MODERN PROBLEMS OF MATHEMATICS TEACHER TRAINING IN RUSSIA

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ABSTRACT

The problem of qualitative changes in the training of a mathematics teacher of a secondary school in Russia is defined. In the course of the experimental psychological research we have found three crucial periods of professional development of students. Teachers should be treated at the formation of an integral system of professional-pedagogical activity: professional knowledge, founding stage, technological structure of professional activity and shaping a teacher’s personality. Conception of founding of school mathematical elements (knowledge, skills, abilities mathematical methods) presupposes development in the process of mathematical training of students. In the proposed didactic system of mathematical education of prospective mathematics teachers a fundamental role is played by a pedagogical technology of visual-model teaching of mathematics and activity of students.

Key words: teacher’s training, mathematics, innovation technology, founding process, visual model technology
1. Introduction

Despite numerous attempts to alter curricula and syllabus, introduction of a state educational standard, development of tendencies to democratise higher pedagogical education, no significant qualitative changes in training of mathematics teachers in secondary school have been taking place in Russia within the last ten years.

In the course of the experimental psychological research of teacher training and independent professional activity of mathematics teachers three crucial periods of their professional development have been defined. The first period is connected with the end of the 1-year at college; the second – with the end of the third and end of the fourth year at college; the third one – the end of the first and beginning of the second year of the independent work at school.

Variance, factor and cluster analysis of the obtained results proves that every crisis is connected with overcoming of corresponding contradiction. The first crisis is connected with contradictions between the form and content of education at school and at college, the second crisis is evidence of contradiction between fundamental and professional-methodological training at college. The third one reflects contradictions between professional preparedness for work at school and professional requirements, which a teacher faces at school. All this results in inadequate professional training of a teacher: formality of knowledge, lack of soundness in professional knowledge of skills and habits in the subject, poor knowledge of methodology and technology of teaching, low creative activity and insignificant interest to innovations in planning and organising educational process at school.

It is necessary to give an adequate characteristic to professional formation, in other words, describe it as a systematic, continuous process, which is determined by a complex of internal and external factors and is being realised on the basis of various psychological mechanisms and pedagogical technologies.

At present the problems under consideration are being approached in various ways. The first way is to increase volume of mathematical and pedagogical courses at colleges. The authors of this approach are probably of an opinion that the more fundamental and pedagogic theories and methods the student are taught, the faster the system of their professional-pedagogical activity is being formed. But practical experience proves that it is not always the case.

The second way, which is more promising, admits that methodological training of students should be started as early as possible – even in their first and second year. This training is to be based on various professional trials and tests, still not grounded either on theory or experiment.

In our opinion, the pedagogical process of training mathematics teachers should be treated at formation of an integral system of professional-pedagogical activity. The first, professional, stage should be devoted to formation of the subject knowledge and skills, aimed at formation of the nearest specific generalisation of basic educational elements of school mathematics. The second stage, when knowledge of mathematics became fundamental, they acquire profound theoretical generalisation, which on the third, methodological stage is integrated into the structure of professional activity as a means of realisation of the pedagogue’s teaching and educational functions. To ensure painless inclusion of the generalised knowledge they must be organised in a way that best suits school children. Founding and visual modelling performs exactly this function of reorganisation of knowledge of a certain subject according to aims and tasks of teaching activity.

2. Psychology of Mathematics

In the last decades, mathematics as a pedagogical problem has been under unprecedented pressure on the part of society concerning the subject matter of teaching as well as teaching methodology. The problem is that the depth of its formalization and following the inner structural regularities contradict both the ontogenesis and socialization of a single individual and needs of society in terms of providing for its visualization, modeling and revealing the social status.
What is mathematics nowadays?

Mathematics as a science presents methodology and language of other subjects, as well as connections between abstract objects, which unsynonymously reflect practical reality. That's why mathematics occupies a special place among the sciences.

Mathematics not only promotes new knowledge about nature and personality, but also finds a practical stimulus for development in applied sciences. Thus the development of the theory of locally convex spaces in functional analysis was stimulated by the physical problems of quantum electrodynamics in finding generalized solutions of equations in mathematical physics, the theory of unboundary operators in the Banach space – by problems of quantum mechanics, tensor analysis – by the problems of mechanics of elastic media, the theory of the function of many complex variables – by the problems of the quantum theory of the field etc.

Therefore consequences of the strenuous tendency of fundamental mathematical knowledge are closely connected with intensive application of mathematical methods in other sciences (including the humanities). Some of them spontaneously influence vital activities and socialization of a personality in the modern world.

On the other hand, during the recent ten years some new areas have been developed in mathematics. These areas have independent subjects and specific methods of research. Among them are: artificial intelligence and the theory of mass service, the theory of random processes and functional analysis, the theory of games and mathematical programming, algebraic geometry and set-theoretical topology etc.

The cardinal means which promote new formations is modeling as a high form of the sign-symbol activity leading to new knowledge about nature and the technical process in production, about the laws of social development and the regularities of human thinking, perception and memory.

During the recent years we have seen reinforcement of the role of mathematics as a means of humanization and socialization of personality-oriented education in modern society as a necessary attribute of the educational paradigm of the 21st century personality.

Considering mathematics as a pedagogical problem, we face the problems of an adequate notion (idea), distinguishing, formation, stable perception and reproduction of mathematical knowledge in all the 3 hypostases of mathematics (Figure 1).

Figure 1

The Model of Three Hypostases of Mathematics

Personality

Study

Mathematics as a Science

Education & Development

Modeling

Practical

Process

Scientific Knowledge About Nature, Society & Thinking

Infrastructure & Vital Activities of Society

Modeling

Application & Structure

Self-Development Essence of Mathematical Knowledge

How can we show in the process of teaching mathematics its role in the substantiation of space flights and security of air transportation? How can we show that physics is a powerful supporting component of vital human activities, which come only to observation and experience without mathematical knowledge and that psychology is only fortune-telling and subjectivism without statistic methods of experimental data analysis and modeling of mental processes? What is the best way to tell a pupil that A. Wiles proved Fermat’s Last Theorem in 1995, and that trisection of an angle and squaring the circle are impossible with a ruler and a pair of compasses? How can
we develop the pupils’ thinking operations (logic, analysis, synthesis, generalization, concrete definition, analogy etc.) in the process of teaching mathematics (which objectively must be the most powerful of the developing means which unfortunately can’t be observed yet) more effectively?

The problem of an integral notion and adequate acquisition of mathematical knowledge under the conditions of immediate perception, developing and making the structures of a pupil more active is of paramount importance in organizing didactic and cognitive processes.

3. Innovative Process of Knowledge Founding

We based our assumptions on the fact that concordance or optimisation of interaction of fundamental and professional components in the general structure of pedagogical education is a key moment in training of a student at a teachers’ training college. It is obvious, that fundamentalisation of mathematical knowledge without considering it as a pedagogical aim will hinder professional training of a teacher. At the same time it is beyond doubt that a teacher is not in able to fulfil his professional functions successfully without a certain volume, structure and quality of fundamental knowledge. Thus, in a nutshell, the problem is to find means, forms and ways to bring to concordance fundamental and professional lines in the process of pedagogical education.

**Founding** is a process of creating conditions (psychological, pedagogical and methodological-organisational) for actualisation of basic structural units, which reveal their essence, integrity, relations between the subjects in the direction of professionalism of knowledge and shaping a teacher’s personality.

**Conception of founding of** school mathematical elements (knowledge, skills, abilities mathematical methods) presupposes development in the process of mathematical training of students in the following components:

- determination of contents of basic schooling element (knowledge, skills, abilities mathematical methods, ideas, algorithms and procedures) level of organisation;
- determination of contents of levels and stages (professional, fundamental and technological) of basic schooling element development at college;
- determination of founding technologies (diagnosable aim-finding, visual modelling of global structure levels, local capacity for modelling, control of students’ creative and cognitive activity, blocks of motivation of basic schooling elements);
- determination of methodological adequacy of basic school and college (founding) schooling elements on the basis of modern methodological concepts.

In order to realise the principle of founding it is necessary to define the basis for helical diagram of the basic knowledge, skills and experience of mathematical training of students at teachers’ training colleges modelling. If founding of various school subjects is to be carried out layer by layer, then volume, content and structure of mathematical training must undergo considerable changes in respect to practical realisation of theoretical generalisation of school knowledge based on the principle of a “boomerang”. If the knowledge is being founded in such a way, the teacher, who possesses knowledge of the subject, together with the student will master the methodological side of teaching. The school knowledge will act as a structure-formation factor, making it possible to select theoretical knowledge from mathematics of a higher level, via which school knowledge has been founded. The layer of founding provides perfection and extension of practical skills, projected by approximate basis of learning activity. In the activity aspect of pedagogical process realisation of foundation principle acquires a helical character, which corresponds to dialectical understanding of a system of knowledge development.

Development of spirals of basic school subject elements via construction of ancestral generalisation and technological comprehension of its specific manifestation render integrity and orientation to the projected didactic system.
For example, this chain of founding can exist.

**Figure 2**

Diagram of school knowledge founding (Concept of derivative)

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Local founding

Concept of derivative
- Concept of derivative on the level of school practical skills

Basic concept
- Concept of derivative
  \[ \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = f'(x); \]
  \[ f' : \mathbb{R} \rightarrow \mathbb{R} \]

Global founding

Concept of partial derivative
- Concept of partial derivative
  \[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \ldots; \]
  \[ \text{function gradient} \]

- Concept of complex derivative
  \[ f' : \mathbb{C} \rightarrow \mathbb{C} \]
  \[ \text{(particularities)} \]

Methods of explaining the concept of derivative at a secondary school

Theoretical generalisation

- Concept of derivative as a linear operator
  \[ f' : X \rightarrow X \]
  \[ X - \text{banach space} \]
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The value of the present model of founding (conception of derivative on the level of “data” to its deep theoretical generalisation on the level of “essentiality”) for training processes at college as well as prospective professional activity for a math student is beyond doubt. It seems such models will find their place in syllabuses of mathematical analysis and individual technologies of teaching mathematics at school.

At the same time construction of such a model absorb in its unique and particular manifestation all main features of theoretical knowledge about foundation process of basic educational elements of maths at school. Creation of system-genetic block of spirals of founding makes it possible to define a stable nucleus of educational information content, which projects elements of approximate basis for educational activity of students.

### 4. Visual Methods of Teaching in Application to Mathematics

In teaching mathematics the teacher encounters the following problems:
- a high degree of abstraction in the perception of new material;
- an information overload of sign-symbolic forms;
- a limited period of time for active perception of new material;
- weak motivation in studies.

The first problem results in getting only formal knowledge, and its isolation from the essence of the basic phenomena and theorems. The second makes it difficult to recognize the object of perception. The third increases conflict in teaching. The fourth means that pupils’ interest in studies is decreasing.
Strange as it may seem, all these problems can be solved when we pay special attention to the first stage of cognitive perception, immediate perception to be exact. An in-depth study of this stage of perception requires a full understanding of neuro-physiological aspects of thinking, a psychological and pedagogical analysis, as well as personal experience and modeling.

The main purpose and result of this paper is the creating of conception on visual modeling of mathematical objects (in particular, complex and symbolic) and creating conditions for suitable perception.

It is essential to know that perception has three stages: stimulation, operation and understanding. Moreover, the process of perception may be concerned with two forms of acting: simultaneous (as an entire view of the essence of an object) and successive (as a sustained process perception is open not only on the level of stimulation but on more higher levels: analysis, choosing pattern, comparison, reaction formation).

According to our concept utilisation of visual methods in teaching of mathematics of a prospective teacher is to be treated as a special property of psychological images of mathematical objects, the essence of which is considered in a integral paradigm of perception of the basis of the following criterions:

- diagnosable aim-finding of integrity of a mathematical object;
- adequate perception (learner’s comprehension of essence of the mathematical object in accordance with aims of teaching);
- stability of the perceptive image and presentation under conditions of direct perception;
- cognitive and creating activity on the basis of comfort, successful teaching.

The process of perception of the given material presupposes all the key qualities of a mathematical object. It is especially important when information is of great volume. It is necessary to keep in mind such actions when separate pieces of knowledge or an arranged set of knowledge are given. We can deal with proving theorems, teaching some parts of mathematical analysis in its various logical correlations, with a single lesson, a lecture etc.

As has already been mentioned, according to A.N. Leontiev, when visual methods of teaching are used, it is necessary to proceed from the psychological role, which they (methods of teaching) play in the perception of new material. He chose two functions of visual methods of teaching:
- the first is aimed at extending the sensible experience;
- the second is aimed at developing the essence of the processes or phenomena under study.

In connection with that, external teacher's actions are divided into bearing and structural actions depending on the orientation of the sensible or rational element of perception.

The external bearing actions can be as follows:
- writing down formulas, tables, displaying models, drawing up graphs, formulating theorems, using text-books or manuals.

The structural external actions can be as follows:
- proving theorems, choosing the main theoretical notions and methods, realizing links between different subjects.

Each external structural action is an arranged set of bearings for external actions connected with each other on equal terms.

Namely, a classification of teacher's external actions makes it possible to choose the following visual methods in teaching mathematics:
- operative visual methods of teaching (e.g. the use of computers, drawings, calculators, schemes, graphs etc.),
- structural visual methods of teaching (e.g. looking at the logical analysis of a theorem or a notion may require a whole lecture)
- formalizing visual methods of teaching,
- background visual methods of teaching (creating the necessary conditions for checking the figure from the background by elements of this background).
Developing these key qualities of the object of perception is the main point of visual teaching methods. That is why when we deal with visual methods of teaching we must not forget that the stage of immediate perception follows the stage of choosing the key qualities in the object of perception (i.e. the goal). This "a priori" point of view presupposes modeling the object of perception by means of the neuro-physiological mechanism of memory and psychology of perception. Some attributes of visual teaching methods may be as follows:

- computer displays, logical analysis of theorems, key theorems, key notions, data banks of problems and investigation methods, bearing code tables, block-schemes for proving theorems, concrete background, a list of topics or parts of mathematics.

Systematic realisation of all means of visualisation in the process of teaching mathematics is an important factor of formation of integral images of mathematical objects, which means that it considerably facilitates understanding of mathematical knowledge and development of cognitive abilities and mathematical thinking.

A mathematics teacher’s activity in the process of teaching due to an abstract character and complexity of mathematical material presupposes detailed concrete definition of utilised principles of teaching and their systematic use. It leads to the necessity to work out a common interpretation of visualisation principles in mathematics, developing techniques of a teacher’s activity in the process of visual teaching, detection of the visualisation specifics in teaching of mathematics. All of this is based on positive experience of progressive teachers and scientists.

We hope that results of present direction of research will be useful not only to university teaching staff, but to secondary school teachers as well. These groups will be able to rely on it as a basis of creative design of teaching processes and samples of innovation methods in teaching mathematics.

REFERENCES