earth is 6380 km, and the radius of the Sun is about 110 times this.
   a) How many square metres is the surface of the Earth?
   b) How many cubic metres is the volume of the Sun?
      Give the results in a normal form.
   c) The shadow of a ball standing on the ground reaches as far as 42.5 cm from its touching point. At the same time, the shadow of a 1 meter-tall child standing next to it is 2 metres. How large is the diameter of the ball?

3. József smokes one packet of cigarettes a day. The price of a box of cigarettes went up from 210 Ft to 250 Ft.
   a) How many percent is the price rise?
   b) If József’s net monthly income is 80 000 Ft, how many percent of his monthly income did he spend on cigarettes after the price rise? (Take into account 30-day months.)
   c) To protect his health and pressed by the recent price rise, József has decided to stop smoking. He will put the price of the 250 Ft. cigarettes in a bank at the beginning of every month. The bank will reinvest the interest, i.e. on the last day of every month they will add it to the actual amount on his account and this increased amount will continue to produce interest. The monthly interest rate is 2%. How big will the amount be that József can receive at the end of the 12th month?

4. a) Every year several thousand people apply for pilot training. They have to undergo 3 tests:
   A — a vision test,
   B — an allergy test, and a
   C — a height-endurance test.

   One year there were 2000 applicants.
   After the tests we have the following data:
   570 of them failed the vision test,
   798 people had some kind of allergy-related problems,
   65 could not endure heights,
   120 people had both vision and allergy problems,
   32 could not endure heights and had a vision problem,
   42 had an allergy and could not endure heights,
   25 of them failed all three tests.

   • How many applicants passed all three tests?
   • How many applicants had only allergy-related problems?
   • How many applicants had exactly two problems?

   b) You can get to and from any of five different airports. The airline runs 2 flights from the first, the second and the third airport, one flight from the fourth, and three from the fifth. Draw a network based on the above information.

Part II

From the next four question (5-8) you will have to choose three to solve.

5. What is more likely? If a regular dice thrown up six times will produce at least one six, or if a regular coin thrown up 10 times will produce at least 5 heads?

6. Before light bulbs were invented the windows of factories were designed to enable as much light as possible to get into them. Some factories used the so-called “Noorman window”. These consisted of a rectangle and a semicircle, the semicircle joined the rectangle on one of its sides and its diameter was as long as this side of the rectangle.

   If the circumference of the window is constant, how wide and how long should the rectangle be to let the largest amount possible of light through it?
7. 65% of Hungarian health officers are women. On one training course there were 100 health officers present. Give the interval that will include the number of women health visitors present with 90% certainty.

8. Solve the following equation on the set of real numbers:

\[
\begin{align*}
&5 - x^2 - y = 0 \\
&x^2 - 6 \sqrt{x^2 + y^2} + y^2 = -5
\end{align*}
\]

Part III

9. A radio tower is sending signals to an engine while it is moving along a line. Placed in a Cartesian coordinate system the radio tower is on the R(1;0) point. The equation of the t line is: \(2x + y = 30\), where all data is given in km.
   
a) Represent the situation assuming a coordinate system where the units on both axes are the same.
   
b) The engine gets its strongest signal in point C, so C is the point of line t that is closest to R. Define the coordinates of point C.
   
c) When the engine is more than 28 km away from the tower, it does not receive the signals anymore. Define the two end points of the section where the radio signals can still be received.
   
d) A further two equally strong radio towers will be set up in such a way that their signals can be received in the greatest possible area. Where should we place these two towers – taking into account the above parameters – so that one of the two towers could be received on the longest continuous line along t? Give their coordinates.
   
e) When building the towers the following iron units are used as supporting elements, whose height is 6 dm. Their other parameters can be read from their pictures in dm. How much will corrosion protection cost considering that the application of 1 square metre costs \(700\) Ft?
   
f) How big will the weight of one unit be if the density of iron is \(7800\) kg/m\(^3\)?
Annex 2: An example of the evaluation guidelines going with one question.

3rd question

a) 250 : 210 = 1.19
The price rise is 19%. 1 point

b) After the price rise
30 · 250 = 7500
7500 : 80 000 = 0.09375
At present, he is spending 9.4% of his wages on them. 1 point

1 point

c) 1 month’s saving: A = 30 · 250 = 7500
Monthly interest rate: 2%.
Therefore q = 1.02
At the end of the 1st month: Aq
At the end of the 2nd month: Aq^2 + Aq

If the student only counts with 1 day, he won’t get these 2 points.
If the yearly amount is individually calculated correctly, he will receive these 4 points. If he makes a mistake while doing it, he gets 1 point for every 3 good amounts.

For the formula of the geometrical series.

At the end of the 12th month:
Aq^{12} + Aq^{11} + Aq^{10} + ... + Aq^2 + Aq =

= Aq(1 + q + q^2 + ... + q^{11}) = Aq \frac{q^{12} - 1}{q - 1} =

= 7500 · 1.02 \frac{1.02^{12} - 1}{1.02 - 1} = 102 602 Ft

At the end of the 12th month he could receive 102 602 Ft.

If he only writes down the last formula, he will still receive 4 points.
If he takes the formula for the annuity from the Collection of Formulas, he will only get the amount he can receive at the beginning of the 12th month - and for this he can get 5 points.

Total: 10 points