TEACHING MATHEMATICS USING THE IDEA OF “RESEARCH PROBLEMS”

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ABSTRACT

Most mathematical tasks found in high school and early college textbooks begin with the words: “simplify the following algebraic expressions …”, “calculate the following…” or “solve the inequality…”. Mathematicians, however, more often deal with more open problems, where the main aim may be to establish whether the object with the given properties exists at all, or whether the given assertion is valid in principle rather than to simplify or calculate something. Undergraduate mathematics courses for preparing high school teachers might benefit from including a number of such “higher – order” tasks.

By the “research problem tasks” concept we shall mean those that are based on subjectively difficult theorems or mathematical constructions that are initially not known to a particular student (or he is unfamiliar with the proof modus operandi). There are such tasks that a student, when solving them, encounters the necessity to investigate mathematical models of configuration which are new to him, non-standard connections, existing between such models, properties of figures, and at the same time he has to find and establish a logical scheme of reasoning. Solution of a research problem task results in the established and well-founded algorithm of solution for the total class of similar problems or heuristic device, the scientific idea that, after being justified and generalized, can be used and recommended for the solution of other similar nonstandard problems. The proposed method is found to considerably intensify and advance the process of students’ mathematical training, to upgrade their knowledge, skills and habits. The conducted investigation has been reflected in Applebaum (2001) Ph. D. Thesis “Research Problem-based Mathematical Training Intensification and Advancement in Gifted Students “.

Rationale

Formation of mathematical thinking in children, training their minds in criticism, development of convergent and divergent faculties in an individual with simultaneous high-level support and enrichment of their knowledge, skills and habits is the key objective, task and challenge of a mathematics teacher.

Many researchers ([2], pp. 222 –248) noted that convergent intellectual faculties reveal themselves, first of all, in the efficiency of information processing, and in the capacity of quick finding the proper way out of the given situation. Divergent intellectual faculties manifest themselves in the ability to put forward a number of equally correct ideas concerning the same problem solution. Convergent and divergent intellectual faculties thus characterize the adaptive opportunities of individual behavior in the hidebound activity conditions.

The researcher A.D. de Groot has come to the conclusion that any creative act or product was in no way the result of intuitive inspiration or inherent geniality, but rather appeared as the result of specific individual development combined with long term accumulation and differentiation of experience, useful for the given sphere of activity. ([3], p. 68)

R. Gardner came to similar conclusions while describing the phenomenon of the "experience crystallization" ([4], p.26). It should be noted that Poincare ([8], p.79) has asserted similar ideas in his famous report in the Psychological Society in Paris. "The thing that surprises us first of all, I mean a visibility of a sudden inspiration, is an obvious result of the long unconscious work of intelligence in the field of the analysis of knowledge and experience that have been received in this time or another…"

Thus summarizing the foregoing, we shall emphasize:

1. The modern community increasingly more demands convergent and divergent intellectual faculties of a personality mental activity. At present, the tendency to enhance the role of these intellectual faculties is especially marked when choosing among the applicants for an office in different areas of human activity.

2. Convergent and divergent intellectual faculties of a personality can not be manifested and realized on "a blank place". The person's skills and habits of work in the chosen field of activity can effectively be manifested and developed only on the basis of solid knowledge mastered at the level of profound comprehension rather then just formally.

3. Convergent and divergent intellectual faculties of a personality are able to essentially improve his mental activity and make it constructive only then when these two branches of an individual facilities develop in parallel, supplementing and enriching each other.

Basis concepts and notations

By the "scholastic research tasks" concept we mean the subjectively difficult theorems or mathematical constructions that are not initially known to a particular student (or he is unfamiliar with the proof modus operandi).

These are such problems that a student, when solving them, encounters the necessity to investigate mathematical models of configuration which are new to him, non-standard connection, existing between such models, property of figures, and at the same time he has to find and establish the logic scheme of reasoning. As it is customary in the majority of mathematics methodical courses, all scholastic tasks can be divided into two main groups – heuristic and algorithmic tasks. Solution of a scholastic research task results in the established and well-founded solution algorithm for the total class of similar problems or heuristic device, the scientific idea that, after being justified and generalized, can be used and recommended for the solution of alternative nonstandard problems.

The technique of each such problem solution assumes that there exists an initial opportunity of splitting given problem into a chain of comparatively easy lemmas. This technique also assumes the opportunity to derive and analyze intermediate results of the received solution both by the student and by his tutor.

Thus, we shall notice that the concept of "scholastic research tasks" is always considered in a relative sense, in the context of a concrete student's personality. It allows individualizing and intensifying the process of intellectual education of a particular pupil or student on the given material. Our experience (that has shown its advantages in the educational structures of various countries) enables us to assert that it is possible to train the pupil in self-education and scientific creativity skills as well as in the elements of experimentalist or researcher work on the research type tasks.

We emphasize that, as it was noticed above, the basic motives for the choice of such tasks were dictated, on the one hand, by the considerations based on our own positive pedagogical experience. On other hand, this choice was the consequence of study and analysis of the results of well-known psychologist L.S.Vygodsky.

Vygodsky has justified the following fact: the condition of person development as a whole, and the level of his mathematical thinking in particular, is determined not only by a personality current state. Not only what the child has already learned to do is essential, but also what he is capable to learn. Here, as Vygodsky has shown, two parameters are necessary to be accounted for:

1) How a student solves the offered tasks independently, by himself.

2) How he solves the same tasks with the help of adults.

Certainly, with the help of adults the child can solve only such tasks that lay in the scope of his own intellectual abilities.

The divergence between these two parameters would also be a parameter that defines the so-called "zone of proximal development".

Tasks that a child is capable to understand or solve with the help of a tutor specify the area of his nearest development.

"What a child can perform with the help of adults today (that is what does currently lay in the area of his nearest development), will tomorrow be the thing he would manage to perform independently (that is, that will tomorrow proceed to the level of his authentic development)". ([10], p.92)

The idea of taking into account not only that was already achieved but that would be achievable in the nearest future as well, that is to work on an advancing, – has appeared rather fruitful not only in the researches of others scientists (such as psychologist V. A. Krutetsky, for instance), but in our everyday pedagogical practice as well.

Expediency and necessity of the students' training in the solution of research tasks. Problem urgency.

1. Essentially always the process of any research task solution reconstructs the atmosphere of scientific work in the most realistic way. It can be ascribed to any analysis in general and to a mathematician's work, in particular. Hence, a child can receive general notion about the research work since his school age. This is obviously rather significant for his vocational guidance. B. N. Delaunay, the brilliant representative of Moscow mathematical school, declared in this connection that "great scientific discovery differs from serious scholastic research task only in one feature: a child spends

several hours or even several days to solve his problem, whereas a real scientific discovery may sometimes take the whole scientist’s life”.

2. Statistical data confirm that mathematicians accomplish their most important discoveries at the age of 22 to 26. Therefore, from our point of view, it is promising enough to teach children the scientific analysis methods at their early school age.

3. In the course of training to solve research tasks students learn to master the special schemes of plausible and provable reasoning and gain high level of knowledge, skills and habits of work from numerous mathematics divisions, as well. That is very important per se, of course.

4. The process of search for the scholastic task solution will demand from a student to undertake corresponding intellectual efforts. Thus, the intellectual facilities of a pupil receive a powerful impulse for development. "You see that anything you are compelled to discover independently, by yourself, leaves a path in your mind which you can always use to take advantage when a necessity would arise".([6], p.23)

5. "Scholastic research tasks" allow individualizing and intensifying the process of intellectual education of a particular pupil or student in the given material. Use of material of the research type tasks makes it possible to train a pupil in self-education and scientific creativity skills as well as to accustom him to the elements of experimentalist or researcher work. ([1], p.5)

6. Course of the research tasks solution, as it is, opens up the majority of heuristic solution procedures that are valuable for the mathematical personality development. Later, the skills obtained can be extended to any mathematical material or to any sphere of scientific interests of a future specialist. From the aforesaid, follow the urgency and practical prospects of the declared problem study.

Selected methods of the students’ training in the research task solution

We shall refer to the whole well-founded assembly of mathematical actions as to an *approach to the mathematical problem solution*. We shall refer to the well-founded logical scheme that lays in the basis of a particular mathematical problem solution as the *method of mathematical problem solution*.

From the declared task point of view, the "Method of Heuristic Training" is of particular interest.

Obviously, G. Polya may be rightfully considered as the author of this method of training in its modern interpretation and justification. (See, for instance, his book "How to solve it"). The essence of the method is that a student is offered to carry out the search for a particular problem solution in accordance with the *sui generis* invariant set of general questions. Answers to these questions should draw the student near to the guesswork or to the solution discovery.

In the due course, some students would manage to master the proposed scheme of "reasoning through substantiated questions", and quite often they would gain success in the solution of scholastic problems.

But... from our point of view, for the necessities of the general mass-scale pedagogical practice, his method in its stated interpretation can sometimes appear as unacceptable.

Let us find the cause of this.

Heuristic schemes, which in their different variants and on different stages were given to the students, have certain common features. But abstract advice of general type such as: "…apprehend an offered problem…" or "formulate a relationship between the known and the unknown…" are of little help to the student, when searching for the concrete problem solution, if any sufficient experience in problem solution is absent.

Below we offer our vision of the development of main ideas for the given method of education. The realization of the following methods and devices of students' education is its base:

- Inductive method of teaching the research task solution.
- Method of teaching the research task solution by the way of analogy.
- Deductive method of teaching the research task solution.

In practice, the choice of teaching method depends mainly on the particular task features and on the particular purposes of the tutor.

Let's briefly consider the essence of each method.

Inductive method of teaching the solution of research tasks.

(This method implies a transition from the specific to the general.)

The inductive method of teaching is based on some mathematical experiment. This method requires much more time compared to the two other approaches, as the greatest difficulty of such an approach is to promote the credible hypothesis. Nevertheless, the advantage of this method lies in its maximum vicinity to the real scientific mathematical activity and in the fact that the inductive method develops intuition and creates conditions for the insight and impressing rise. The inductive method of teaching can considerably activate the pupils' creative activity. As the psychologist V. A. Krutetsky has shown in his investigations: "Despite the primitive character of the trail and error approaches, they are underlying the large class of creative processes at the research task solution… It should be noted that the trails could be taken at any level of analytical or synthetic activity. Only at the lowest level of trails these trails are blind – that is they are just guessing, when the pupils fail to realize why namely this test is conducted and what they should receive as a result of this trail." ([5], p. 510).

In practice, we aspire to organize an inductive method of teaching as an experimental step-by-step pedagogical process.

Let's illustrate our method on several variations of the same particular problem solution. The problem was offered to the 12 – 13 year-old children who attended lessons at the mathematical club in a Beer-Sheva (Israel) school.

Example 1. For anyone natural n calculate the sum

$$S(n)=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+...+\frac{1}{n\cdot(n+1)}$$

The first stage. **Experiment.**

At the first stage, we offered to analyze and study the values of the sums S, as the functions of number n: $S = S(n)$. At this stage, we didn't establish the amount of tests and didn't restrict pupils in doing those tests.

Results of these tests are collected in the table 1.

n	1	2	3	4	5	6	7	8	9	10	11
S(n)	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{7}{8}$	$\frac{8}{9}$	$\frac{9}{10}$	$\frac{10}{11}$	$\frac{11}{12}$

The second stage. **Promotion of a hypothesis.**

At this particular stage it is extremely important to enable the pupil to realize his own independence and scientific activity. After performing a set of experiments, comes the stage of a hypothesis promotion. It is clear that this is a crucial step. The speed of the task solution depends on a correct hypothesis. In the given example it is really easy to notice a rule: numerator of any fraction in the second row of Table 1 is equal to the number of addends of the sum *S (n)*, and the denominator of this fraction is by a unit more than the numerator.

Thus, we receive a working hypothesis: prospective answer is described by the correlation:

$$S(n) = \frac{n}{n+1}$$

The third stage. **Experimental check of the hypothesis validity**

The purpose of this stage is to ensure the necessary feeling of reliance in the logic completeness of the task solution in a pupil. Our pupils acquire rather quickly the firm confidence that to assert invalidity of any hypothesis that has been put forward, sufficient is to show that it is invalid in one particular case. Hypothesis validity must be proved by rigorous deduction. It will be accomplished at the next stage.

The fourth stage. **Proof of the hypothesis validity**

In general it is possible to prove the correlation:

$$S(n) = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + ... + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$$

by the incremental method or with the help of some other scheme. The pupils get acquainted with these ideas earlier and are trained to use these schemes carefully.

Method of teaching solution of the research tasks by the way of analogy

The method for construction of the theory of a research task solving by the way of analogy is one of the major methods of training in our pedagogical practice. The value of mastering this approach is not only the investigation of a particular educational material, but also a valuable opportunity to teach the mathematically gifted schoolchildren to the framework of scientific activity and to develop their mathematical thinking. The most difficult and important for the teacher is to pick up the maximum convenient source of analogy.

Example 2. Prove the correlation:

$$S(n) = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + ... + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1} \qquad (1)$$

At a training stage, the initiative in the choice of a suitable theory or analogy belongs to the teacher. In this case, it is expedient to explain an idea and scheme of application of an incremental method of the sum calculation for numerical series to the pupil.

By the way, the pupils can easily apprehend an idea of the incremental method application by the following problem solution:

Problem. Calculate the sum of numbers:

$$S(n) = 1^2 + 2^2 + 3^2 + ... + n^2$$

Solution. Let's consider an identity:

$$(n+1)^3 = n^3 + 3 \cdot n^2 + 3 \cdot n + 1$$

It follows from this identity that

$$(n+1)^3 - n^3 = 3 \cdot n^2 + 3 \cdot n + 1.$$

In the left-hand part of the last identity we have received the difference of cubes of two consecutive natural numbers. By substituting consecutive numbers 1, 2, 3,... , n instead of n, we shall receive n identities of the same kind:

$$2^3 - 1^3 = 3 \cdot 1^2 + 3 \cdot 1 + 1$$
$$3^3 - 2^3 = 3 \cdot 2^2 + 3 \cdot 2 + 1$$
$$4^3 - 3^3 = 3 \cdot 3^2 + 3 \cdot 3 + 1$$
$$5^3 - 4^3 = 3 \cdot 3^2 + 3 \cdot 4 + 1$$

....

$$(n+1)^3 - n^3 = 3 \cdot n^2 + 3 \cdot n + 1$$

The preparatory work is completed. What remains now – to total the sequentially left parts of equalities and separately their right parts. Thus in the left part of the new identity all members, with the exception of the two of them, will be mutually cancelled out. So we shall receive:

$$(n+1)^3 - 1^3 = 3 \cdot (1^2 + 2^2 + 3^2 + ... + n^2) + 3 \cdot (1 + 2 + 3 + ... + n) +$$
$$+ (1 + 1 + ... + 1)$$

Now, it is easy to receive the correlation for any natural n:

$$S(n) = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n \cdot (2 \cdot n + 1) \cdot (n+1)}{6}$$

Applying the method of teaching by analogy, we aim the pupil to look for the appropriate decomposition of the common term of numerical series from *example 2*:

$$a_n = \frac{1}{n \cdot (n+1)} = c_{n+1} - c_n$$

in the difference of two consecutive terms of some new sequence with common term C_n. It is easy to show that in our case $c_n = \frac{1}{n}$.

So, to prove the correlation (1) an equality

$$a_n = \frac{1}{n \cdot (n+1)} = c_n - c_{n+1} = \frac{1}{n} - \frac{1}{n+1} \qquad (2)$$

can be used just as equality

$$a_n = c_{n+1} - c_n = (n+1)^3 - n^3 = 3 \cdot n^2 + 3 \cdot n + 1$$

was used in the previous problem solution.

By substituting consecutive numbers 1, 2, 3... n instead of n in the left-hand side of (1) and using each time the decomposition (2), we shall obtain the desired answer. The solution was reached by analogy with the previous problem solution:

$$S(n) = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + ... + \frac{1}{n \cdot (n+1)} = 1 - \frac{1}{n+1},$$ QED.

Deductive method of teaching the research task solution
(From general — to partial!)

While training to understand the idea of approach to the research task solution by this method, we offer the pupil consecutive series of theoretical tasks, resulting in the construction of the theory elements. This technique is extremely important because it simulates and sometimes reconstructs the way of reasoning along which the pioneer scientist has gone. This method teaches to pick out the major stages of the proof. As a rule, it is possible to save training time by this pedagogical technique, because introduction of the new material takes place simultaneously with its consolidation.

Example 3. Calculate the sum:

$$S = \frac{1}{98 \cdot 99} + \frac{1}{99 \cdot 100} + \frac{1}{100 \cdot 101} + \ldots + \frac{1}{1998 \cdot 1999} + \frac{1}{1999 \cdot 2000}$$

a) Realizing a deductive method of teaching, we initially aim the pupil to search for "the Whole" by its visible, that is, by its known "Part".

In this instance the student first should try to complement his task condition up to a general form, which is to write down the general dependence of sum S on number n.

After that, the student should prove his correlation validness by using appropriate technique and calculate the difference between two values of $S(n)$ for number $n = 1999$ and $n = 97$, correspondingly.

According to the above-mentioned, we shall formulate the general task:

Calculate the sum for any natural n:

$$S(n) = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n \cdot (n+1)}$$

b) As it was already shown,

$$S(n) = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n \cdot (n+1)} = 1 - \frac{1}{n+1}$$

Therefore, $S(1999) = \frac{1999}{2000}$, similarly, $S(97) = \frac{97}{98}$.

The final answer is:

$$S(1999) - S(97) = \frac{1999}{2000} - \frac{97}{98} = \frac{951}{98000}$$

To accomplish this brief review of the teaching methods, it is necessary to emphasize the presence of developed internal philosophical bonds between those methods. In other words, if, for example, the method of training by analogy is selected as the main method, then all other methods are used indirectly. It was demonstrated in the analyzed examples.

Didactic support for the scientific research process of teaching and development of pupils

Here we are going to show some representative examples that illustrate our basic principles of selection of the didactic contents for the effective development of convergent and divergent intellectual faculties of the personality mathematical thinking.

Problems of the solution existence definition.

During our 20-year pedagogical practice we have carried out a series of experiments that brought us to the conclusion that if our aim is to teach a pupil the technique of the research task solution, it would be advantageous to begin with the problems that sound as "*whether it is possible or no*?"

Let's dwell on this type of tasks in detail.

Previously it is necessary to explain to the pupils the following:

Most of mathematical tasks that may be found in a school textbook begin with the words: “simplify…”, “calculate this…” or “find what is…“. In mathematical sciences, however, investigators very often deal with the research problems, where the main aim is rather to establish whether the object with the given properties exists at all or whether the given assertion is valid in principle than to simplify or calculate something.

This type of tasks usually begins with words: “whether it is possible to (do something)", or “whether (something) exists?" and so on.

When searching and justifying the solution of such tasks, it is necessary to stick to the following rule:

1) If we assert that something can be made it is well enough to specify the concrete way that allows fulfilling that.

2) If we assert that under no conditions something can be made, then the examples by themselves wouldn’t help here. It is necessary to construct a rigorous deduction in this case.

Let's consider some representative examples of problems that we used in our work with the children.

Example 4. Several marble blocks with overall weight of 14 tons are to be transported from one site to another. Exact weights of individual pieces are unknown, but it is known that none of these blocks weighs more than 400 kg. Three questions are set:

1. May it be asserted that 12 trucks with the weight-carrying ability of up to 1500 kg will be actually enough to cope with this task?

2. What is the minimal number of trucks with the weight-carrying ability equal to 1500 kg each to be ordered to transport the cargo?

3. If 9 identical trucks are actually enough to transport this cargo, what should be the minimal weight-carrying ability of one truck?

Solution (with the elements of discussion).

Answer to the first question. We shall "throw a trial stone", that is we shall begin with an attempt to construct an example demonstrating that 12 trucks with weight-carrying ability equal to1500 kg each will cope with the given task. The problem lies in the fact that the weight of each individual block is unknown. However, it is intuitively obvious that the maximum efficiency of the trucks used means maximum possible loading of the each truck. That results in the minimum number of trucks needed. On the other hand, the less is the weight of each piece, the larger may be the weight-carrying ability of each truck used (ideal case would be if each marble block would have a grain size). The simple calculation shows that if all marble blocks have the same weight, for example, 350 kg, one truck could carry 4 blocks with $350 \cdot 4 = 1400kg$ gross weight. And this means that 10 machines would be enough to transport all marble. This calculation shows that under certain conditions we can "save" minimum two trucks of 12. We tried to think up a situation facilitating an effective utilization of transport. But it is also clear also that the calculation was carried out under "favorable conditions".

How would the situation be solved if those favorable conditions would not realized? Evidently, such favorable conditions do not take place in the general case. Let's try to construct a suitable example, reasoning from the end.

Let us assume that 12 trucks would not be enough for the given task. For example, we can imagine a situation when each truck transfers **less** than $14000:12 = 1166\frac{2}{3}kg$

of a marble. It is possible if the weight of each block does not differ much from, for example, 388 kg. In this case, the truck is able to carry three blocks as $388 \cdot 3 = 1164 < 1500 kg$ but it cannot carry four as $388 \cdot 4 = 1552 > 1500 kg$.

In that case, 12 trucks wouldn't be actually enough to cope with the task as:
$1164 \cdot 12 = 13968 < 14000 kg$.

Now we have only to define the figures more accurately. If the weight of each block is assumed to be equal $14000 : 36 = 388\frac{8}{9} kg$ and the consignment consists of 36 marble blocks, then 3 blocks can be loaded on a truck: $388\frac{8}{9} \cdot 3 = 1166\frac{2}{3} < 1500 kg$

(and $388\frac{8}{9} \cdot 4 = 1555\frac{5}{9} > 1500 kg$) and we obtain that 12 trucks would be enough. But if there would be, for example, 37 blocks of identical weight, then:

a) the weight of each piece would be equal to $378\frac{14}{37}$ kg;

b) no more than 3 blocks may be loaded on each truck because
$378\frac{14}{37} \cdot 3 = 1135\frac{5}{37} < 1500 kg$ and $378\frac{14}{37} \cdot 4 = 1513\frac{19}{37} > 1500 kg$

This means that 12 machines can carry only $12 \cdot 3 = 36$ such blocks. One block will not be transported. The answer to the first question is negative.

Answer to the second question

As by the problem conditions, the weight of each block does not exceed 400 kg, any truck is able to carry above 1100 kg of a cargo. This permits to claim that 13 trucks would be actually enough for the transportation of all marble blocks:
$13 \cdot 1100 = 14300 > 14000 kg$

Answer to the third question

If the weight-carrying ability of each truck is equal to M and the underload should not exceed 400 kg, then the maximum load that the truck should transport can exceed
$(M - 400) kg$. From an inequality $9 \cdot (M - 400) > 14000 kg$ we shall obtain $M > 1955\frac{5}{9} kg$. So, we got the lower limit of the truck weight-carrying ability.

Thus, the answers to all questions set in the problem are obtained.

Conclusion

Theoretical prospects of the given problem research.

The results of this study are expected to stimulate the development of the major methods and principles of the mathematical education organization. For instance, the investigation and development of the heuristic educational methods. We believe that the most interesting line of investigation consists in the study of motivations and reinforcement of interest to the problem solution. The central point is the stimulating atmosphere of scholastic process created by a teacher. We believe that the existing organization of students' activity does not sufficiently promote the development of deep personal interest in such an activity in the majority of students.

Students' training in the scientific methods of research problem solution stimulates the process of shaping and development of person's mathematical thinking, promotes the quality of his knowledge, brings up and drills his intellectual endurance, arouses a young person’s deep personal interest in his own mental activity, and prompts a person to self-education.([1], p.21)

In conclusion, we would like to emphasize and direct reader’s attention to the following: any method of teaching should be used creatively, in view of the interests of the learning person. In this connection an idea stated by Maier N.R. ([7], p.65): "the person can fail to solve a problem not because he is not capable to find the solution but rather because the habitual mode of action restrains the correct decision development", deserves merit.

From this, the need for the improvement of obsolete teaching methodology as well as the necessity of search for the new effective methods of teaching which are able to develop a creative person are naturally ensued.

REFERENCES

[1] Applebaum, M.V., 2001, Teaching Mathematics using the idea of “Research problems”, Ph.D. Thesisses. Azerbaijanian State Pedagogical University, Baku, Azerbaijan.

[2] Cholodnja M. A. " Psychology of intelligence: paradoxes of research " M., 1997 (In Russian)

[3] De Groot, A. D. (1965) Thought and Choice in Chess. The Haugue: Mouton

[4] Gardner, R.W., Holzman P.S., Klein G.S., Linton H. B., Sprence D.P., (1959), Cognitive control. A study of individual consistencies in consistencies in cognitive behaviour. Psychological Issues. Monograph 4. V.1 (4)

[5] Krutetskii, V. A. (1976), The Psychology of the Mathematical Abilities of the Schoolchildren. Chicago, University of Chicago Press.

[6] G. Litenberg, G., "Hphorismen" [Berlin, 1902-1906.])

[7] Maier, N.R.,(1933). An aspect of human reasoning. “British journal of psychology”, vol. 24.

[8] Poincare, A.: 1983, About Science. Moscow. Nauka.

[9] Polya, G. (1957), How to Solve the Problem. New York - London John Wiley&Sons, INC

[10] Vigodski, L. S. (1956)" the Elected psychological researches”. Ì ., APN RSFSR edition.