

ELEMENTS BOOK 3

Fundamentals of Plane Geometry Involving Circles

Ὅροι.

α'. Ἴσοι κύκλοι εἰσίν, ὧν αἱ διάμετροι ἴσαι εἰσίν, ἢ ὧν αἱ ἐκ τῶν κέντρων ἴσαι εἰσίν.

β'. Εὐθεία κύκλου ἐφάπτεσθαι λέγεται, ἣτις ἀπτομένη τοῦ κύκλου καὶ ἐκβαλλομένη οὐ τέμνει τὸν κύκλον.

γ'. Κύκλοι ἐφάπτεσθαι ἀλλήλων λέγονται οἵτινες ἀπτόμενοι ἀλλήλων οὐ τέμνουσιν ἀλλήλους.

δ'. Ἐν κύκλῳ ἴσον ἀπέχειν ἀπὸ τοῦ κέντρου εὐθεῖαι λέγονται, ὅταν αἱ ἀπὸ τοῦ κέντρου ἐπ' αὐτάς κάθετοι ἀγόμεναι ἴσαι ὦσιν.

ε'. Μείζων δὲ ἀπέχειν λέγεται, ἐφ' ἣν ἡ μείζων κάθετος πίπτει.

ς'. Τμήμα κύκλου ἐστὶ τὸ περιεχόμενον σχῆμα ὑπὸ τε εὐθείας καὶ κύκλου περιφερείας.

ζ'. Τμήματος δὲ γωνία ἐστὶν ἡ περιεχομένη ὑπὸ τε εὐθείας καὶ κύκλου περιφερείας.

η'. Ἐν τμήματι δὲ γωνία ἐστίν, ὅταν ἐπὶ τῆς περιφερείας τοῦ τμήματος ληφθῆ τι σημεῖον καὶ ἀπ' αὐτοῦ ἐπὶ τὰ πέρατα τῆς εὐθείας, ἢ ἐστὶ βάσις τοῦ τμήματος, ἐπιζευχθῶσιν εὐθεῖαι, ἡ περιεχομένη γωνία ὑπὸ τῶν ἐπιζευχθεισῶν εὐθειῶν.

θ'. Ὅταν δὲ αἱ περιέχουσαι τὴν γωνίαν εὐθεῖαι ἀπολαμβάνωσι τινα περιφέρειαν, ἐπ' ἐκείνης λέγεται βεβηκέναι ἡ γωνία.

ι'. Τομεὺς δὲ κύκλου ἐστίν, ὅταν πρὸς τῷ κέντρῳ τοῦ κύκλου συσταθῆ γωνία, τὸ περιεχόμενον σχῆμα ὑπὸ τε τῶν τὴν γωνίαν περιεχουσῶν εὐθειῶν καὶ τῆς ἀπολαμβανομένης ὑπ' αὐτῶν περιφερείας.

ια'. Ὅμοια τμήματα κύκλων ἐστὶ τὰ δεχόμενα γωνίας ἴσας, ἢ ἐν οἷς αἱ γωνίαι ἴσαι ἀλλήλαις εἰσίν.

α'.

Τοῦ δοθέντος κύκλου τὸ κέντρον εὐρεῖν.

Ἐστω ὁ δοθείς κύκλος ὁ $ABΓ$. δεῖ δὴ τοῦ $ABΓ$ κύκλου τὸ κέντρον εὐρεῖν.

Διήχθω τις εἰς αὐτόν, ὡς ἔτυχεν, εὐθεῖα ἡ AB , καὶ τετμήσθω δίχα κατὰ τὸ Δ σημεῖον, καὶ ἀπὸ τοῦ Δ τῆ AB πρὸς ὀρθὰς ἤχθω ἡ $\Delta\Gamma$ καὶ διήχθω ἐπὶ τὸ E , καὶ τετμήσθω ἡ $ΓE$ δίχα κατὰ τὸ Z . λέγω, ὅτι τὸ Z κέντρον ἐστὶ τοῦ $ABΓ$ [κύκλου].

Μὴ γάρ, ἀλλ' εἰ δυνατόν, ἔστω τὸ H , καὶ ἐπεζεύχθωσαν αἱ HA , $H\Delta$, HB . καὶ ἐπεὶ ἴση ἐστὶν ἡ $A\Delta$ τῆ ΔB , κοινὴ δὲ ἡ ΔH , δύο δὲ αἱ $A\Delta$, ΔH δύο ταῖς $H\Delta$, ΔB ἴσαι εἰσίν ἑκατέρᾳ ἑκατέρᾳ· καὶ βάσις ἡ HA βάσει τῆ HB ἐστὶν ἴση· ἐκ κέντρου γάρ· γωνία ἄρα ἡ ὑπὸ $A\Delta H$ γωνία τῆ $\Delta H B$ ἴση ἐστίν.

Definitions

1. Equal circles are (circles) whose diameters are equal, or whose (distances) from the centers (to the circumferences) are equal (i.e., whose radii are equal).

2. A straight-line said to touch a circle is any (straight-line) which, meeting the circle and being produced, does not cut the circle.

3. Circles said to touch one another are any (circles) which, meeting one another, do not cut one another.

4. In a circle, straight-lines are said to be equally far from the center when the perpendiculars drawn to them from the center are equal.

5. And (that straight-line) is said to be further (from the center) on which the greater perpendicular falls (from the center).

6. A segment of a circle is the figure contained by a straight-line and a circumference of a circle.

7. And the angle of a segment is that contained by a straight-line and a circumference of a circle.

8. And the angle in a segment is the angle contained by the joined straight-lines, when any point is taken on the circumference of a segment, and straight-lines are joined from it to the ends of the straight-line which is the base of the segment.

9. And when the straight-lines containing an angle cut off some circumference, the angle is said to stand upon that (circumference).

10. And a sector of a circle is the figure contained by the straight-lines surrounding an angle, and the circumference cut off by them, when the angle is constructed at the center of a circle.

11. Similar segments of circles are those accepting equal angles, or in which the angles are equal to one another.

Proposition 1

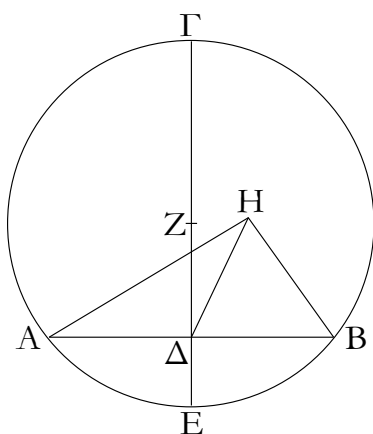
To find the center of a given circle.

Let ABC be the given circle. So it is required to find the center of circle ABC .

Let some straight-line AB have been drawn through (ABC), at random, and let (AB) have been cut in half at point D [Prop. 1.9]. And let DC have been drawn from D , at right-angles to AB [Prop. 1.11]. And let (CD) have been drawn through to E . And let CE have been cut in half at F [Prop. 1.9]. I say that (point) F is the center of the [circle] ABC .

For (if) not then, if possible, let G (be the center of the circle), and let GA , GD , and GB have been joined. And since AD is equal to DB , and DG (is) common, the two

ὅταν δὲ εὐθεΐα ἐπ' εὐθεΐαν σταθεῖσα τὰς ἐφεξῆς γωνίας ἴσας ἀλλήλαις ποιῆ, ὀρθὴ ἑκατέρα τῶν ἴσων γωνιῶν ἐστὶν ὀρθή ἄρα ἐστὶν ἡ ὑπὸ $H\Delta B$. ἐστὶ δὲ καὶ ἡ ὑπὸ $Z\Delta B$ ὀρθή· ἴση ἄρα ἡ ὑπὸ $Z\Delta B$ τῇ ὑπὸ $H\Delta B$, ἡ μείζων τῇ ἐλάττωι· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα τὸ H κέντρον ἐστὶ τοῦ $AB\Gamma$ κύκλου. ὁμοίως δὲ δείξομεν, ὅτι οὐδ' ἄλλο τι πλὴν τοῦ Z .



Τὸ Z ἄρα σημεῖον κέντρον ἐστὶ τοῦ $AB\Gamma$ [κύκλου].

Πόρισμα.

Ἐκ δὴ τούτου φανερόν, ὅτι ἐὰν ἐν κύκλῳ εὐθεΐα τις εὐθεϊάν τινα δίχα καὶ πρὸς ὀρθὰς τέμνη, ἐπὶ τῆς τεμνούσης ἐστὶ τὸ κέντρον τοῦ κύκλου. — ὅπερ ἔδει ποιῆσαι.

† The Greek text has “ GD, DB ”, which is obviously a mistake.

β'.

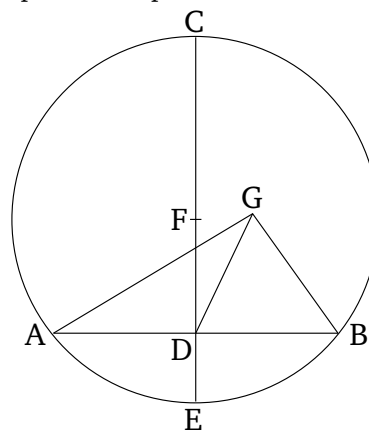
Ἐὰν κύκλου ἐπὶ τῆς περιφερείας ληφθῆ δύο τυχόντα σημεῖα, ἢ ἐπὶ τὰ σημεῖα ἐπιζευγνυμένη εὐθεΐα ἐντὸς πεσεῖται τοῦ κύκλου.

Ἐστω κύκλος ὁ $AB\Gamma$, καὶ ἐπὶ τῆς περιφερείας αὐτοῦ εἰληφθῶ δύο τυχόντα σημεῖα τὰ A, B · λέγω, ὅτι ἡ ἀπὸ τοῦ A ἐπὶ τὸ B ἐπιζευγνυμένη εὐθεΐα ἐντὸς πεσεῖται τοῦ κύκλου.

Μὴ γάρ, ἀλλ' εἰ δυνατόν, πιπτέτω ἐκτὸς ὡς ἡ AEB , καὶ εἰληφθῶ τὸ κέντρον τοῦ $AB\Gamma$ κύκλου, καὶ ἔστω τὸ Δ , καὶ ἐπεζεύχθωσαν αἱ $\Delta A, \Delta B$, καὶ διήχθω ἡ ΔZE .

Ἐπεὶ οὖν ἴση ἐστὶν ἡ ΔA τῇ ΔB , ἴση ἄρα καὶ γωνία ἡ ὑπὸ ΔAE τῇ ὑπὸ ΔBE · καὶ ἐπεὶ τριγώνου τοῦ ΔAE μία

(straight-lines) AD, DG are equal to the two (straight-lines) BD, DG ,[†] respectively. And the base GA is equal to the base GB . For (they are both) radii. Thus, angle ADG is equal to angle GDB [Prop. 1.8]. And when a straight-line stood upon (another) straight-line make adjacent angles (which are) equal to one another, each of the equal angles is a right-angle [Def. 1.10]. Thus, GDB is a right-angle. And FDB is also a right-angle. Thus, FDB (is) equal to GDB , the greater to the lesser. The very thing is impossible. Thus, (point) G is not the center of the circle ABC . So, similarly, we can show that neither is any other (point) except F .



Thus, point F is the center of the [circle] ABC .

Corollary

So, from this, (it is) manifest that if any straight-line in a circle cuts any (other) straight-line in half, and at right-angles, then the center of the circle is on the former (straight-line). — (Which is) the very thing it was required to do.

Proposition 2

If two points are taken at random on the circumference of a circle then the straight-line joining the points will fall inside the circle.

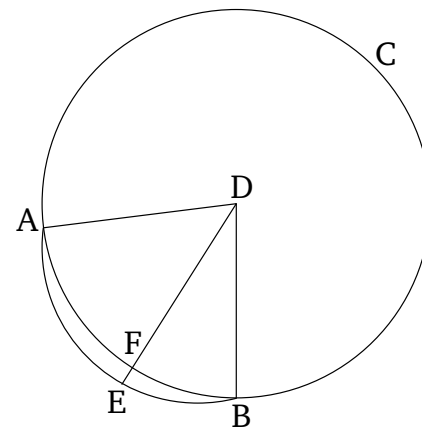
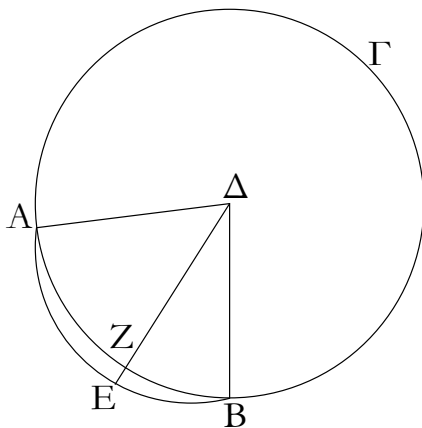
Let ABC be a circle, and let two points A and B have been taken at random on its circumference. I say that the straight-line joining A to B will fall inside the circle.

For (if) not then, if possible, let it fall outside (the circle), like AEB (in the figure). And let the center of the circle ABC have been found [Prop. 3.1], and let it be (at point) D . And let DA and DB have been joined, and let DFE have been drawn through.

Therefore, since DA is equal to DB , the angle DAE

πλευρά προσεκβέβληται ἡ AEB , μείζων ἄρα ἡ ὑπὸ ΔEB γωνία τῆς ὑπὸ ΔAE . ἴση δὲ ἡ ὑπὸ ΔAE τῇ ὑπὸ ΔBE · μείζων ἄρα ἡ ὑπὸ ΔEB τῆς ὑπὸ ΔBE . ὑπὸ δὲ τὴν μείζονα γωνίαν ἡ μείζων πλευρὰ ὑποτείνει· μείζων ἄρα ἡ ΔB τῆς ΔE . ἴση δὲ ἡ ΔB τῇ ΔZ . μείζων ἄρα ἡ ΔZ τῆς ΔE ἢ ἐλάττων τῆς μείζονος· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα ἡ ἀπὸ τοῦ A ἐπὶ τὸ B ἐπιζευγνυμένη εὐθεῖα ἐκτὸς πεσεῖται τοῦ κύκλου. ὁμοίως δὴ δεῖξομεν, ὅτι οὐδὲ ἐπ' αὐτῆς τῆς περιφερείας· ἐντὸς ἄρα.

(is) thus also equal to DBE [Prop. 1.5]. And since in triangle DAE the one side, AEB , has been produced, angle DEB (is) thus greater than DAE [Prop. 1.16]. And DAE (is) equal to DBE [Prop. 1.5]. Thus, DEB (is) greater than DBE . And the greater angle is subtended by the greater side [Prop. 1.19]. Thus, DB (is) greater than DE . And DB (is) equal to DF . Thus, DF (is) greater than DE , the lesser than the greater. The very thing is impossible. Thus, the straight-line joining A to B will not fall outside the circle. So, similarly, we can show that neither (will it fall) on the circumference itself. Thus, (it will fall) inside (the circle).



Ἐάν ἄρα κύκλου ἐπὶ τῆς περιφερείας ληφθῆ δύο τυχόντα σημεία, ἢ ἐπὶ τὰ σημεία ἐπιζευγνυμένη εὐθεῖα ἐντὸς πεσεῖται τοῦ κύκλου· ὅπερ ἔδει δεῖξαι.

Thus, if two points are taken at random on the circumference of a circle then the straight-line joining the points will fall inside the circle. (Which is) the very thing it was required to show.

γ'.

Proposition 3

Ἐάν ἐν κύκλῳ εὐθεῖα τις διὰ τοῦ κέντρου εὐθειάν τινα μὴ διὰ τοῦ κέντρου δίχα τέμνη, καὶ πρὸς ὀρθὰς αὐτὴν τέμνει· καὶ ἐάν πρὸς ὀρθὰς αὐτὴν τέμνη, καὶ δίχα αὐτὴν τέμνει.

In a circle, if any straight-line through the center cuts in half any straight-line not through the center then it also cuts it at right-angles. And (conversely) if it cuts it at right-angles then it also cuts it in half.

Ἐστω κύκλος ὁ $AB\Gamma$, καὶ ἐν αὐτῷ εὐθεῖα τις διὰ τοῦ κέντρου ἢ $\Gamma\Delta$ εὐθειάν τινα μὴ διὰ τοῦ κέντρου τὴν AB δίχα τεμνέτω κατὰ τὸ Z σημεῖον· λέγω, ὅτι καὶ πρὸς ὀρθὰς αὐτὴν τέμνει.

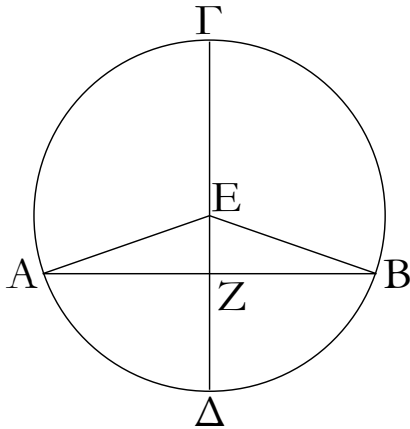
Let ABC be a circle, and, within it, let some straight-line through the center, CD , cut in half some straight-line not through the center, AB , at the point F . I say that (CD) also cuts (AB) at right-angles.

Εἰλήφθω γὰρ τὸ κέντρον τοῦ $AB\Gamma$ κύκλου, καὶ ἔστω τὸ E , καὶ ἐπεξεύχθωσαν αἱ EA , EB .

For let the center of the circle ABC have been found [Prop. 3.1], and let it be (at point) E , and let EA and EB have been joined.

Καὶ ἐπεὶ ἴση ἐστὶν ἡ AZ τῇ ZB , κοινὴ δὲ ἡ ZE , δύο δυσὶν ἴσαι [εἰσίν]· καὶ βάσις ἡ EA βάσει τῇ EB ἴση· γωνία ἄρα ἡ ὑπὸ AZE γωνία τῇ ὑπὸ BZE ἴση ἐστίν. ὅταν δὲ εὐθεῖα ἐπ' εὐθειᾶν σταθεῖσα τὰς ἐφεξῆς γωνίας ἴσας ἀλλήλαις ποιῆ, ὀρθὴ ἑκατέρω τῶν ἴσων γωνιῶν ἐστίν· ἑκατέρω ἄρα τῶν ὑπὸ AZE , BZE ὀρθὴ ἐστίν. ἢ $\Gamma\Delta$ ἄρα διὰ τοῦ κέντρου οὕσα τὴν AB μὴ διὰ τοῦ κέντρου οὕσα δίχα τέμνουσα καὶ πρὸς ὀρθὰς τέμνει.

And since AF is equal to FB , and FE (is) common, two (sides of triangle AFE) [are] equal to two (sides of triangle BFE). And the base EA (is) equal to the base EB . Thus, angle AFE is equal to angle BFE [Prop. 1.8]. And when a straight-line stood upon (another) straight-line makes adjacent angles (which are) equal to one another, each of the equal angles is a right-angle [Def. 1.10]. Thus, AFE and BFE are each right-angles. Thus, the



Ἀλλὰ δὴ ἡ ΓΔ τὴν ΑΒ πρὸς ὀρθὰς τεμνέτω· λέγω, ὅτι καὶ δίχα αὐτὴν τέμνει, τουτέστιν, ὅτι ἴση ἐστὶν ἡ ΑΖ τῆ ΖΒ.

Τῶν γὰρ αὐτῶν κατασκευασθέντων, ἐπεὶ ἴση ἐστὶν ἡ ΕΑ τῆ ΕΒ, ἴση ἐστὶ καὶ γωνία ἡ ὑπὸ ΕΑΖ τῆ ὑπὸ ΕΒΖ. ἐστὶ δὲ καὶ ὀρθὴ ἡ ὑπὸ ΑΖΕ ὀρθὴ τῆ ὑπὸ ΒΖΕ ἴση· δύο ἄρα τρίγωνά ἐστι ΕΑΖ, ΕΖΒ τὰς δύο γωνίας δυσὶ γωνίαις ἴσας ἔχοντα καὶ μίαν πλευρὰν μιᾷ πλευρᾷ ἴσην κοινὴν αὐτῶν τὴν ΕΖ ὑποτείνουσιν ὑπὸ μίαν τῶν ἴσων γωνιῶν· καὶ τὰς λοιπὰς ἄρα πλευρὰς ταῖς λοιπαῖς πλευραῖς ἴσας ἔξει· ἴση ἄρα ἡ ΑΖ τῆ ΖΒ.

Ἐὰν ἄρα ἐν κύκλῳ εὐθεῖα τις διὰ τοῦ κέντρου εὐθεῖάν τινα μὴ διὰ τοῦ κέντρου δίχα τέμνη, καὶ πρὸς ὀρθὰς αὐτὴν τέμνει· καὶ ἐὰν πρὸς ὀρθὰς αὐτὴν τέμνη, καὶ δίχα αὐτὴν τέμνει· ὅπερ ἔδει δεῖξαι.

δ'.

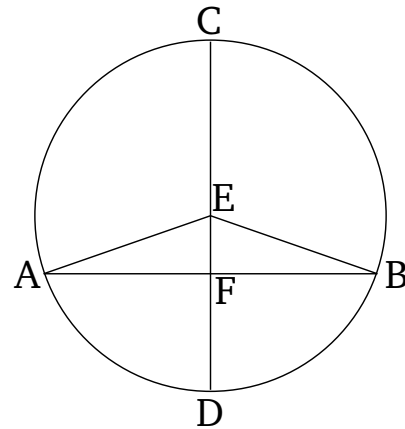
Ἐὰν ἐν κύκλῳ δύο εὐθεῖαι τέμνωσιν ἀλλήλας μὴ διὰ τοῦ κέντρου οὔσαι, οὐ τέμνουσιν ἀλλήλας δίχα.

Ἐστω κύκλος ὁ ΑΒΓΔ, καὶ ἐν αὐτῷ δύο εὐθεῖαι αἱ ΑΓ, ΒΔ τεμνέτωσαν ἀλλήλας κατὰ τὸ Ε μὴ διὰ τοῦ κέντρου οὔσαι· λέγω, ὅτι οὐ τέμνουσιν ἀλλήλας δίχα.

Εἰ γὰρ δυνατόν, τεμνέτωσαν ἀλλήλας δίχα ὥστε ἴσην εἶναι τὴν μὲν ΑΕ τῆ ΕΓ, τὴν δὲ ΒΕ τῆ ΕΔ· καὶ εἰλήφθω τὸ κέντρον τοῦ ΑΒΓΔ κύκλου, καὶ ἔστω τὸ Ζ, καὶ ἐπεζεύχθω ἡ ΖΕ.

Ἐπεὶ οὖν εὐθεῖα τις διὰ τοῦ κέντρου ἡ ΖΕ εὐθεῖάν τινα μὴ διὰ τοῦ κέντρου τὴν ΑΓ δίχα τέμνει, καὶ πρὸς ὀρθὰς αὐτὴν τέμνει· ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ ΖΕΑ· πάλιν, ἐπεὶ εὐθεῖα τις ἡ ΖΕ εὐθεῖάν τινα τὴν ΒΔ δίχα τέμνει, καὶ πρὸς ὀρθὰς αὐτὴν τέμνει· ὀρθὴ ἄρα ἡ ὑπὸ ΖΕΒ. ἐδείχθη δὲ καὶ ἡ ὑπὸ ΖΕΑ ὀρθὴ· ἴση ἄρα ἡ ὑπὸ ΖΕΑ τῆ ὑπὸ ΖΕΒ ἡ ἐλάττων τῆ

(straight-line) CD , which is through the center and cuts in half the (straight-line) AB , which is not through the center, also cuts (AB) at right-angles.



And so let CD cut AB at right-angles. I say that it also cuts (AB) in half. That is to say, that AF is equal to FB .

For, with the same construction, since EA is equal to EB , angle EAF is also equal to EBF [Prop. 1.5]. And the right-angle AFE is also equal to the right-angle BFE . Thus, EAF and EFB are two triangles having two angles equal to two angles, and one side equal to one side—(namely), their common (side) EF , subtending one of the equal angles. Thus, they will also have the remaining sides equal to the (corresponding) remaining sides [Prop. 1.26]. Thus, AF (is) equal to FB .

Thus, in a circle, if any straight-line through the center cuts in half any straight-line not through the center then it also cuts it at right-angles. And (conversely) if it cuts it at right-angles then it also cuts it in half. (Which is) the very thing it was required to show.

Proposition 4

In a circle, if two straight-lines, which are not through the center, cut one another then they do not cut one another in half.

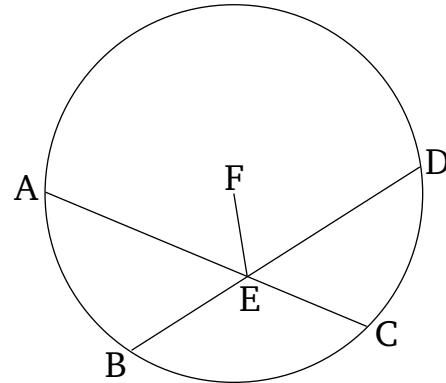
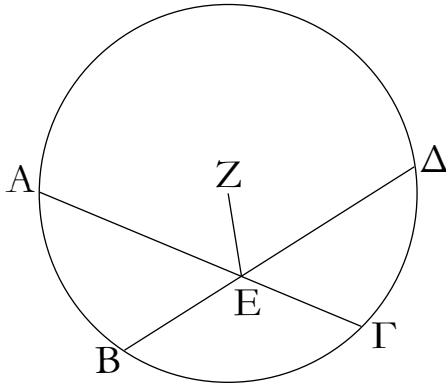
Let $ABCD$ be a circle, and within it, let two straight-lines, AC and BD , which are not through the center, cut one another at (point) E . I say that they do not cut one another in half.

For, if possible, let them cut one another in half, such that AE is equal to EC , and BE to ED . And let the center of the circle $ABCD$ have been found [Prop. 3.1], and let it be (at point) F , and let FE have been joined.

Therefore, since some straight-line through the center, FE , cuts in half some straight-line not through the center, AC , it also cuts it at right-angles [Prop. 3.3]. Thus, FEA is a right-angle. Again, since some straight-line FE

μείζονι· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα αἱ $ΑΓ$, $ΒΔ$ τέμνουσιν ἀλλήλας δίχα.

cuts in half some straight-line BD , it also cuts it at right-angles [Prop. 3.3]. Thus, FEB (is) a right-angle. But FEA was also shown (to be) a right-angle. Thus, FEA (is) equal to FEB , the lesser to the greater. The very thing is impossible. Thus, AC and BD do not cut one another in half.



Ἐὰν ἄρα ἐν κύκλῳ δύο εὐθεῖαι τέμνωσιν ἀλλήλας μὴ διὰ τοῦ κέντρου οὔσαι, οὐ τέμνουσιν ἀλλήλας δίχα· ὅπερ ἔδει δεῖξαι.

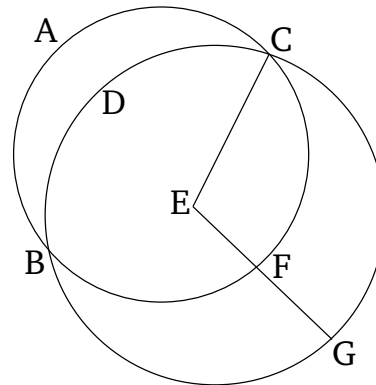
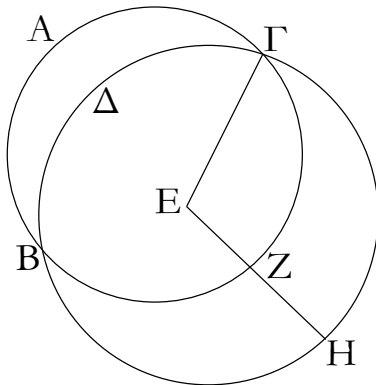
Thus, in a circle, if two straight-lines, which are not through the center, cut one another then they do not cut one another in half. (Which is) the very thing it was required to show.

ε'.

Proposition 5

Ἐὰν δύο κύκλοι τέμνωσιν ἀλλήλους, οὐκ ἔσται αὐτῶν τὸ αὐτὸ κέντρον.

If two circles cut one another then they will not have the same center.



Δύο γὰρ κύκλοι οἱ $ΑΒΓ$, $ΓΔΗ$ τεμνέτωσαν ἀλλήλους κατὰ τὰ $Β$, $Γ$ σημεία. λέγω, ὅτι οὐκ ἔσται αὐτῶν τὸ αὐτὸ κέντρον.

For let the two circles ABC and CDG cut one another at points B and C . I say that they will not have the same center.

Εἰ γὰρ δυνατόν, ἔστω τὸ $Ε$, καὶ ἐπεζεύχθω ἡ $ΕΓ$, καὶ διήχθω ἡ $ΕΖΗ$, ὡς ἔτυχεν. καὶ ἐπεὶ τὸ $Ε$ σημεῖον κέντρον ἐστὶ τοῦ $ΑΒΓ$ κύκλου, ἴση ἐστὶν ἡ $ΕΓ$ τῇ $ΕΖ$. πάλιν, ἐπεὶ τὸ $Ε$ σημεῖον κέντρον ἐστὶ τοῦ $ΓΔΗ$ κύκλου, ἴση ἐστὶν ἡ $ΕΓ$ τῇ $ΕΗ$: ἐδείχθη δὲ ἡ $ΕΓ$ καὶ τῇ $ΕΖ$ ἴση: καὶ ἡ $ΕΖ$ ἄρα τῇ $ΕΗ$ ἐστὶν ἴση ἢ ἐλάσσων τῇ μείζονι: ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα τὸ $Ε$ σημεῖον κέντρον ἐστὶ τῶν $ΑΒΓ$, $ΓΔΗ$ κύκλων.

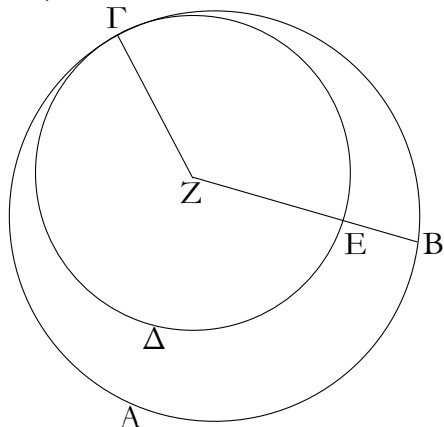
For, if possible, let E be (the common center), and let EC have been joined, and let EFG have been drawn through (the two circles), at random. And since point E is the center of the circle ABC , EC is equal to EF . Again, since point E is the center of the circle CDG , EC is equal to EG . But EC was also shown (to be) equal to EF . Thus, EF is also equal to EG , the lesser to the greater. The very thing is impossible. Thus, point E is not

Ἐὰν ἄρα δύο κύκλοι τέμνωσιν ἀλλήλους, οὐκ ἔστιν

αὐτῶν τὸ αὐτὸ κέντρον· ὅπερ ἔδει δεῖξαι.

ϛ'.

Ἐὰν δύο κύκλοι ἐφάπτωνται ἀλλήλων, οὐκ ἔσται αὐτῶν τὸ αὐτὸ κέντρον.



Δύο γὰρ κύκλοι οἱ ABΓ, ΓΔΕ ἐφάπτεσθωσαν ἀλλήλων κατὰ τὸ Γ σημεῖον· λέγω, ὅτι οὐκ ἔσται αὐτῶν τὸ αὐτὸ κέντρον.

Εἰ γὰρ δυνατόν, ἔστω τὸ Z, καὶ ἐπεζεύχθω ἡ ZΓ, καὶ διήχθω, ὡς ἔτυχεν, ἡ ZEB.

Ἐπεὶ οὖν τὸ Z σημεῖον κέντρον ἐστὶ τοῦ ABΓ κύκλου, ἴση ἐστὶν ἡ ZΓ τῇ ZB. πάλιν, ἐπεὶ τὸ Z σημεῖον κέντρον ἐστὶ τοῦ ΓΔΕ κύκλου, ἴση ἐστὶν ἡ ZΓ τῇ ZE. ἐδείχθη δὲ ἡ ZΓ τῇ ZB ἴση· καὶ ἡ ZE ἄρα τῇ ZB ἐστὶν ἴση, ἢ ἐλάττω τῇ μείζονι· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα τὸ Z σημεῖον κέντρον ἐστὶ τῶν ABΓ, ΓΔΕ κύκλων.

Ἐὰν ἄρα δύο κύκλοι ἐφάπτωνται ἀλλήλων, οὐκ ἔσται αὐτῶν τὸ αὐτὸ κέντρον· ὅπερ ἔδει δεῖξαι.

ζ'.

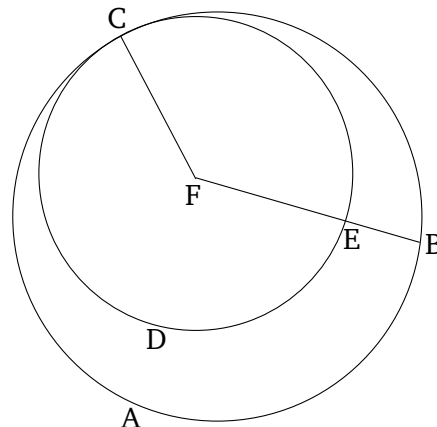
Ἐὰν κύκλου ἐπὶ τῆς διαμέτρου ληφθῆ τι σημεῖον, ὃ μὴ ἐστὶ κέντρον τοῦ κύκλου, ἀπὸ δὲ τοῦ σημείου πρὸς τὸν κύκλον προσπίπτωσιν εὐθεῖαι τινες, μεγίστη μὲν ἔσται, ἐφ' ἧς τὸ κέντρον, ἐλάχιστη δὲ ἡ λοιπή, τῶν δὲ ἄλλων αἰεὶ ἢ ἕγγιον τῆς διὰ τοῦ κέντρου τῆς ἀπώτερον μείζων ἐστίν, δύο δὲ μόνον ἴσα ἀπὸ τοῦ σημείου προσπεσοῦνται πρὸς τὸν κύκλον ἐφ' ἑκάτερα τῆς ἐλάχιστης.

the (common) center of the circles *ABC* and *CDG*.

Thus, if two circles cut one another then they will not have the same center. (Which is) the very thing it was required to show.

Proposition 6

If two circles touch one another then they will not have the same center.



For let the two circles *ABC* and *CDE* touch one another at point *C*. I say that they will not have the same center.

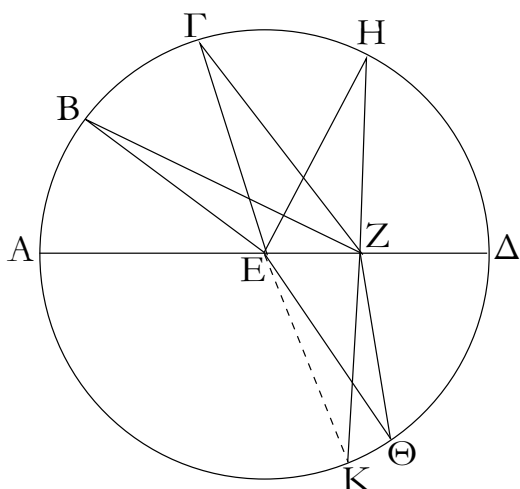
For, if possible, let *F* be (the common center), and let *FC* have been joined, and let *FEB* have been drawn through (the two circles), at random.

Therefore, since point *F* is the center of the circle *ABC*, *FC* is equal to *FB*. Again, since point *F* is the center of the circle *CDE*, *FC* is equal to *FE*. But *FC* was shown (to be) equal to *FB*. Thus, *FE* is also equal to *FB*, the lesser to the greater. The very thing is impossible. Thus, point *F* is not the (common) center of the circles *ABC* and *CDE*.

Thus, if two circles touch one another then they will not have the same center. (Which is) the very thing it was required to show.

Proposition 7

If some point, which is not the center of the circle, is taken on the diameter of a circle, and some straight-lines radiate from the point towards the (circumference of the) circle, then the greatest (straight-line) will be that on which the center (lies), and the least the remainder (of the same diameter). And for the others, a (straight-line) nearer[†] to the (straight-line) through the center is always greater than a (straight-line) further away. And only two equal (straight-lines) will radiate from the point towards the (circumference of the) circle, (one) on each



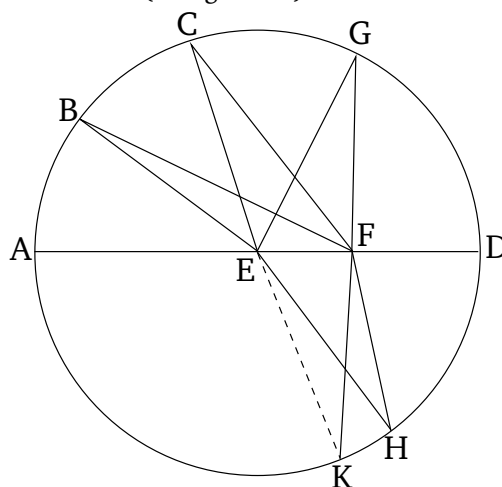
Ἐστω κύκλος ὁ $AB\Gamma\Delta$, διάμετρος δὲ αὐτοῦ ἔστω ἡ $A\Delta$, καὶ ἐπὶ τῆς $A\Delta$ εἰλήφθω τι σημεῖον τὸ Z , ὃ μὴ ἔστι κέντρον τοῦ κύκλου, κέντρον δὲ τοῦ κύκλου ἔστω τὸ E , καὶ ἀπὸ τοῦ Z πρὸς τὸν $AB\Gamma\Delta$ κύκλον προσπιπέτωσαν εὐθεῖαι τινες αἱ $ZB, Z\Gamma, ZH$. λέγω, ὅτι μεγίστη μὲν ἔστιν ἡ ZA , ἐλαχίστη δὲ ἡ $Z\Delta$, τῶν δὲ ἄλλων ἢ μὲν ZB τῆς $Z\Gamma$ μείζων, ἢ δὲ $Z\Gamma$ τῆς ZH .

Ἐπεζεύχθωσαν γὰρ αἱ $BE, \Gamma E, HE$. καὶ ἐπεὶ παντὸς τριγώνου αἱ δύο πλευραὶ τῆς λοιπῆς μείζονές εἰσιν, αἱ ἄρα EB, EZ τῆς BZ μείζονές εἰσιν. ἴση δὲ ἡ AE τῆ BE [αἱ ἄρα BE, EZ ἴσαι εἰσὶ τῇ AZ]: μείζων ἄρα ἡ AZ τῆς BZ . πάλιν, ἐπεὶ ἴση ἔστιν ἡ BE τῆ ΓE , κοινὴ δὲ ἡ ZE , δύο δὲ αἱ BE, EZ δυοῖ ταῖς $\Gamma E, EZ$ ἴσαι εἰσίν. ἀλλὰ καὶ γωνία ἡ ὑπὸ BEZ γωνίας τῆς ὑπὸ ΓEZ μείζων: βάσις ἄρα ἡ BZ βάσεως τῆς ΓZ μείζων ἔστιν. διὰ τὰ αὐτὰ δὴ καὶ ἡ ΓZ τῆς ZH μείζων ἔστιν.

Πάλιν, ἐπεὶ αἱ HZ, ZE τῆς EH μείζονές εἰσιν, ἴση δὲ ἡ EH τῆ $E\Delta$, αἱ ἄρα HZ, ZE τῆς $E\Delta$ μείζονές εἰσιν. κοινὴ ἀφρηθήσθω ἡ EZ : λοιπὴ ἄρα ἡ HZ λοιπῆς τῆς $Z\Delta$ μείζων ἔστιν. μεγίστη μὲν ἄρα ἡ ZA , ἐλαχίστη δὲ ἡ $Z\Delta$, μείζων δὲ ἢ μὲν ZB τῆς $Z\Gamma$, ἢ δὲ $Z\Gamma$ τῆς ZH .

Λέγω, ὅτι καὶ ἀπὸ τοῦ Z σημείου δύο μόνον ἴσαι προσπεσοῦνται πρὸς τὸν $AB\Gamma\Delta$ κύκλον ἐφ' ἑκάτερα τῆς $Z\Delta$ ἐλαχίστης. συνεστάτω γὰρ πρὸς τῇ EZ εὐθείᾳ καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ E τῆ ὑπὸ HEZ γωνία ἴση ἢ ὑπὸ $ZE\Theta$, καὶ ἐπεζεύχθω ἡ $Z\Theta$. ἐπεὶ οὖν ἴση ἔστιν ἡ HE τῆ $E\Theta$, κοινὴ δὲ ἡ EZ , δύο δὲ αἱ HE, EZ δυοῖ ταῖς $\Theta E, EZ$ ἴσαι εἰσίν: καὶ γωνία ἡ ὑπὸ HEZ γωνία τῆ ὑπὸ ΘEZ ἴση: βάσις ἄρα ἡ ZH βάσει τῆ $Z\Theta$ ἴση ἔστιν. λέγω δὴ, ὅτι τῆ ZH ἄλλη ἴση οὐ προσπεσεῖται πρὸς τὸν κύκλον ἀπὸ τοῦ Z σημείου. εἰ γὰρ δυνατόν, προσπιπέτω ἡ ZK . καὶ ἐπεὶ ἡ ZK τῆ ZH ἴση ἔστιν, ἀλλὰ ἡ $Z\Theta$ τῆ ZH [ἴση ἔστιν], καὶ ἡ ZK ἄρα τῆ $Z\Theta$ ἔστιν ἴση, ἢ ἕγγιον τῆς διὰ τοῦ κέντρου τῆ ἀπώτερον ἴση: ὅπερ ἀδύνατον. οὐκ ἄρα ἀπὸ τοῦ Z σημείου ἕτερα τις

(side) of the least (straight-line).



Let $ABCD$ be a circle, and let AD be its diameter, and let some point F , which is not the center of the circle, have been taken on AD . Let E be the center of the circle. And let some straight-lines, FB, FC , and FG , radiate from F towards (the circumference of) circle $ABCD$. I say that FA is the greatest (straight-line), FD the least, and of the others, FB (is) greater than FC , and FC than FG .

For let BE, CE , and GE have been joined. And since for every triangle (any) two sides are greater than the remaining (side) [Prop. 1.20], EB and EF is thus greater than BF . And AE (is) equal to BE [thus, BE and EF is equal to AF]. Thus, AF (is) greater than BF . Again, since BE is equal to CE , and FE (is) common, the two (straight-lines) BE, EF are equal to the two (straight-lines) CE, EF (respectively). But, angle BEF (is) also greater than angle CEF .[‡] Thus, the base BF is greater than the base CF . Thus, the base BF is greater than the base CF [Prop. 1.24]. So, for the same (reasons), CF is also greater than FG .

Again, since GF and FE are greater than EG [Prop. 1.20], and EG (is) equal to ED , GF and FE are thus greater than ED . Let EF have been taken from both. Thus, the remainder GF is greater than the remainder FD . Thus, FA (is) the greatest (straight-line), FD the least, and FB (is) greater than FC , and FC than FG .

I also say that from point F only two equal (straight-lines) will radiate towards (the circumference of) circle $ABCD$, (one) on each (side) of the least (straight-line) FD . For let the (angle) FEH , equal to angle GEF , have been constructed on the straight-line EF , at the point E on it [Prop. 1.23], and let FH have been joined. Therefore, since GE is equal to EH , and EF (is) common,

προσπεσεῖται πρὸς τὸν κύκλον ἴση τῇ HZ : μία ἄρα μόνη.

Ἐάν ἄρα κύκλου ἐπὶ τῆς διαμέτρου ληφθῆ τι σημεῖον, ὃ μὴ ἔστι κέντρον τοῦ κύκλου, ἀπὸ δὲ τοῦ σημείου πρὸς τὸν κύκλον προσπίπτωσιν εὐθεεῖαι τινες, μεγίστη μὲν ἔσται, ἐφ' ἧς τὸ κέντρον, ἐλαχίστη δὲ ἡ λοιπή, τῶν δὲ ἄλλων ἀεὶ ἡ ἔγγιον τῆς διὰ τοῦ κέντρου τῆς ἀπώτερον μείζων ἔστιν, δύο δὲ μόνον ἴσαι ἀπὸ τοῦ αὐτοῦ σημείου προσπεσοῦνται πρὸς τὸν κύκλον ἐφ' ἑκάτερα τῆς ἐλαχίστης· ὅπερ ἔδει δεῖξαι.

the two (straight-lines) GE , EF are equal to the two (straight-lines) HE , EF (respectively). And angle GEF (is) equal to angle HEF . Thus, the base FG is equal to the base FH [Prop. 1.4]. So I say that another (straight-line) equal to FG will not radiate towards (the circumference of) the circle from point F . For, if possible, let FK (so) radiate. And since FK is equal to FG , but FH [is equal] to FG , FK is thus also equal to FH , the nearer to the (straight-line) through the center equal to the further away. The very thing (is) impossible. Thus, another (straight-line) equal to GF will not radiate from the point F towards (the circumference of) the circle. Thus, (there is) only one (such straight-line).

Thus, if some point, which is not the center of the circle, is taken on the diameter of a circle, and some straight-lines radiate from the point towards the (circumference of the) circle, then the greatest (straight-line) will be that on which the center (lies), and the least the remainder (of the same diameter). And for the others, a (straight-line) nearer to the (straight-line) through the center is always greater than a (straight-line) further away. And only two equal (straight-lines) will radiate from the same point towards the (circumference of the) circle, (one) on each (side) of the least (straight-line). (Which is) the very thing it was required to show.

† Presumably, in an angular sense.

‡ This is not proved, except by reference to the figure.

η'.

Ἐάν κύκλου ληφθῆ τι σημεῖον ἐκτός, ἀπὸ δὲ τοῦ σημείου πρὸς τὸν κύκλον διαχθῶσιν εὐθεεῖαι τινες, ὧν μία μὲν διὰ τοῦ κέντρου, αἱ δὲ λοιπαί, ὡς ἔτυχεν, τῶν μὲν πρὸς τὴν κοίλην περιφέρειαν προσπιπτουσῶν εὐθειῶν μεγίστη μὲν ἔστιν ἡ διὰ τοῦ κέντρου, τῶν δὲ ἄλλων ἀεὶ ἡ ἔγγιον τῆς διὰ τοῦ κέντρου τῆς ἀπώτερον μείζων ἔστιν, τῶν δὲ πρὸς τὴν κυρτὴν περιφέρειαν προσπιπτουσῶν εὐθειῶν ἐλαχίστη μὲν ἔστιν ἡ μεταξὺ τοῦ τε σημείου καὶ τῆς διαμέτρου, τῶν δὲ ἄλλων ἀεὶ ἡ ἔγγιον τῆς ἐλαχίστης τῆς ἀπώτερον ἔστιν ἐλάττων, δύο δὲ μόνον ἴσαι ἀπὸ τοῦ σημείου προσπεσοῦνται πρὸς τὸν κύκλον ἐφ' ἑκάτερα τῆς ἐλαχίστης.

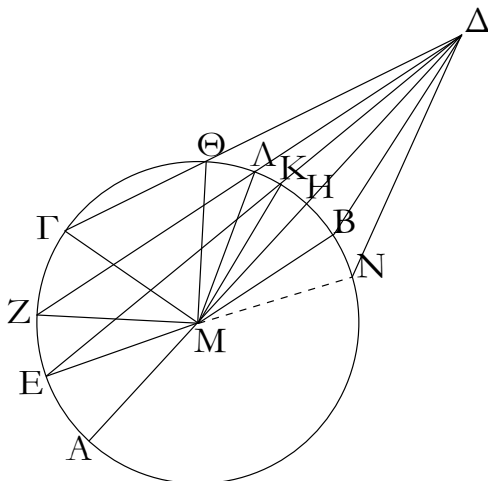
Ἔστω κύκλος ὁ $ABΓ$, καὶ τοῦ $ABΓ$ εἰλήφθω τι σημεῖον ἐκτός τὸ Δ , καὶ ἀπ' αὐτοῦ διήχθωσαν εὐθεεῖαι τινες αἱ ΔA , ΔE , ΔZ , $\Delta Γ$, ἔστω δὲ ἡ ΔA διὰ τοῦ κέντρου. λέγω, ὅτι τῶν μὲν πρὸς τὴν $AEZΓ$ κοίλην περιφέρειαν προσπιπτουσῶν εὐθειῶν μεγίστη μὲν ἔστιν ἡ διὰ τοῦ κέντρου ἡ ΔA , μείζων δὲ ἡ μὲν ΔE τῆς ΔZ ἢ δὲ ΔZ τῆς $\Delta Γ$, τῶν δὲ πρὸς τὴν $\Theta\Lambda K\text{H}$ κυρτὴν περιφέρειαν προσπιπτουσῶν εὐθειῶν ἐλαχίστη μὲν ἔστιν ἡ ΔH ἢ μεταξὺ τοῦ σημείου καὶ τῆς διαμέτρου τῆς AH , ἀεὶ δὲ ἡ ἔγγιον τῆς ΔH ἐλαχίστης ἐλάττων ἔστι τῆς ἀπώτερον, ἢ μὲν ΔK τῆς $\Delta\Lambda$, ἢ δὲ $\Delta\Lambda$

Proposition 8

If some point is taken outside a circle, and some straight-lines are drawn from the point to the (circumference of the) circle, one of which (passes) through the center, the remainder (being) random, then for the straight-lines radiating towards the concave (part of the) circumference, the greatest is that (passing) through the center. For the others, a (straight-line) nearer[†] to the (straight-line) through the center is always greater than one further away. For the straight-lines radiating towards the convex (part of the) circumference, the least is that between the point and the diameter. For the others, a (straight-line) nearer to the least (straight-line) is always less than one further away. And only two equal (straight-lines) will radiate from the point towards the (circumference of the) circle, (one) on each (side) of the least (straight-line).

Let ABC be a circle, and let some point D have been taken outside ABC , and from it let some straight-lines, DA , DE , DF , and DC , have been drawn through (the circle), and let DA be through the center. I say that for the straight-lines radiating towards the concave (part of

τῆς ΔΘ.



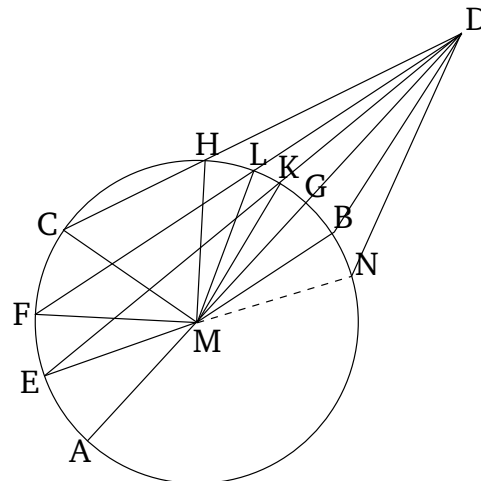
Εἰλήφθω γὰρ τὸ κέντρον τοῦ ΑΒΓ κύκλου καὶ ἔστω τὸ Μ· καὶ ἐπεζεύχθωσαν αἱ ΜΕ, ΜΖ, ΜΓ, ΜΚ, ΜΑ, ΜΘ.

Καὶ ἐπεὶ ἴση ἐστὶν ἡ ΑΜ τῇ ΕΜ, κοινὴ προσκείσθω ἡ ΜΔ· ἡ ἄρα ΑΔ ἴση ἐστὶ ταῖς ΕΜ, ΜΔ. ἄλλ' αἱ ΕΜ, ΜΔ τῆς ΕΔ μείζονές εἰσιν· καὶ ἡ ΑΔ ἄρα τῆς ΕΔ μείζων ἐστίν. πάλιν, ἐπεὶ ἴση ἐστὶν ἡ ΜΕ τῇ ΜΖ, κοινὴ δὲ ἡ ΜΔ, αἱ ΕΜ, ΜΔ ἄρα ταῖς ΖΜ, ΜΔ ἴσαι εἰσίν· καὶ γωνία ἡ ὑπὸ ΕΜΔ γωνίας τῆς ὑπὸ ΖΜΔ μείζων ἐστίν. βάσις ἄρα ἡ ΕΔ βάσεως τῆς ΖΔ μείζων ἐστίν· ὁμοίως δὲ δεῖξομεν, ὅτι καὶ ἡ ΖΔ τῆς ΓΔ μείζων ἐστίν· μεγίστη μὲν ἄρα ἡ ΔΑ, μείζων δὲ ἡ μὲν ΔΕ τῆς ΔΖ, ἡ δὲ ΔΖ τῆς ΔΓ.

Καὶ ἐπεὶ αἱ ΜΚ, ΚΔ τῆς ΜΔ μείζονές εἰσιν, ἴση δὲ ἡ ΜΗ τῇ ΜΚ, λοιπὴ ἄρα ἡ ΚΔ λοιπῆς τῆς ΗΔ μείζων ἐστίν· ὥστε ἡ ΗΔ τῆς ΚΔ ἐλάττων ἐστίν· καὶ ἐπεὶ τριγώνου τοῦ ΜΑΔ ἐπὶ μιᾶς τῶν πλευρῶν τῆς ΜΔ δύο εὐθεῖαι ἐντὸς συνεστάθησαν αἱ ΜΚ, ΚΔ, αἱ ἄρα ΜΚ, ΚΔ τῶν ΜΑ, ΛΔ ἐλάττονές εἰσιν· ἴση δὲ ἡ ΜΚ τῇ ΜΑ· λοιπὴ ἄρα ἡ ΔΚ λοιπῆς τῆς ΔΑ ἐλάττων ἐστίν. ὁμοίως δὲ δεῖξομεν, ὅτι καὶ ἡ ΔΑ τῆς ΔΘ ἐλάττων ἐστίν· ἐλαχίστη μὲν ἄρα ἡ ΔΗ, ἐλάττων δὲ ἡ μὲν ΔΚ τῆς ΔΑ ἡ δὲ ΔΑ τῆς ΔΘ.

Λέγω, ὅτι καὶ δύο μόνον ἴσαι ἀπὸ τοῦ Δ σημείου προσπεσοῦνται πρὸς τὸν κύκλον ἐφ' ἑκάτερα τῆς ΔΗ ἐλαχίστης· συνεστάτω πρὸς τῇ ΜΔ εὐθεῖα καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ Μ τῇ ὑπὸ ΚΜΔ γωνία ἴση γωνία ἡ ὑπὸ ΔΜΒ, καὶ ἐπεζεύχθω ἡ ΔΒ. καὶ ἐπεὶ ἴση ἐστὶν ἡ ΜΚ τῇ ΜΒ, κοινὴ δὲ ἡ ΜΔ, δύο δὲ αἱ ΚΜ, ΜΔ δύο ταῖς ΒΜ, ΜΔ

the) circumference, *AEFC*, the greatest is the one (passing) through the center, (namely) *AD*, and (that) *DE* (is) greater than *DF*, and *DF* than *DC*. For the straight-lines radiating towards the convex (part of the) circumference, *HLKG*, the least is the one between the point and the diameter *AG*, (namely) *DG*, and a (straight-line) nearer to the least (straight-line) *DG* is always less than one farther away, (so that) *DK* (is less) than *DL*, and *DL* than than *DH*.



For let the center of the circle have been found [Prop. 3.1], and let it be (at point) *M* [Prop. 3.1]. And let *ME*, *MF*, *MC*, *MK*, *ML*, and *MH* have been joined.

And since *AM* is equal to *EM*, let *MD* have been added to both. Thus, *AD* is equal to *EM* and *MD*. But, *EM* and *MD* is greater than *ED* [Prop. 1.20]. Thus, *AD* is also greater than *ED*. Again, since *ME* is equal to *MF*, and *MD* (is) common, the (straight-lines) *EM*, *MD* are thus equal to *FM*, *MD*. And angle *EMD* is greater than angle *FMD*.[‡] Thus, the base *ED* is greater than the base *FD* [Prop. 1.24]. So, similarly, we can show that *FD* is also greater than *CD*. Thus, *AD* (is) the greatest (straight-line), and *DE* (is) greater than *DF*, and *DF* than *DC*.

And since *MK* and *KD* is greater than *MD* [Prop. 1.20], and *MG* (is) equal to *MK*, the remainder *KD* is thus greater than the remainder *GD*. So *GD* is less than *KD*. And since in triangle *MLD*, the two internal straight-lines *MK* and *KD* were constructed on one of the sides, *MD*, then *MK* and *KD* are thus less than *ML* and *LD* [Prop. 1.21]. And *MK* (is) equal to *ML*. Thus, the remainder *DK* is less than the remainder *DL*. So, similarly, we can show that *DL* is also less than *DH*. Thus, *DG* (is) the least (straight-line), and *DK* (is) less than *DL*, and *DL* than *DH*.

I also say that only two equal (straight-lines) will radi-

ἴσαι εἰσὶν ἑκατέρα ἑκατέρᾳ· καὶ γωνία ἡ ὑπὸ KMD γωνία τῆ ὑπὸ BMD ἴση· βάσις ἄρα ἡ DK βάσει τῆ DB ἴση ἐστίν. λέγω [δὴ], ὅτι τῆ DK εὐθείᾳ ἄλλη ἴση οὐ προσπεσεῖται πρὸς τὸν κύκλον ἀπὸ τοῦ Δ σημείου. εἰ γὰρ δυνατόν, προσπιπτέτω καὶ ἔστω ἡ ΔN . ἐπεὶ οὖν ἡ DK τῆ ΔN ἐστὶν ἴση, ἀλλ' ἡ DK τῆ DB ἐστὶν ἴση, καὶ ἡ DB ἄρα τῆ ΔN ἐστὶν ἴση, ἡ ἔγγιον τῆς ΔH ἐλαχίστης τῆ ἀπώτερον [ἐστὶν] ἴση· ὅπερ ἀδύνατον ἐδείχθη. οὐκ ἄρα πλείους ἢ δύο ἴσαι πρὸς τὸν $AB\Gamma$ κύκλον ἀπὸ τοῦ Δ σημείου ἐφ' ἑκάτερα τῆς ΔH ἐλαχίστης προσπεσοῦνται.

Ἐὰν ἄρα κύκλου ληφθῆ τι σημεῖον ἐκτός, ἀπὸ δὲ τοῦ σημείου πρὸς τὸν κύκλον διαχθῶσιν εὐθεῖαι τινες, ὧν μία μὲν διὰ τοῦ κέντρου αἱ δὲ λοιπαί, ὡς ἔτυχεν, τῶν μὲν πρὸς τὴν κοίλην περιφέρειαν προσπιπτουσῶν εὐθειῶν μεγίστη μὲν ἐστὶν ἡ διὰ τοῦ κέντρου, τῶν δὲ ἄλλων αἰεὶ ἡ ἔγγιον τῆς διὰ τοῦ κέντρου τῆς ἀπώτερον μείζων ἐστίν, τῶν δὲ πρὸς τὴν κυρτὴν περιφέρειαν προσπιπτουσῶν εὐθειῶν ἐλαχίστη μὲν ἐστὶν ἡ μεταξὺ τοῦ τε σημείου καὶ τῆς διαμέτρου, τῶν δὲ ἄλλων αἰεὶ ἡ ἔγγιον τῆς ἐλαχίστης τῆς ἀπώτερον ἐστὶν ἐλάττων, δύο δὲ μόνον ἴσαι ἀπὸ τοῦ σημείου προσπεσοῦνται πρὸς τὸν κύκλον ἐφ' ἑκάτερα τῆς ἐλαχίστης· ὅπερ ἔδει δεῖξαι.

ate from point D towards (the circumference of) the circle, (one) on each (side) on the least (straight-line), DG . Let the angle DMB , equal to angle KMD , have been constructed on the straight-line MD , at the point M on it [Prop. 1.23], and let DB have been joined. And since MK is equal to MB , and MD (is) common, the two (straight-lines) KM , MD are equal to the two (straight-lines) BM , MD , respectively. And angle KMD (is) equal to angle BMD . Thus, the base DK is equal to the base DB [Prop. 1.4]. [So] I say that another (straight-line) equal to DK will not radiate towards the (circumference of the) circle from point D . For, if possible, let (such a straight-line) radiate, and let it be DN . Therefore, since DK is equal to DN , but DK is equal to DB , then DB is thus also equal to DN , (so that) a (straight-line) nearer to the least (straight-line) DG [is] equal to one further away. The very thing was shown (to be) impossible. Thus, not more than two equal (straight-lines) will radiate towards (the circumference of) circle ABC from point D , (one) on each side of the least (straight-line) DG .

Thus, if some point is taken outside a circle, and some straight-lines are drawn from the point to the (circumference of the) circle, one of which (passes) through the center, the remainder (being) random, then for the straight-lines radiating towards the concave (part of the) circumference, the greatest is that (passing) through the center. For the others, a (straight-line) nearer to the (straight-line) through the center is always greater than one further away. For the straight-lines radiating towards the convex (part of the) circumference, the least is that between the point and the diameter. For the others, a (straight-line) nearer to the least (straight-line) is always less than one further away. And only two equal (straight-lines) will radiate from the point towards the (circumference of the) circle, (one) on each (side) of the least (straight-line). (Which is) the very thing it was required to show.

† Presumably, in an angular sense.

‡ This is not proved, except by reference to the figure.

θ'.

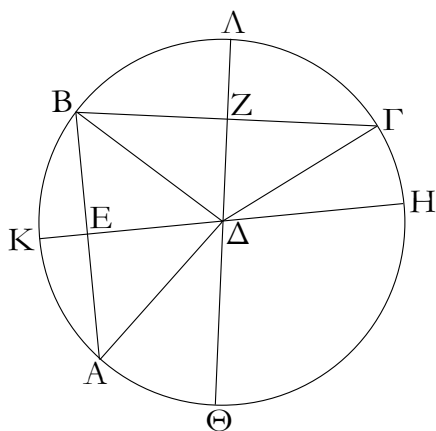
Ἐὰν κύκλου ληφθῆ τι σημεῖον ἐντός, ἀπὸ δὲ τοῦ σημείου πρὸς τὸν κύκλον προσπίπτωσι πλείους ἢ δύο ἴσαι εὐθεῖαι, τὸ ληφθὲν σημεῖον κέντρον ἐστὶ τοῦ κύκλου.

Ἐστω κύκλος ὁ $AB\Gamma$, ἐντός δὲ αὐτοῦ σημεῖον τὸ Δ , καὶ ἀπὸ τοῦ Δ πρὸς τὸν $AB\Gamma$ κύκλον προσπιπτέωσαν πλείους ἢ δύο ἴσαι εὐθεῖαι αἱ ΔA , ΔB , $\Delta \Gamma$. λέγω, ὅτι τὸ Δ σημεῖον κέντρον ἐστὶ τοῦ $AB\Gamma$ κύκλου.

Proposition 9

If some point is taken inside a circle, and more than two equal straight-lines radiate from the point towards the (circumference of the) circle, then the point taken is the center of the circle.

Let ABC be a circle, and D a point inside it, and let more than two equal straight-lines, DA , DB , and DC , radiate from D towards (the circumference of) circle ABC .



Ἐπεζεύχθωσαν γὰρ αἰ AB, BΓ καὶ τετμήσθωσαν δίχα κατὰ τὰ E, Z σημεῖα, καὶ ἐπιζευχθεῖσαι αἰ EΔ, ZΔ διήχθωσαν ἐπὶ τὰ H, K, Θ, Λ σημεῖα.

Ἐπεὶ οὖν ἴση ἐστὶν ἡ AE τῇ EB, κοινὴ δὲ ἡ EΔ, δύο δὴ αἰ AE, EΔ δύο ταῖς BE, EΔ ἴσαι εἰσὶν· καὶ βάσις ἡ ΔA βάσει τῇ ΔB ἴση· γωνία ἄρα ἡ ὑπὸ AED γωνία τῇ ὑπὸ BED ἴση ἐστίν· ὀρθὴ ἄρα ἑκατέρα τῶν ὑπὸ AED, BED γωνιῶν· ἡ HK ἄρα τὴν AB τέμνει δίχα καὶ πρὸς ὀρθάς. καὶ ἐπεὶ, ἐὰν ἐν κύκλῳ εὐθείᾳ τις εὐθείαν τινα δίχα τε καὶ πρὸς ὀρθὰς τέμνη, ἐπὶ τῆς τεμνούσης ἐστὶ τὸ κέντρον τοῦ κύκλου, ἐπὶ τῆς HK ἄρα ἐστὶ τὸ κέντρον τοῦ κύκλου. διὰ τὰ αὐτὰ δὴ καὶ ἐπὶ τῆς ΘΛ ἐστὶ τὸ κέντρον τοῦ ABΓ κύκλου. καὶ οὐδὲν ἕτερον κοινὸν ἔχουσιν αἰ HK, ΘΛ εὐθεῖαι ἢ τὸ Δ σημεῖον· τὸ Δ ἄρα σημεῖον κέντρον ἐστὶ τοῦ ABΓ κύκλου.

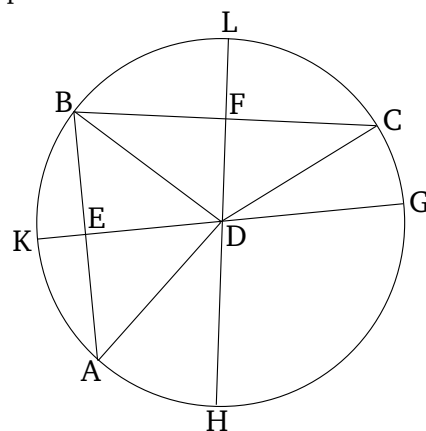
Ἐὰν ἄρα κύκλου ληφθῆ τι σημεῖον ἐντός, ἀπὸ δὲ τοῦ σημείου πρὸς τὸν κύκλον προσπίπτωσι πλείους ἢ δύο ἴσαι εὐθεῖαι, τὸ ληφθὲν σημεῖον κέντρον ἐστὶ τοῦ κύκλου· ὅπερ εἶδει δεῖξαι.

ι'.

Κύκλος κύκλον οὐ τέμνει κατὰ πλείονα σημεῖα ἢ δύο.

Εἰ γὰρ δυνατόν, κύκλος ὁ ABΓ κύκλον τὸν ΔEZ τεμνέτω κατὰ πλείονα σημεῖα ἢ δύο τὰ B, H, Z, Θ, καὶ ἐπιζευχθεῖσαι αἰ BΘ, BH δίχα τεμνέσθωσαν κατὰ τὰ K, Λ σημεῖα· καὶ ἀπὸ τῶν K, Λ ταῖς BΘ, BH πρὸς ὀρθὰς ἀχθεῖσαι αἰ KΓ, ΛM διήχθωσαν ἐπὶ τὰ A, E σημεῖα.

I say that point *D* is the center of circle *ABC*.



For let *AB* and *BC* have been joined, and (then) have been cut in half at points *E* and *F* (respectively) [Prop. 1.10]. And *ED* and *FD* being joined, let them have been drawn through to points *G*, *K*, *H*, and *L*.

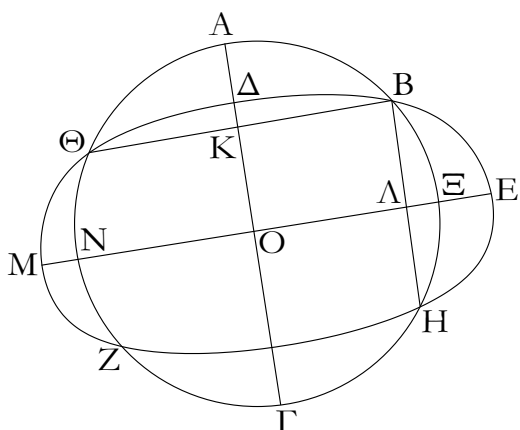
Therefore, since *AE* is equal to *EB*, and *ED* (is) common, the two (straight-lines) *AE*, *ED* are equal to the two (straight-lines) *BE*, *ED* (respectively). And the base *DA* (is) equal to the base *DB*. Thus, angle *AED* is equal to angle *BED* [Prop. 1.8]. Thus, angles *AED* and *BED* (are) each right-angles [Def. 1.10]. Thus, *GK* cuts *AB* in half, and at right-angles. And since, if some straight-line in a circle cuts some (other) straight-line in half, and at right-angles, then the center of the circle is on the former (straight-line) [Prop. 3.1 corr.], the center of the circle is thus on *GK*. So, for the same (reasons), the center of circle *ABC* is also on *HL*. And the straight-lines *GK* and *HL* have no common (point) other than point *D*. Thus, point *D* is the center of circle *ABC*.

Thus, if some point is taken inside a circle, and more than two equal straight-lines radiate from the point towards the (circumference of the) circle, then the point taken is the center of the circle. (Which is) the very thing it was required to show.

Proposition 10

A circle does not cut a(nother) circle at more than two points.

For, if possible, let the circle *ABC* cut the circle *DEF* at more than two points, *B*, *G*, *F*, and *H*. And *BH* and *BG* being joined, let them (then) have been cut in half at points *K* and *L* (respectively). And *KC* and *LM* being drawn at right-angles to *BH* and *BG* from *K* and *L* (respectively) [Prop. 1.11], let them (then) have been drawn through to points *A* and *E* (respectively).



Ἐπει οὖν ἐν κύκλῳ τῷ $AB\Gamma$ εὐθεΐά τις ἢ AG εὐθεϊάν τινα τὴν $B\Theta$ δίχα καὶ πρὸς ὀρθὰς τέμνει, ἐπὶ τῆς AG ἄρα ἐστὶ τὸ κέντρον τοῦ $AB\Gamma$ κύκλου. πάλιν, ἐπεὶ ἐν κύκλῳ τῷ αὐτῷ τῷ $AB\Gamma$ εὐθεΐά τις ἢ NE εὐθεϊάν τινα τὴν BH δίχα καὶ πρὸς ὀρθὰς τέμνει, ἐπὶ τῆς NE ἄρα ἐστὶ τὸ κέντρον τοῦ $AB\Gamma$ κύκλου. ἐδείχθη δὲ καὶ ἐπὶ τῆς AG , καὶ κατ' οὐδὲν συμβάλλουσιν αἱ AG , NE εὐθεΐαι ἢ κατὰ τὸ O . τὸ O ἄρα σημεῖον κέντρον ἐστὶ τοῦ $AB\Gamma$ κύκλου. ὁμοίως δὴ δεῖξομεν, ὅτι καὶ τοῦ ΔEZ κύκλου κέντρον ἐστὶ τὸ O . δύο ἄρα κύκλων τεμνόντων ἀλλήλους τῶν $AB\Gamma$, ΔEZ τὸ αὐτὸ ἐστὶ κέντρον τὸ O . ὅπερ ἐστὶν ἀδύνατον.

Οὕκ ἄρα κύκλος κύκλον τέμνει κατὰ πλείονα σημεῖα ἢ δύο. ὅπερ ἔδει δεῖξαι.

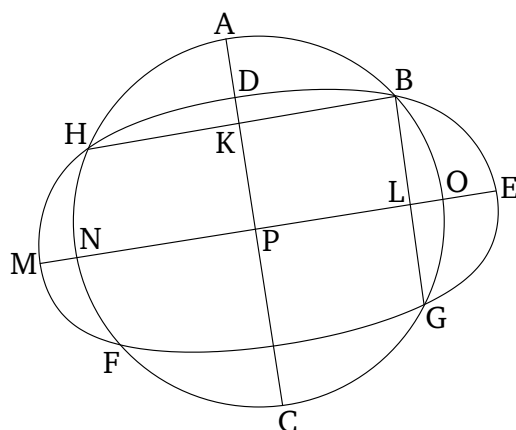
ια'.

Ἐὰν δύο κύκλοι ἐφάπτωνται ἀλλήλων ἐντός, καὶ ληφθῆ αὐτῶν τὰ κέντρα, ἢ ἐπὶ τὰ κέντρα αὐτῶν ἐπιζευγνυμένη εὐθεΐα καὶ ἐκβαλλομένη ἐπὶ τὴν συναφὴν πεσεῖται τῶν κύκλων.

Δύο γὰρ κύκλοι οἱ $AB\Gamma$, $A\Delta E$ ἐφαπτέσθωσαν ἀλλήλων ἐντός κατὰ τὸ A σημεῖον, καὶ εἰλήφθω τοῦ μὲν $AB\Gamma$ κύκλου κέντρον τὸ Z , τοῦ δὲ $A\Delta E$ τὸ H . λέγω, ὅτι ἢ ἀπὸ τοῦ H ἐπὶ τὸ Z ἐπιζευγνυμένη εὐθεΐα ἐκβαλλομένη ἐπὶ τὸ A πεσεῖται.

Μὴ γάρ, ἀλλ' εἰ δυνατόν, πιπτέτω ὡς ἡ $ZH\Theta$, καὶ ἐπεζεύχθωσαν αἱ AZ , AH .

Ἐπει οὖν αἱ AH , HZ τῆς ZA , τουτέστι τῆς $Z\Theta$, μείζονές εἰσιν, κοινὴ ἀφηρήσθω ἡ ZH . λοιπὴ ἄρα ἡ AH λοιπῆς τῆς $H\Theta$ μείζων ἐστίν. ἴση δὲ ἡ AH τῇ $H\Delta$. καὶ ἡ $H\Delta$ ἄρα τῆς $H\Theta$ μείζων ἐστίν ἢ ἐλάττων τῆς μείζονος. ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα ἢ ἀπὸ τοῦ Z ἐπὶ τὸ H ἐπιζευγνυμένη εὐθεΐα ἐκτὸς πεσεῖται. κατὰ τὸ A ἄρα ἐπὶ τῆς συναφῆς πεσεῖται.



Therefore, since in circle ABC some straight-line AC cuts some (other) straight-line BH in half, and at right-angles, the center of circle ABC is thus on AC [Prop. 3.1 corr.]. Again, since in the same circle ABC some straight-line NO cuts some (other straight-line) BG in half, and at right-angles, the center of circle ABC is thus on NO [Prop. 3.1 corr.]. And it was also shown (to be) on AC . And the straight-lines AC and NO meet at no other (point) than P . Thus, point P is the center of circle ABC . So, similarly, we can show that P is also the center of circle DEF . Thus, two circles cutting one another, ABC and DEF , have the same center P . The very thing is impossible [Prop. 3.5].

Thus, a circle does not cut a(nother) circle at more than two points. (Which is) the very thing it was required to show.

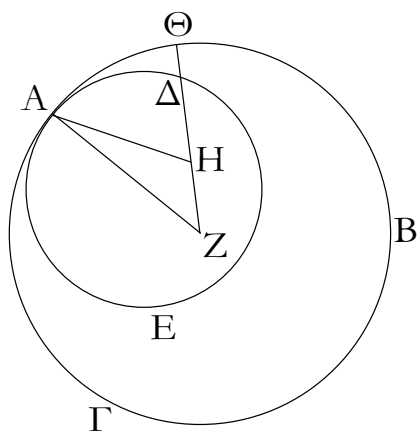
Proposition 11

If two circles touch one another internally, and their centers are found, then the straight-line joining their centers, being produced, will fall upon the point of union of the circles.

For let two circles, ABC and ADE , touch one another internally at point A , and let the center F of circle ABC have been found [Prop. 3.1], and (the center) G of (circle) ADE [Prop. 3.1]. I say that the straight-line joining G to F , being produced, will fall on A .

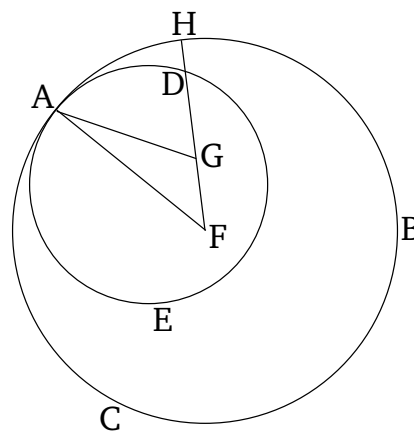
For (if) not then, if possible, let it fall like FGH (in the figure), and let AF and AG have been joined.

Therefore, since AG and GF is greater than FA , that is to say FH [Prop. 1.20], let FG have been taken from both. Thus, the remainder AG is greater than the remainder GH . And AG (is) equal to GD . Thus, GD is also greater than GH , the lesser than the greater. The very thing is impossible. Thus, the straight-line joining F to G will not fall outside (one circle but inside the other). Thus, it will fall upon the point of union (of the circles)



Ἐάν ἄρα δύο κύκλοι ἐφάπτωνται ἀλλήλων ἐντός, [καὶ ληφθῆ αὐτῶν τὰ κέντρα], ἢ ἐπὶ τὰ κέντρα αὐτῶν ἐπιζευγνυμένη εὐθεῖα [καὶ ἐκβαλλομένη] ἐπὶ τὴν συναφὴν πεσεῖται τῶν κύκλων· ὅπερ ἔδει δεῖξαι.

at point A.



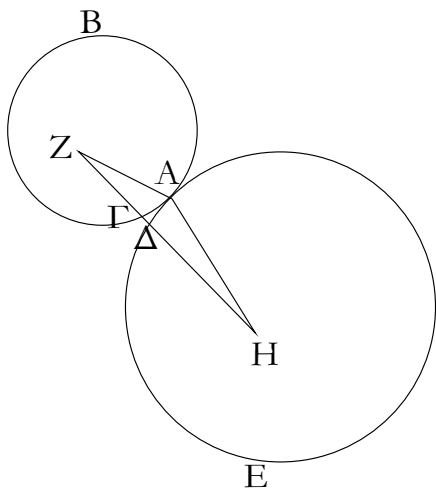
Thus, if two circles touch one another internally, [and their centers are found], then the straight-line joining their centers, [being produced], will fall upon the point of union of the circles. (Which is) the very thing it was required to show.

ιβ'.

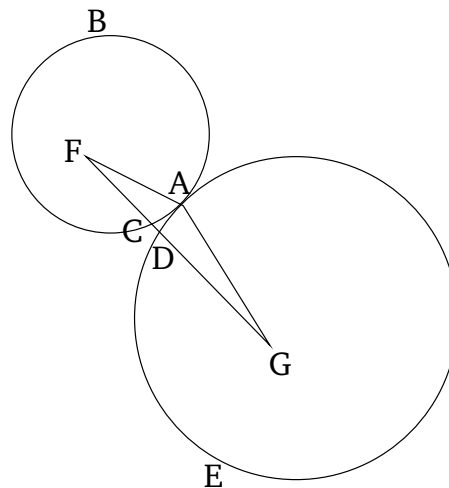
Ἐάν δύο κύκλοι ἐφάπτωνται ἀλλήλων ἐκτός, ἢ ἐπὶ τὰ κέντρα αὐτῶν ἐπιζευγνυμένη διὰ τῆς ἐπαφῆς ἐλεύσεται.

Proposition 12

If two circles touch one another externally then the (straight-line) joining their centers will go through the point of union.



Δύο γὰρ κύκλοι οἱ $AB\Gamma$, $A\Delta E$ ἐφαπτέσθωσαν ἀλλήλων ἐκτός κατὰ τὸ A σημεῖον, καὶ εἰλήφθω τοῦ μὲν $AB\Gamma$ κέντρον τὸ Z , τοῦ δὲ $A\Delta E$ τὸ H . λέγω, ὅτι ἡ ἀπὸ τοῦ Z ἐπὶ τὸ H ἐπιζευγνυμένη εὐθεῖα διὰ τῆς κατὰ τὸ A ἐπαφῆς ἐλεύσεται.



For let two circles, ABC and ADE , touch one another externally at point A , and let the center F of ABC have been found [Prop. 3.1], and (the center) G of ADE [Prop. 3.1]. I say that the straight-line joining F to G will go through the point of union at A .

Μὴ γάρ, ἀλλ' εἰ δυνατόν, ἐρχέσθω ὡς ἡ $Z\Gamma\Delta H$, καὶ ἐπεζεύχθωσαν αἱ AZ , AH .

For (if) not then, if possible, let it go like $FCDG$ (in the figure), and let AF and AG have been joined.

Ἐπεὶ οὖν τὸ Z σημεῖον κέντρον ἐστὶ τοῦ $AB\Gamma$ κύκλου, ἴση ἐστὶν ἡ ZA τῇ $Z\Gamma$. πάλιν, ἐπεὶ τὸ H σημεῖον κέντρον ἐστὶ τοῦ $A\Delta E$ κύκλου, ἴση ἐστὶν ἡ HA τῇ $H\Delta$. ἐδείχθη

Therefore, since point F is the center of circle ABC , FA is equal to FC . Again, since point G is the center of circle ADE , GA is equal to GD . And FA was also shown

δὲ καὶ ἡ ΖΑ τῆ ΖΓ ἴση· αἱ ἄρα ΖΑ, ΑΗ ταῖς ΖΓ, ΗΔ ἴσαι εἰσίν· ὥστε ὅλη ἡ ΖΗ τῶν ΖΑ, ΑΗ μείζων ἐστίν· ἀλλὰ καὶ ἐλάττων· ὅπερ ἐστὶν ἀδύνατον· οὐκ ἄρα ἡ ἀπὸ τοῦ Ζ ἐπὶ τὸ Η ἐπιζευγνυμένη εὐθεῖα διὰ τῆς κατὰ τὸ Α ἐπαφῆς οὐκ ἐλεύσεται· δι' αὐτῆς ἄρα.

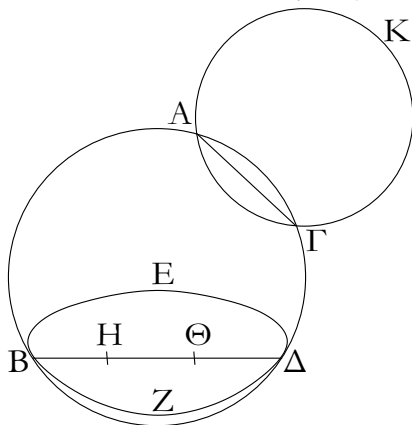
Ἐάν ἄρα δύο κύκλοι ἐφάπτωνται ἀλλήλων ἐκτός, ἡ ἐπὶ τὰ κέντρα αὐτῶν ἐπιζευγνυμένη [εὐθεῖα] διὰ τῆς ἐπαφῆς ἐλεύσεται· ὅπερ ἔδει δεῖξαι.

(to be) equal to FC . Thus, the (straight-lines) FA and AG are equal to the (straight-lines) FC and GD . So the whole of FG is greater than FA and AG . But, (it is) also less [Prop. 1.20]. The very thing is impossible. Thus, the straight-line joining F to G cannot not go through the point of union at A . Thus, (it will go) through it.

Thus, if two circles touch one another externally then the [straight-line] joining their centers will go through the point of union. (Which is) the very thing it was required to show.

ιγ'.

Κύκλος κύκλου οὐκ ἐφάπτεται κατὰ πλείονα σημεῖα ἢ καθ' ἓν, ἐάν τε ἐντός ἐάν τε ἐκτός ἐφάπτηται.



Εἰ γὰρ δυνατόν, κύκλος ὁ ΑΒΓΔ κύκλου τοῦ ΕΒΖΔ ἐφαπτέσθω πρότερον ἐντός κατὰ πλείονα σημεῖα ἢ ἐν τὰ Δ, Β.

Καὶ εἰλήφθω τοῦ μὲν ΑΒΓΔ κύκλου κέντρον τὸ Η, τοῦ δὲ ΕΒΖΔ τὸ Θ.

Ἡ ἄρα ἀπὸ τοῦ Η ἐπὶ τὸ Θ ἐπιζευγνυμένη ἐπὶ τὰ Β, Δ πεσεῖται. πιπτέτω ὡς ἡ ΒΗΘΔ. καὶ ἐπεὶ τὸ Η σημεῖον κέντρον ἐστὶ τοῦ ΑΒΓΔ κύκλου, ἴση ἐστὶν ἡ ΒΗ τῆς ΗΔ· μείζων ἄρα ἡ ΒΗ τῆς ΘΔ· πολλῶ ἄρα μείζων ἡ ΒΘ τῆς ΘΔ. πάλιν, ἐπεὶ τὸ Θ σημεῖον κέντρον ἐστὶ τοῦ ΕΒΖΔ κύκλου, ἴση ἐστὶν ἡ ΒΘ τῆς ΘΔ· ἐδείχθη δὲ αὐτῆς καὶ πολλῶ μείζων· ὅπερ ἀδύνατον· οὐκ ἄρα κύκλος κύκλου ἐφάπτεται ἐντός κατὰ πλείονα σημεῖα ἢ ἓν.

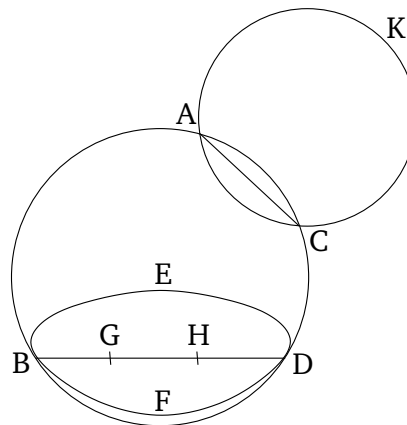
Λέγω δὴ, ὅτι οὐδὲ ἐκτός.

Εἰ γὰρ δυνατόν, κύκλος ὁ ΑΓΚ κύκλου τοῦ ΑΒΓΔ ἐφαπτέσθω ἐκτός κατὰ πλείονα σημεῖα ἢ ἐν τὰ Α, Γ, καὶ ἐπεζεύχθω ἡ ΑΓ.

Ἐπεὶ οὖν κύκλων τῶν ΑΒΓΔ, ΑΓΚ εἰληπται ἐπὶ τῆς περιφερείας ἑκατέρου δύο τυχόντα σημεῖα τὰ Α, Γ, ἡ ἐπὶ τὰ σημεῖα ἐπιζευγνυμένη εὐθεῖα ἐντός ἑκατέρου πεσεῖται· ἀλλὰ τοῦ μὲν ΑΒΓΔ ἐντός ἔπεσεν, τοῦ δὲ ΑΓΚ ἐκτός· ὅπερ ἄτοπον· οὐκ ἄρα κύκλος κύκλου ἐφάπτεται ἐκτός κατὰ πλείονα σημεῖα ἢ ἓν. ἐδείχθη δέ, ὅτι οὐδὲ ἐντός.

Proposition 13

A circle does not touch a(nother) circle at more than one point, whether they touch internally or externally.



For, if possible, let circle $ABDC$ † touch circle $EBFD$ —first of all, internally—at more than one point, D and B .

And let the center G of circle $ABDC$ have been found [Prop. 3.1], and (the center) H of $EBFD$ [Prop. 3.1].

Thus, the (straight-line) joining G and H will fall on B and D [Prop. 3.11]. Let it fall like $BGHD$ (in the figure). And since point G is the center of circle $ABDC$, BG is equal to GD . Thus, BG (is) greater than HD . Thus, BH (is) much greater than HD . Again, since point H is the center of circle $EBFD$, BH is equal to HD . But it was also shown (to be) much greater than it. The very thing (is) impossible. Thus, a circle does not touch a(nother) circle internally at more than one point.

So, I say that neither (does it touch) externally (at more than one point).

For, if possible, let circle ACK touch circle $ABDC$ externally at more than one point, A and C . And let AC have been joined.

Therefore, since two points, A and C , have been taken at random on the circumference of each of the circles $ABDC$ and ACK , the straight-line joining the points will fall inside each (circle) [Prop. 3.2]. But, it fell inside $ABDC$, and outside ACK [Def. 3.3]. The very thing

Κύκλος ἄρα κύκλου οὐκ ἐφάπτεται κατὰ πλείονα σημεῖα ἢ [καθ'] ἓν, ἐάν τε ἐντὸς ἐάν τε ἐκτὸς ἐφάπτηται· ὅπερ ἔδει δεῖξαι.

(is) absurd. Thus, a circle does not touch a(nother) circle externally at more than one point. And it was shown that neither (does it) internally.

Thus, a circle does not touch a(nother) circle at more than one point, whether they touch internally or externally. (Which is) the very thing it was required to show.

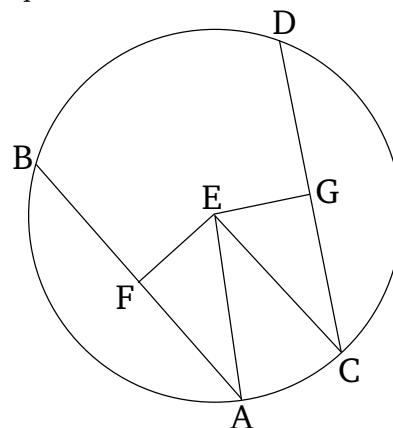
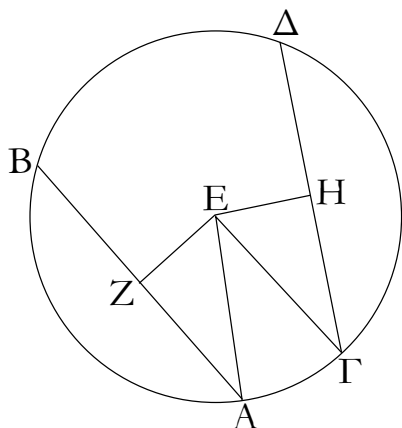
† The Greek text has “*ABCD*”, which is obviously a mistake.

ιδ'.

Ἐν κύκλῳ αἱ ἴσαι εὐθεῖαι ἴσον ἀπέχουσιν ἀπὸ τοῦ κέντρου, καὶ αἱ ἴσον ἀπέχουσαι ἀπὸ τοῦ κέντρου ἴσαι ἀλλήλαις εἰσίν.

Proposition 14

In a circle, equal straight-lines are equally far from the center, and (straight-lines) which are equally far from the center are equal to one another.



Ἐστω κύκλος ὁ *ABGD*, καὶ ἐν αὐτῷ ἴσαι εὐθεῖαι ἔστωσαν αἱ *AB*, *GD*. λέγω, ὅτι αἱ *AB*, *GD* ἴσον ἀπέχουσιν ἀπὸ τοῦ κέντρου.

Let *ABDC*[†] be a circle, and let *AB* and *CD* be equal straight-lines within it. I say that *AB* and *CD* are equally far from the center.

Εἰλήφθω γὰρ τὸ κέντρον τοῦ *ABGD* κύκλου καὶ ἔστω τὸ *E*, καὶ ἀπὸ τοῦ *E* ἐπὶ τὰς *AB*, *GD* κάθεται ἡχθωσαν αἱ *EZ*, *EH*, καὶ ἐπεζεύχθωσαν αἱ *AE*, *EG*.

For let the center of circle *ABDC* have been found [Prop. 3.1], and let it be (at) *E*. And let *EF* and *EG* have been drawn from (point) *E*, perpendicular to *AB* and *CD* (respectively) [Prop. 1.12]. And let *AE* and *EC* have been joined.

Ἐπεὶ οὖν εὐθεῖα τις δια τοῦ κέντρου ἢ *EZ* εὐθεῖάν τινα μὴ δια τοῦ κέντρου τὴν *AB* πρὸς ὀρθὰς τέμνει, καὶ δίχα αὐτὴν τέμνει. ἴση ἄρα ἡ *AZ* τῇ *ZB*. διπλῆ ἄρα ἡ *AB* τῆς *AZ*. διὰ τὰ αὐτὰ δὴ καὶ ἡ *GD* τῆς *GH* ἐστὶ διπλῆ· καὶ ἐστὶν ἴση ἡ *AB* τῇ *GD*. ἴση ἄρα καὶ ἡ *AZ* τῇ *GH*. καὶ ἐπεὶ ἴση ἐστὶν ἡ *AE* τῇ *EG*, ἴσον καὶ τὸ ἀπὸ τῆς *AE* τῷ ἀπὸ τῆς *EG*. ἀλλὰ τῷ μὲν ἀπὸ τῆς *AE* ἴσα τὰ ἀπὸ τῶν *AZ*, *EZ*. ὀρθὴ γὰρ ἡ πρὸς τῷ *Z* γωνία· τῷ δὲ ἀπὸ τῆς *EG* ἴσα τὰ ἀπὸ τῶν *EH*, *HG*. ὀρθὴ γὰρ ἡ πρὸς τῷ *H* γωνία· τὰ ἄρα ἀπὸ τῶν *AZ*, *ZE* ἴσα ἐστὶ τοῖς ἀπὸ τῶν *GH*, *HE*, ὡν τὸ ἀπὸ τῆς *AZ* ἴσον ἐστὶ τῷ ἀπὸ τῆς *GH*. ἴση γὰρ ἐστὶν ἡ *AZ* τῇ *GH*. λοιπὸν ἄρα τὸ ἀπὸ τῆς *ZE* τῷ ἀπὸ τῆς *EH* ἴσον ἐστίν· ἴση ἄρα ἡ *EZ* τῇ *EH*. ἐν δὲ κύκλῳ ἴσον ἀπέχειν ἀπὸ τοῦ κέντρου εὐθεῖαι λέγονται, ὅταν αἱ ἀπὸ τοῦ κέντρου ἐπ' αὐτὰς κάθεται ἀγόμεναι ἴσαι ὦσιν· αἱ ἄρα *AB*, *GD* ἴσον ἀπέχουσιν ἀπὸ τοῦ κέντρου.

Therefore, since some straight-line, *EF*, through the center (of the circle), cuts some (other) straight-line, *AB*, not through the center, at right-angles, it also cuts it in half [Prop. 3.3]. Thus, *AF* (is) equal to *FB*. Thus, *AB* (is) double *AF*. So, for the same (reasons), *CD* is also double *CG*. And *AB* is equal to *CD*. Thus, *AF* (is) also equal to *CG*. And since *AE* is equal to *EC*, the (square) on *AE* (is) also equal to the (square) on *EC*. But, the (sum of the squares) on *AF* and *EF* (is) equal to the (square) on *AE*. For the angle at *F* (is) a right-angle [Prop. 1.47]. And the (sum of the squares) on *EG* and *GC* (is) equal to the (square) on *EC*. For the angle at *G* (is) a right-angle [Prop. 1.47]. Thus, the (sum of the squares) on *AF* and *FE* is equal to the (sum of the squares) on *CG* and *GE*, of which the (square) on *AF* is equal to the (square) on *CG*. For *AF* is equal to *CG*.

Ἀλλὰ δὴ αἱ *AB*, *GD* εὐθεῖαι ἴσον ἀπέχεται ἀπὸ τοῦ κέντρου, τουτέστιν ἴση ἔστω ἡ *EZ* τῇ *EH*. λέγω, ὅτι ἴση ἐστὶ καὶ ἡ *AB* τῇ *GD*.

Τῶν γὰρ αὐτῶν κατασκευασθέντων ὁμοίως δείζομεν, ὅτι διπλῆ ἐστὶν ἡ μὲν AB τῆς AZ , ἡ δὲ $\Gamma\Delta$ τῆς ΓH . καὶ ἐπεὶ ἴση ἐστὶν ἡ AE τῆ ΓE , ἴσον ἐστὶ τὸ ἀπὸ τῆς AE τῶ ἀπὸ τῆς ΓE . ἀλλὰ τῶ μὲν ἀπὸ τῆς AE ἴσα ἐστὶ τὰ ἀπὸ τῶν EZ , ZA , τῶ δὲ ἀπὸ τῆς ΓE ἴσα τὰ ἀπὸ τῶν EH , $\text{H}\Gamma$. τὰ ἄρα ἀπὸ τῶν EZ , ZA ἴσα ἐστὶ τοῖς ἀπὸ τῶν EH , $\text{H}\Gamma$. ὦν τὸ ἀπὸ τῆς EZ τῶ ἀπὸ τῆς EH ἐστὶν ἴσον. ἴση γὰρ ἡ EZ τῆ EH . λοιπὸν ἄρα τὸ ἀπὸ τῆς AZ ἴσον ἐστὶ τῶ ἀπὸ τῆς ΓH . ἴση ἄρα ἡ AZ τῆ ΓH . καὶ ἐστὶ τῆς μὲν AZ διπλῆ ἡ AB , τῆς δὲ ΓH διπλῆ ἡ $\Gamma\Delta$. ἴση ἄρα ἡ AB τῆ $\Gamma\Delta$.

Ἐν κύκλῳ ἄρα αἱ ἴσαι εὐθεῖαι ἴσον ἀπέχουσιν ἀπὸ τοῦ κέντρου, καὶ αἱ ἴσον ἀπέχουσαι ἀπὸ τοῦ κέντρου ἴσαι ἀλλήλαις εἰσίν. ὅπερ ἔδει δείξαι.

Thus, the remaining (square) on FE is equal to the (remaining square) on EG . Thus, EF (is) equal to EG . And straight-lines in a circle are said to be equally far from the center when perpendicular (straight-lines) which are drawn to them from the center are equal [Def. 3.4]. Thus, AB and CD are equally far from the center.

So, let the straight-lines AB and CD be equally far from the center. That is to say, let EF be equal to EG . I say that AB is also equal to CD .

For, with the same construction, we can, similarly, show that AB is double AF , and CD (double) CG . And since AE is equal to CE , the (square) on AE is equal to the (square) on CE . But, the (sum of the squares) on EF and FA is equal to the (square) on AE [Prop. 1.47]. And the (sum of the squares) on EG and GC (is) equal to the (square) on CE [Prop. 1.47]. Thus, the (sum of the squares) on EF and FA is equal to the (sum of the squares) on EG and GC , of which the (square) on EF is equal to the (square) on EG . For EF (is) equal to EG . Thus, the remaining (square) on AF is equal to the (remaining square) on CG . Thus, AF (is) equal to CG . And AB is double AF , and CD double CG . Thus, AB (is) equal to CD .

Thus, in a circle, equal straight-lines are equally far from the center, and (straight-lines) which are equally far from the center are equal to one another. (Which is) the very thing it was required to show.

† The Greek text has “ $ABCD$ ”, which is obviously a mistake.

ιε΄.

Proposition 15

Ἐν κύκλῳ μεγίστη μὲν ἡ διάμετρος, τῶν δὲ ἄλλων αἰεὶ ἡ ἕγγιον τοῦ κέντρου τῆς ἀπώτερον μείζων ἐστίν.

Ἐστω κύκλος ὁ $AB\Gamma\Delta$, διάμετρος δὲ αὐτοῦ ἔστω ἡ $A\Delta$, κέντρον δὲ τὸ E , καὶ ἕγγιον μὲν τῆς $A\Delta$ διαμέτρου ἔστω ἡ $B\Gamma$, ἀπώτερον δὲ ἡ $Z\text{H}$. λέγω, ὅτι μεγίστη μὲν ἐστὶν ἡ $A\Delta$, μείζων δὲ ἡ $B\Gamma$ τῆς $Z\text{H}$.

Ἦχθωσαν γὰρ ἀπὸ τοῦ E κέντρου ἐπὶ τὰς $B\Gamma$, $Z\text{H}$ κάθετοι αἱ $E\Theta$, $E\text{K}$. καὶ ἐπεὶ ἕγγιον μὲν τοῦ κέντρου ἐστὶν ἡ $B\Gamma$, ἀπώτερον δὲ ἡ $Z\text{H}$, μείζων ἄρα ἡ $E\text{K}$ τῆς $E\Theta$. κείσθω τῆ $E\Theta$ ἴση ἡ $E\Lambda$, καὶ διὰ τοῦ Λ τῆ $E\text{K}$ πρὸς ὀρθὰς ἀχθεῖσα ἡ ΛM διήχθω ἐπὶ τὸ N , καὶ ἐπεξεύχθωσαν αἱ ME , EN , ZE , EH .

Καὶ ἐπεὶ ἴση ἐστὶν ἡ $E\Theta$ τῆ $E\Lambda$, ἴση ἐστὶ καὶ ἡ $B\Gamma$ τῆ $M\text{N}$. πάλιν, ἐπεὶ ἴση ἐστὶν ἡ μὲν AE τῆ EM , ἡ δὲ $E\Delta$ τῆ EN , ἡ ἄρα $A\Delta$ ταῖς ME , EN ἴση ἐστίν. ἀλλ’ αἱ μὲν ME , EN τῆς $M\text{N}$ μείζονές εἰσιν [καὶ ἡ $A\Delta$ τῆς $M\text{N}$ μείζων ἐστίν], ἴση δὲ ἡ $M\text{N}$ τῆ $B\Gamma$. ἡ $A\Delta$ ἄρα τῆς $B\Gamma$ μείζων ἐστίν. καὶ ἐπεὶ δύο αἱ ME , EN δύο ταῖς ZE , EH ἴσαι εἰσίν, καὶ γωνία ἡ ὑπὸ MEN γωνίας τῆς ὑπὸ ZEH μείζων [ἐστίν], βᾶσις ἄρα

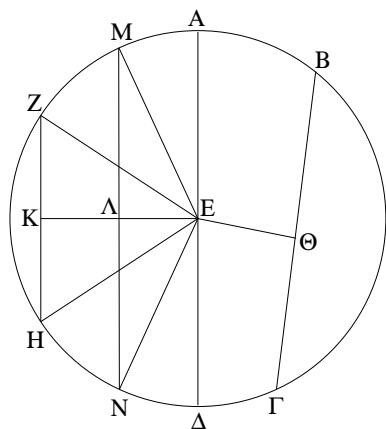
In a circle, a diameter (is) the greatest (straight-line), and for the others, a (straight-line) nearer to the center is always greater than one further away.

Let $ABCD$ be a circle, and let AD be its diameter, and E (its) center. And let BC be nearer to the diameter AD ,[†] and FG further away. I say that AD is the greatest (straight-line), and BC (is) greater than FG .

For let EH and $E\text{K}$ have been drawn from the center E , at right-angles to BC and FG (respectively) [Prop. 1.12]. And since BC is nearer to the center, and FG further away, $E\text{K}$ (is) thus greater than EH [Def. 3.5]. Let EL be made equal to EH [Prop. 1.3]. And LM being drawn through L , at right-angles to $E\text{K}$ [Prop. 1.11], let it have been drawn through to N . And let ME , EN , FE , and EG have been joined.

And since EH is equal to EL , BC is also equal to $M\text{N}$ [Prop. 3.14]. Again, since AE is equal to EM , and ED to EN , AD is thus equal to ME and EN . But, ME and EN is greater than $M\text{N}$ [Prop. 1.20] [also AD is

ἡ MN βάσεως τῆς ZH μείζων ἐστίν. ἀλλὰ ἡ MN τῆ $BΓ$ ἐδείχθη ἴση [καὶ ἡ $BΓ$ τῆς ZH μείζων ἐστίν]. μεγίστη μὲν ἄρα ἡ $ΑΔ$ διάμετρος, μείζων δὲ ἡ $BΓ$ τῆς ZH .



Ἐν κύκλῳ ἄρα μεγίστη μὲν ἐστίν ἡ διάμετρος, τῶν δὲ ἄλλων αἰεὶ ἡ ἔγγιον τοῦ κέντρου τῆς ἀπώτερον μείζων ἐστίν· ὅπερ ἔδει δεῖξαι.

† Euclid should have said “to the center”, rather than “to the diameter AD ”, since BC , AD and FG are not necessarily parallel.

‡ This is not proved, except by reference to the figure.

15'.

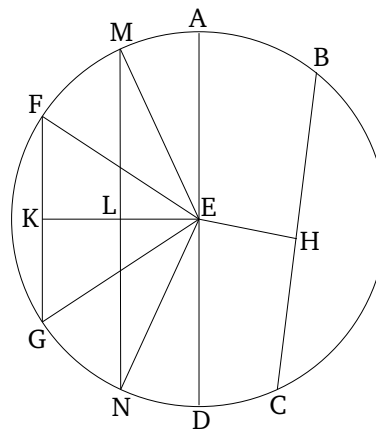
Ἡ τῆ AD διαμέτρῳ τοῦ κύκλου πρὸς ὀρθὰς ἀπ' ἄκρας ἀγομένη ἐκτὸς πεσεῖται τοῦ κύκλου, καὶ εἰς τὸν μεταξὺ τόπον τῆς τε εὐθείας καὶ τῆς περιφερείας ἕτερα εὐθεῖα οὐ παρεμπεσεῖται, καὶ ἡ μὲν τοῦ ἡμικυκλίου γωνία ἀπάσης γωνίας ὀξείας εὐθυγράμμου μείζων ἐστίν, ἡ δὲ λοιπὴ ἐλάττω.

Ἐστω κύκλος ὁ $ΑΒΓ$ περὶ κέντρον τὸ $Δ$ καὶ διάμετρον τὴν $ΑΒ$ · λέγω, ὅτι ἡ ἀπὸ τοῦ $Α$ τῆ $ΑΒ$ πρὸς ὀρθὰς ἀπ' ἄκρας ἀγομένη ἐκτὸς πεσεῖται τοῦ κύκλου.

Μὴ γάρ, ἀλλ' εἰ δυνατόν, πιπτέτω ἐντὸς ὡς ἡ $ΓΑ$, καὶ ἐπεζεύχθω ἡ $ΔΓ$.

Ἐπεὶ ἴση ἐστίν ἡ $ΔΑ$ τῆ $ΔΓ$, ἴση ἐστὶ καὶ γωνία ἡ ὑπὸ $ΔΑΓ$ γωνία τῆ $ὑπὸ ΑΓΔ$. ὀρθὴ δὲ ἡ ὑπὸ $ΔΑΓ$ · ὀρθὴ ἄρα καὶ ἡ ὑπὸ $ΑΓΔ$ · τριγώνου δὴ τοῦ $ΑΓΔ$ αἱ δύο γωνίαι αἱ ὑπὸ $ΔΑΓ$, $ΑΓΔ$ δύο ὀρθαῖς ἴσαι εἰσίν· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα ἡ ἀπὸ τοῦ $Α$ σημείου τῆ $ΒΑ$ πρὸς ὀρθὰς ἀγομένη ἐντὸς πεσεῖται τοῦ κύκλου. ὁμοίως δὲ δεῖξομεν, ὅτι οὐδ' ἐπὶ τῆς περιφερείας· ἐκτὸς ἄρα.

greater than MN], and MN (is) equal to BC . Thus, AD is greater than BC . And since the two (straight-lines) ME , EN are equal to the two (straight-lines) FE , EG (respectively), and angle MEN [is] greater than angle FEG ,[‡] the base MN is thus greater than the base FG [Prop. 1.24]. But, MN was shown (to be) equal to BC [(so) BC is also greater than FG]. Thus, the diameter AD (is) the greatest (straight-line), and BC (is) greater than FG .



Thus, in a circle, a diameter (is) the greatest (straight-line), and for the others, a (straight-line) nearer to the center is always greater than one further away. (Which is) the very thing it was required to show.

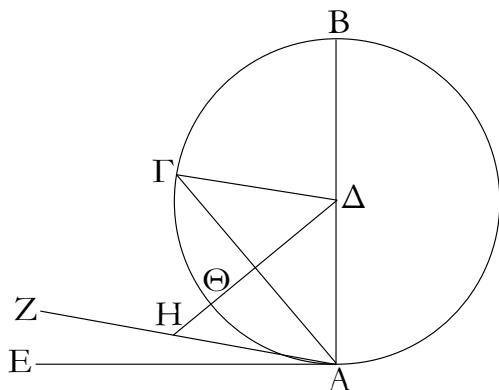
Proposition 16

A (straight-line) drawn at right-angles to the diameter of a circle, from its end, will fall outside the circle. And another straight-line cannot be inserted into the space between the (aforementioned) straight-line and the circumference. And the angle of the semi-circle is greater than any acute rectilinear angle whatsoever, and the remaining (angle is) less (than any acute rectilinear angle).

Let ABC be a circle around the center D and the diameter AB . I say that the (straight-line) drawn from A , at right-angles to AB [Prop 1.11], from its end, will fall outside the circle.

For (if) not then, if possible, let it fall inside, like CA (in the figure), and let DC have been joined.

Since DA is equal to DC , angle DAC is also equal to angle ACD [Prop. 1.5]. And DAC (is) a right-angle. Thus, ACD (is) also a right-angle. So, in triangle ACD , the two angles DAC and ACD are equal to two right-angles. The very thing is impossible [Prop. 1.17]. Thus, the (straight-line) drawn from point A , at right-angles



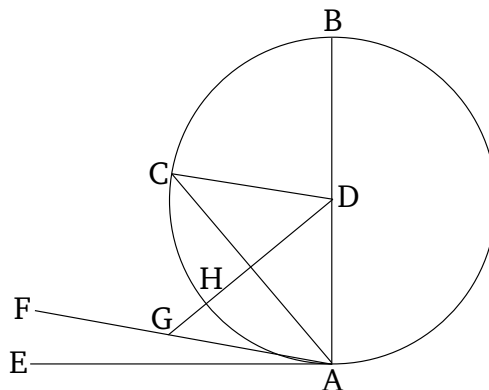
Πιπέτω ως ή ΑΕ· λέγω δή, ότι εις τόν μεταξύ τόπον τῆς τε ΑΕ εὐθείας καί τῆς ΓΘΑ περιφερείας ἑτέρα εὐθεΐα οὐ παρεμπεσεΐται.

Εἰ γάρ δυνατόν, παρεμπιπέτω ως ή ΖΑ, καί ἤχθω ἀπό τοῦ Δ σημείου ἐπὶ τῆν ΖΑ κάθετος ή ΔΗ. καί ἐπει ὀρθή ἐστίν ή ὑπὸ ΑΗΔ, ἐλάττων δὲ ὀρθῆς ή ὑπὸ ΔΑΗ, μείζων ἄρα ή ΑΔ τῆς ΔΗ. ἴση δὲ ή ΔΑ τῆ ΔΘ· μείζων ἄρα ή ΔΘ τῆς ΔΗ, ή ἐλάττων τῆς μείζονος· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα εις τόν μεταξύ τόπον τῆς τε εὐθείας καί τῆς περιφερείας ἑτέρα εὐθεΐα παρεμπεσεΐται.

Λέγω, ὅτι καί ή μὲν τοῦ ἡμικυκλίου γωνία ή περιεχομένη ὑπὸ τε τῆς ΒΑ εὐθείας καί τῆς ΓΘΑ περιφερείας ἀπάσης γωνίας ὀξείας εὐθυγράμμου μείζων ἐστίν, ή δὲ λοιπή ή περιεχομένη ὑπὸ τε τῆς ΓΘΑ περιφερείας καί τῆς ΑΕ εὐθείας ἀπάσης γωνίας ὀξείας εὐθυγράμμου ἐλάττων ἐστίν.

Εἰ γάρ ἐστὶ τις γωνία εὐθύγραμμος μείζων μὲν τῆς περιεχομένης ὑπὸ τε τῆς ΒΑ εὐθείας καί τῆς ΓΘΑ περιφερείας, ἐλάττων δὲ τῆς περιεχομένης ὑπὸ τε τῆς ΓΘΑ περιφερείας καί τῆς ΑΕ εὐθείας, εἰς τόν μεταξύ τόπον τῆς τε ΓΘΑ περιφερείας καί τῆς ΑΕ εὐθείας εὐθεΐα παρεμπεσεΐται, ἥτις ποιήσει μείζονα μὲν τῆς περιεχομένης ὑπὸ τε τῆς ΒΑ εὐθείας καί τῆς ΓΘΑ περιφερείας ὑπὸ εὐθειῶν περιεχομένην, ἐλάττονα δὲ τῆς περιεχομένης ὑπὸ τε τῆς ΓΘΑ περιφερείας καί τῆς ΑΕ εὐθείας. οὐ παρεμπίπτει δέ· οὐκ ἄρα τῆς περιεχομένης γωνίας ὑπὸ τε τῆς ΒΑ εὐθείας καί τῆς ΓΘΑ περιφερείας ἔσται μείζων ὀξεία ὑπὸ εὐθειῶν περιεχομένη, οὐδὲ μὴν ἐλάττων τῆς περιεχομένης ὑπὸ τε τῆς ΓΘΑ περιφερείας καί τῆς ΑΕ εὐθείας.

to BA , will not fall inside the circle. So, similarly, we can show that neither (will it fall) on the circumference. Thus, (it will fall) outside (the circle).



Let it fall like AE (in the figure). So, I say that another straight-line cannot be inserted into the space between the straight-line AE and the circumference CHA .

For, if possible, let it be inserted like FA (in the figure), and let DG have been drawn from point D , perpendicular to FA [Prop. 1.12]. And since AGD is a right-angle, and DAG (is) less than a right-angle, AD (is) thus greater than DG [Prop. 1.19]. And DA (is) equal to DH . Thus, DH (is) greater than DG , the lesser than the greater. The very thing is impossible. Thus, another straight-line cannot be inserted into the space between the straight-line (AE) and the circumference.

And I also say that the semi-circular angle contained by the straight-line BA and the circumference CHA is greater than any acute rectilinear angle whatsoever, and the remaining (angle) contained by the circumference CHA and the straight-line AE is less than any acute rectilinear angle whatsoever.

For if any rectilinear angle is greater than the (angle) contained by the straight-line BA and the circumference CHA , or less than the (angle) contained by the circumference CHA and the straight-line AE , then a straight-line can be inserted into the space between the circumference CHA and the straight-line AE —anything which will make (an angle) contained by straight-lines greater than the angle contained by the straight-line BA and the circumference CHA , or less than the (angle) contained by the circumference CHA and the straight-line AE . But (such a straight-line) cannot be inserted. Thus, an acute (angle) contained by straight-lines cannot be greater than the angle contained by the straight-line BA and the circumference CHA , neither (can it be) less than the (angle) contained by the circumference CHA and the straight-line AE .

Πόρισμα.

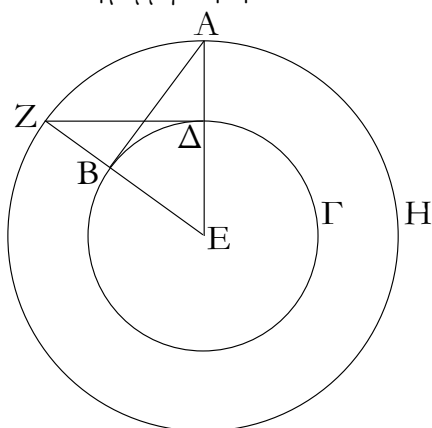
Ἐκ δὴ τούτου φανερόν, ὅτι ἡ τῆς διαμέτρου τοῦ κύκλου πρὸς ὀρθὰς ἀπ' ἄκρας ἀγομένη ἐφάπτεται τοῦ κύκλου [καὶ ὅτι εὐθεῖα κύκλου καθ' ἓν μόνον ἐφάπτεται σημεῖον, ἐπειδὴ περ καὶ ἡ κατὰ δύο αὐτῶ συμβάλλουσα ἐντὸς αὐτοῦ πίπτουσα ἐδείχθη]. ὅπερ ἔδει δεῖξαι.

Corollary

So, from this, (it is) manifest that a (straight-line) drawn at right-angles to the diameter of a circle, from its extremity, touches the circle [and that the straight-line touches the circle at a single point, inasmuch as it was also shown that a (straight-line) meeting (the circle) at two (points) falls inside it [Prop. 3.2]]. (Which is) the very thing it was required to show.

ιζ'.

Ἀπὸ τοῦ δοθέντος σημείου τοῦ δοθέντος κύκλου ἐφαπτομένην εὐθεῖαν γραμμὴν ἀγαγεῖν.



Ἐστω τὸ μὲν δοθὲν σημεῖον τὸ A, ὁ δὲ δοθεὶς κύκλος ὁ BΓΔ· δεῖ δὴ ἀπὸ τοῦ A σημείου τοῦ BΓΔ κύκλου ἐφαπτομένην εὐθεῖαν γραμμὴν ἀγαγεῖν.

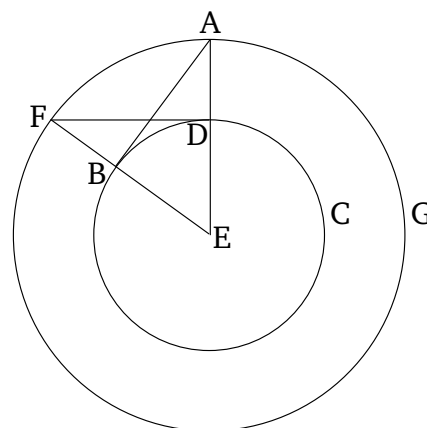
Εἰλήφθω γὰρ τὸ κέντρον τοῦ κύκλου τὸ E, καὶ ἐπεζεύχθω ἡ AE, καὶ κέντρον μὲν τῷ E διαστήματι δὲ τῷ EA κύκλος γεγράφθω ὁ AZH, καὶ ἀπὸ τοῦ Δ τῆς EA πρὸς ὀρθὰς ἤχθω ἡ ΔZ, καὶ ἐπεζεύχθωσαν αἱ EZ, AB· λέγω, ὅτι ἀπὸ τοῦ A σημείου τοῦ BΓΔ κύκλου ἐφαπτομένη ἦκται ἡ AB.

Ἐπεὶ γὰρ τὸ E κέντρον ἐστὶ τῶν BΓΔ, AZH κύκλων, ἴση ἄρα ἐστὶν ἡ μὲν EA τῆς EZ, ἡ δὲ EΔ τῆς EB· δύο δὲ αἱ AE, EB δύο ταῖς ZE, EΔ ἴσαι εἰσὶν· καὶ γωνίαν κοινὴν περιέχουσι τὴν πρὸς τῷ E· βάσις ἄρα ἡ ΔZ βάσει τῆς AB ἴση ἐστίν, καὶ τὸ ΔEZ τρίγωνον τῷ EBA τριγώνῳ ἴσον ἐστίν, καὶ αἱ λοιπαὶ γωνίαι ταῖς λοιπαῖς γωνίαις· ἴση ἄρα ἡ ὑπὸ EΔZ τῆς ὑπὸ EBA. ὀρθὴ δὲ ἡ ὑπὸ EΔZ· ὀρθὴ ἄρα καὶ ἡ ὑπὸ EBA. καὶ ἐστὶν ἡ EB ἐκ τοῦ κέντρου· ἡ δὲ τῆς διαμέτρου τοῦ κύκλου πρὸς ὀρθὰς ἀπ' ἄκρας ἀγομένη ἐφάπτεται τοῦ κύκλου· ἡ AB ἄρα ἐφάπτεται τοῦ BΓΔ κύκλου.

Ἀπὸ τοῦ ἄρα δοθέντος σημείου τοῦ A τοῦ δοθέντος κύκλου τοῦ BΓΔ ἐφαπτομένην εὐθεῖαν γραμμὴν ἦκται ἡ AB· ὅπερ ἔδει ποιῆσαι.

Proposition 17

To draw a straight-line touching a given circle from a given point.



Let A be the given point, and BCD the given circle. So it is required to draw a straight-line touching circle BCD from point A.

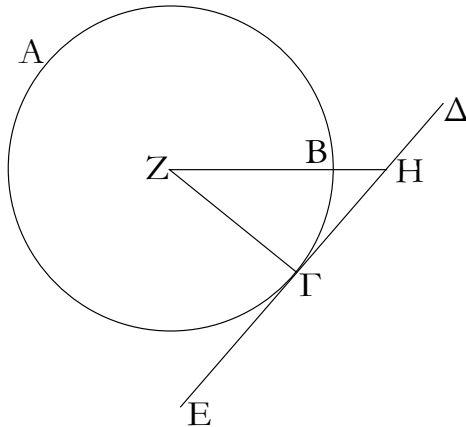
For let the center E of the circle have been found [Prop. 3.1], and let AE have been joined. And let (the circle) AFG have been drawn with center E and radius EA. And let DF have been drawn from (point) D, at right-angles to EA [Prop. 1.11]. And let EF and AB have been joined. I say that the (straight-line) AB has been drawn from point A touching circle BCD.

For since E is the center of circles BCD and AFG, EA is thus equal to EF, and ED to EB. So the two (straight-lines) AE, EB are equal to the two (straight-lines) FE, ED (respectively). And they contain a common angle at E. Thus, the base DF is equal to the base AB, and triangle DEF is equal to triangle EBA, and the remaining angles (are equal) to the (corresponding) remaining angles [Prop. 1.4]. Thus, (angle) EDF (is) equal to EBA. And EDF (is) a right-angle. Thus, EBA (is) also a right-angle. And EB is a radius. And a (straight-line) drawn at right-angles to the diameter of a circle, from its extremity, touches the circle [Prop. 3.16 corr.]. Thus, AB touches circle BCD.

Thus, the straight-line AB has been drawn touching

ιη'.

Ἐάν κύκλου ἐφάπτηται τις εὐθεΐα, ἀπό δὲ τοῦ κέντρου ἐπὶ τὴν ἀφὴν ἐπιζευχθῆ τις εὐθεΐα, ἢ ἐπιζευχθεῖσα κάθετος ἔσται ἐπὶ τὴν ἐφαπτομένην.



Κύκλου γὰρ τοῦ $ABΓ$ ἐφαπτέσθω τις εὐθεΐα ἢ $ΔΕ$ κατὰ τὸ $Γ$ σημεῖον, καὶ εἰλήφθω τὸ κέντρον τοῦ $ABΓ$ κύκλου τὸ Z , καὶ ἀπὸ τοῦ Z ἐπὶ τὸ $Γ$ ἐπεζεύχθω ἢ $ZΓ$. λέγω, ὅτι ἢ $ZΓ$ κάθετός ἐστιν ἐπὶ τὴν $ΔΕ$.

Εἰ γὰρ μή, ἤχθω ἀπὸ τοῦ Z ἐπὶ τὴν $ΔΕ$ κάθετος ἢ ZH .

Ἐπεὶ οὖν ἢ ὑπὸ $ZHΓ$ γωνία ὀρθή ἐστιν, ὁξεία ἄρα ἐστὶν ἢ ὑπὸ $ZΓH$. ὑπὸ δὲ τὴν μείζονα γωνίαν ἢ μείζων πλευρὰ ὑποτείνει· μείζων ἄρα ἢ $ZΓ$ τῆς ZH . ἴση δὲ ἢ $ZΓ$ τῆ ZB . μείζων ἄρα καὶ ἢ ZB τῆς ZH ἢ ἐλάττων τῆς μείζονος· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα ἢ ZH κάθετός ἐστιν ἐπὶ τὴν $ΔΕ$. ὁμοίως δὲ δεῖξομεν, ὅτι οὐδ' ἄλλη τις πλὴν τῆς $ZΓ$. ἢ $ZΓ$ ἄρα κάθετός ἐστιν ἐπὶ τὴν $ΔΕ$.

Ἐάν ἄρα κύκλου ἐφάπτηται τις εὐθεΐα, ἀπὸ δὲ τοῦ κέντρου ἐπὶ τὴν ἀφὴν ἐπιζευχθῆ τις εὐθεΐα, ἢ ἐπιζευχθεῖσα κάθετος ἔσται ἐπὶ τὴν ἐφαπτομένην· ὅπερ ἔδει δεῖξαι.

ιθ'.

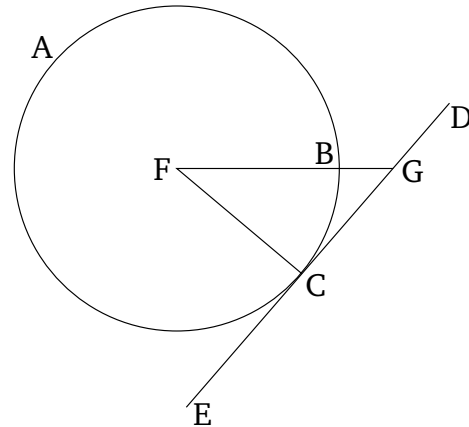
Ἐάν κύκλου ἐφάπτηται τις εὐθεΐα, ἀπὸ δὲ τῆς ἀφῆς τῆ ἐφαπτομένη πρὸς ὀρθὰς [γωνίας] εὐθεΐα γραμμὴ ἀχθῆ, ἐπὶ τῆς ἀχθείσης ἔσται τὸ κέντρον τοῦ κύκλου.

Κύκλου γὰρ τοῦ $ABΓ$ ἐφαπτέσθω τις εὐθεΐα ἢ $ΔΕ$ κατὰ τὸ $Γ$ σημεῖον, καὶ ἀπὸ τοῦ $Γ$ τῆ $ΔΕ$ πρὸς ὀρθὰς ἤχθω ἢ $ΓΑ$. λέγω, ὅτι ἐπὶ τῆς $ΑΓ$ ἐστὶ τὸ κέντρον τοῦ κύκλου.

the given circle BCD from the given point A . (Which is) the very thing it was required to do.

Proposition 18

If some straight-line touches a circle, and some (other) straight-line is joined from the center (of the circle) to the point of contact, then the (straight-line) so joined will be perpendicular to the tangent.



For let some straight-line DE touch the circle ABC at point C , and let the center F of circle ABC have been found [Prop. 3.1], and let FC have been joined from F to C . I say that FC is perpendicular to DE .

For if not, let FG have been drawn from F , perpendicular to DE [Prop. 1.12].

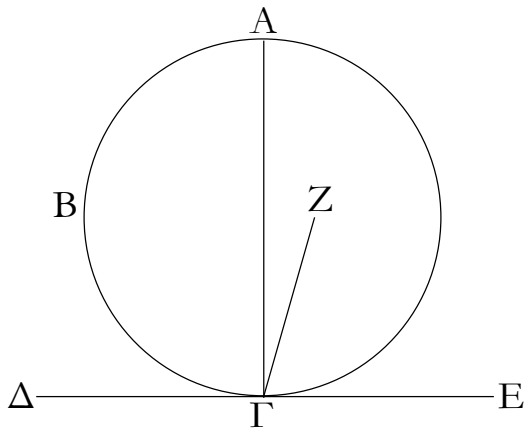
Therefore, since angle FGC is a right-angle, (angle) FCG is thus acute [Prop. 1.17]. And the greater angle is subtended by the greater side [Prop. 1.19]. Thus, FC (is) greater than FG . And FC (is) equal to FB . Thus, FB (is) also greater than FG , the lesser than the greater. The very thing is impossible. Thus, FG is not perpendicular to DE . So, similarly, we can show that neither (is) any other (straight-line) except FC . Thus, FC is perpendicular to DE .

Thus, if some straight-line touches a circle, and some (other) straight-line is joined from the center (of the circle) to the point of contact, then the (straight-line) so joined will be perpendicular to the tangent. (Which is) the very thing it was required to show.

Proposition 19

If some straight-line touches a circle, and a straight-line is drawn from the point of contact, at right-[angles] to the tangent, then the center (of the circle) will be on the (straight-line) so drawn.

For let some straight-line DE touch the circle ABC at point C . And let CA have been drawn from C , at right-

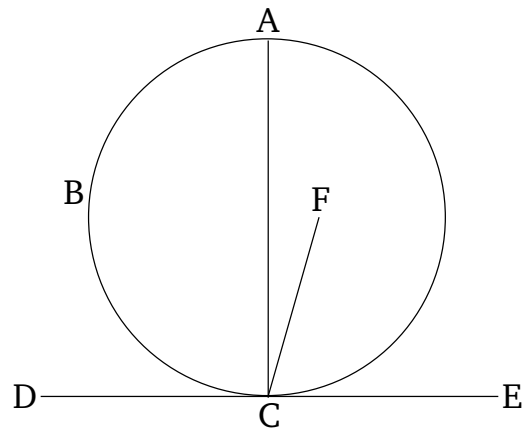


Μή γάρ, ἀλλ' εἰ δυνατόν, ἔστω τὸ Z, καὶ ἐπεζεύχθω ἡ ΓZ.

Ἐπεὶ [οὖν] κύκλου τοῦ ABΓ ἐφάπτεται τις εὐθεΐα ἡ ΔΕ, ἀπὸ δὲ τοῦ κέντρου ἐπὶ τὴν ἀφὴν ἐπέζευκται ἡ ΖΓ, ἡ ΖΓ ἄρα κάθετός ἐστιν ἐπὶ τὴν ΔΕ· ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ ΖΓΕ. ἐστὶ δὲ καὶ ἡ ὑπὸ ΑΓΕ ὀρθή· ἴση ἄρα ἐστὶν ἡ ὑπὸ ΖΓΕ τῇ ὑπὸ ΑΓΕ ἢ ἐλάττων τῇ μείζονι· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα τὸ Z κέντρον ἐστὶ τοῦ ABΓ κύκλου. ὁμοίως δὲ δεῖξομεν, ὅτι οὐδ' ἄλλο τι πλὴν ἐπὶ τῆς ΑΓ.

Ἐάν ἄρα κύκλου ἐφάπτηται τις εὐθεΐα, ἀπὸ δὲ τῆς ἀφῆς τῆ ἐφαπτομένη πρὸς ὀρθὰς εὐθεΐα γραμμὴ ἀχθῆ, ἐπὶ τῆς ἀχθείσης ἔσται τὸ κέντρον τοῦ κύκλου· ὅπερ ἔδει δεῖξαι.

angles to DE [Prop. 1.11]. I say that the center of the circle is on AC .



For (if) not, if possible, let F be (the center of the circle), and let CF have been joined.

[Therefore], since some straight-line DE touches the circle ABC , and FC has been joined from the center to the point of contact, FC is thus perpendicular to DE [Prop. 3.18]. Thus, FCE is a right-angle. And ACE is also a right-angle. Thus, FCE is equal to ACE , the lesser to the greater. The very thing is impossible. Thus, F is not the center of circle ABC . So, similarly, we can show that neither is any (point) other (than one) on AC .

Thus, if some straight-line touches a circle, and a straight-line is drawn from the point of contact, at right-angles to the tangent, then the center (of the circle) will be on the (straight-line) so drawn. (Which is) the very thing it was required to show.

κ'.

Ἐν κύκλῳ ἡ πρὸς τῶ κέντρῳ γωνία διπλασίων ἐστὶ τῆς πρὸς τῇ περιφερείᾳ, ὅταν τὴν αὐτὴν περιφέρειαν βάσιν ἔχωσιν αἱ γωνίαι.

Ἐστω κύκλος ὁ ABΓ, καὶ πρὸς μὲν τῶ κέντρῳ αὐτοῦ γωνία ἔστω ἡ ὑπὸ BEΓ, πρὸς δὲ τῇ περιφερείᾳ ἡ ὑπὸ BAΓ, ἐχέτωσαν δὲ τὴν αὐτὴν περιφέρειαν βάσιν τὴν ΒΓ· λέγω, ὅτι διπλασίων ἐστὶν ἡ ὑπὸ BEΓ γωνία τῆς ὑπὸ BAΓ.

Ἐπιζευχθεῖσα γὰρ ἡ ΑΕ διήχθω ἐπὶ τὸ Z.

Ἐπεὶ οὖν ἴση ἐστὶν ἡ ΕΑ τῇ ΕΒ, ἴση καὶ γωνία ἡ ὑπὸ ΕΑΒ τῇ ὑπὸ ΕΒΑ· αἱ ἄρα ὑπὸ ΕΑΒ, ΕΒΑ γωνίαι τῆς ὑπὸ ΕΑΒ διπλασίους εἰσίν. ἴση δὲ ἡ ὑπὸ BEZ ταῖς ὑπὸ ΕΑΒ, ΕΒΑ· καὶ ἡ ὑπὸ BEZ ἄρα τῆς ὑπὸ ΕΑΒ ἐστὶ διπλῆ. διὰ τὰ αὐτὰ δὲ καὶ ἡ ὑπὸ ZEG τῆς ὑπὸ ΕΑΓ ἐστὶ διπλῆ. ὅλη ἄρα ἡ ὑπὸ BEΓ ὅλης τῆς ὑπὸ BAΓ ἐστὶ διπλῆ.

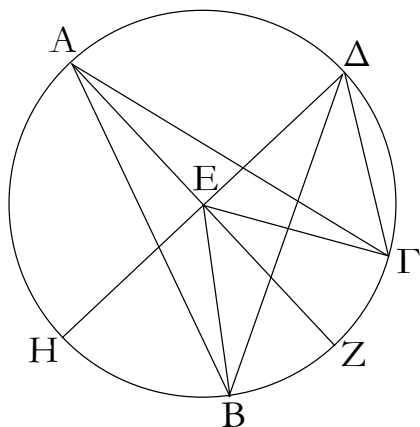
Proposition 20

In a circle, the angle at the center is double that at the circumference, when the angles have the same circumference base.

Let ABC be a circle, and let BEC be an angle at its center, and BAC (one) at (its) circumference. And let them have the same circumference base BC . I say that angle BEC is double (angle) BAC .

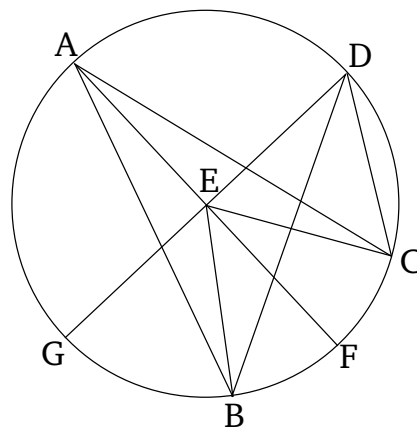
For being joined, let AE have been drawn through to F .

Therefore, since EA is equal to EB , angle EAB (is) also equal to EBA [Prop. 1.5]. Thus, angle EAB and EBA is double (angle) EAB . And BEF (is) equal to EAB and EBA [Prop. 1.32]. Thus, BEF is also double EAB . So, for the same (reasons), FEC is also double EAC . Thus, the whole (angle) BEC is double the whole (angle) BAC .



Κεκλάσθω δὴ πάλιν, καὶ ἔστω ἑτέρα γωνία ἡ ὑπὸ ΒΔΓ, καὶ ἐπιζευχθεῖσα ἡ ΔΕ ἐκβεβλήσθω ἐπὶ τὸ Η. ὁμοίως δὴ δείξομεν, ὅτι διπλῆ ἔστιν ἡ ὑπὸ ΗΕΓ γωνία τῆς ὑπὸ ΕΔΓ, ὧν ἡ ὑπὸ ΗΕΒ διπλῆ ἔστι τῆς ὑπὸ ΕΔΒ· λοιπὴ ἄρα ἡ ὑπὸ ΒΕΓ διπλῆ ἔστι τῆς ὑπὸ ΒΔΓ.

Ἐν κύκλῳ ἄρα ἡ πρὸς τῷ κέντρῳ γωνία διπλασίον ἐστὶ τῆς πρὸς τῇ περιφερείᾳ, ὅταν τὴν αὐτὴν περιφέρειαν βάσιν ἔχωσιν [αἱ γωνίαι]. Ὅπερ ἔδει δεῖξαι.



So let another (straight-line) have been inflected, and let there be another angle, BDC . And DE being joined, let it have been produced to G . So, similarly, we can show that angle GEC is double EDC , of which GEB is double EDB . Thus, the remaining (angle) BEC is double the (remaining angle) BDC .

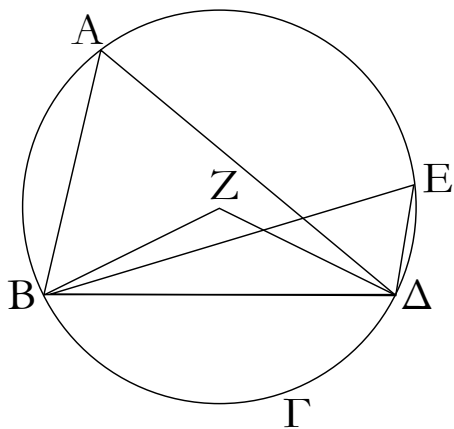
Thus, in a circle, the angle at the center is double that at the circumference, when [the angles] have the same circumference base. (Which is) the very thing it was required to show.

κα'.

Proposition 21

Ἐν κύκλῳ αἱ ἐν τῷ αὐτῷ τμήματι γωνίαι ἴσαι ἀλλήλαις εἰσίν.

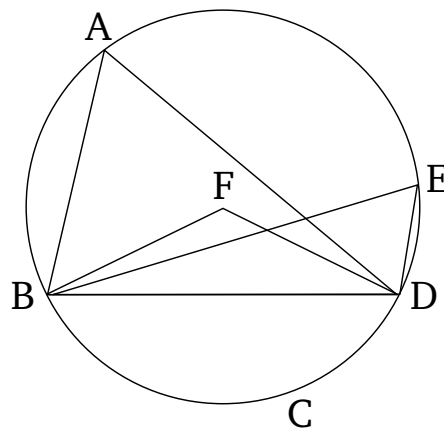
In a circle, angles in the same segment are equal to one another.



Ἐστω κύκλος ὁ ΑΒΓΔ, καὶ ἐν τῷ αὐτῷ τμήματι τῷ ΒΑΕΔ γωνίαι ἔστωσαν αἱ ὑπὸ ΒΑΔ, ΒΕΔ· λέγω, ὅτι αἱ ὑπὸ ΒΑΔ, ΒΕΔ γωνίαι ἴσαι ἀλλήλαις εἰσίν.

Εἰλήφθω γὰρ τοῦ ΑΒΓΔ κύκλου τὸ κέντρον, καὶ ἔστω τὸ Ζ, καὶ ἐπεζεύχθωσαν αἱ ΒΖ, ΖΔ.

Καὶ ἐπεὶ ἡ μὲν ὑπὸ ΒΖΔ γωνία πρὸς τῷ κέντρῳ ἐστίν, ἡ δὲ ὑπὸ ΒΑΔ πρὸς τῇ περιφερείᾳ, καὶ ἔχουσι τὴν αὐτὴν περιφέρειαν βάσιν τὴν ΒΓΔ, ἡ ἄρα ὑπὸ ΒΖΔ γωνία διπλασίον ἐστὶ τῆς ὑπὸ ΒΑΔ. διὰ τὰ αὐτὰ δὴ ἡ ὑπὸ ΒΖΔ καὶ τῆς ὑπὸ



Let $ABCD$ be a circle, and let BAD and BED be angles in the same segment $BAED$. I say that angles BAD and BED are equal to one another.

For let the center of circle $ABCD$ have been found [Prop. 3.1], and let it be (at point) F . And let BF and FD have been joined.

And since angle BFD is at the center, and BAD at the circumference, and they have the same circumference base BCD , angle BFD is thus double BAD [Prop. 3.20].

$BE\Delta$ ἐστὶ διπλῶν· ἴση ἄρα ἢ ὑπὸ $BA\Delta$ τῆ ὑπὸ $BE\Delta$.

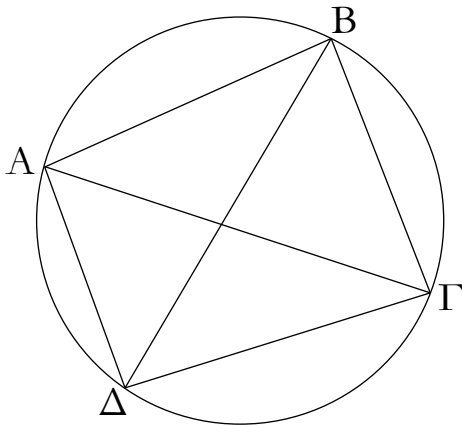
Ἐν κύκλῳ ἄρα αἱ ἐν τῷ αὐτῷ τμήματι γωνίαι ἴσαι ἀλλήλαις εἰσὶν· ὅπερ ἔδει δεῖξαι.

So, for the same (reasons), BFD is also double BED . Thus, BAD (is) equal to BED .

Thus, in a circle, angles in the same segment are equal to one another. (Which is) the very thing it was required to show.

κβ'.

Τῶν ἐν τοῖς κύκλοις τετραπλεύρων αἱ ἀπεναντίον γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσὶν.



Ἐστω κύκλος ὁ $ABG\Delta$, καὶ ἐν αὐτῷ τετράπλευρον ἔστω τὸ $ABG\Delta$ · λέγω, ὅτι αἱ ἀπεναντίον γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσὶν.

Ἐπεζεύχθωσαν αἱ AG , $B\Delta$.

Ἐπεὶ οὖν παντὸς τριγώνου αἱ τρεῖς γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσὶν, τοῦ ABG ἄρα τριγώνου αἱ τρεῖς γωνίαι αἱ ὑπὸ GAB , ABG , BGA δυσὶν ὀρθαῖς ἴσαι εἰσὶν. ἴση δὲ ἢ μὲν ὑπὸ GAB τῆ ὑπὸ $B\Delta G$ · ἐν γὰρ τῷ αὐτῷ τμήματι εἰσι τῷ $BA\Delta G$ · ἢ δὲ ὑπὸ AGB τῆ ὑπὸ $A\Delta B$ · ἐν γὰρ τῷ αὐτῷ τμήματι εἰσι τῷ $A\Delta G B$ · ὅλη ἄρα ἢ ὑπὸ $A\Delta G$ ταῖς ὑπὸ BAG , AGB ἴση ἐστίν. κοινὴ προσκείσθω ἢ ὑπὸ ABG · αἱ ἄρα ὑπὸ ABG , BAG , AGB ταῖς ὑπὸ ABG , $A\Delta G$ ἴσαι εἰσὶν. ἀλλ' αἱ ὑπὸ ABG , BAG , AGB δυσὶν ὀρθαῖς ἴσαι εἰσὶν. καὶ αἱ ὑπὸ ABG , $A\Delta G$ ἄρα δυσὶν ὀρθαῖς ἴσαι εἰσὶν. ὁμοίως δὲ δείξομεν, ὅτι καὶ αἱ ὑπὸ $BA\Delta$, ΔGB γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσὶν.

Τῶν ἄρα ἐν τοῖς κύκλοις τετραπλεύρων αἱ ἀπεναντίον γωνίαι δυσὶν ὀρθαῖς ἴσαι εἰσὶν· ὅπερ ἔδει δεῖξαι.

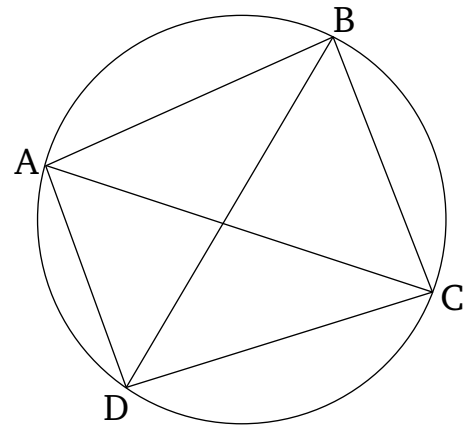
κγ'.

Ἐπὶ τῆς αὐτῆς εὐθείας δύο τμήματα κύκλων ὅμοια καὶ ἄνισα οὐ συσταθήσεται ἐπὶ τὰ αὐτὰ μέρη.

Εἰ γὰρ δυνατόν, ἐπὶ τῆς αὐτῆς εὐθείας τῆς AB δύο τμήματα κύκλων ὅμοια καὶ ἄνισα συνεστάτω ἐπὶ τὰ αὐτὰ μέρη τὰ AGB , $A\Delta B$, καὶ διήχθω ἢ $AG\Delta$, καὶ ἐπεζεύχθωσαν

Proposition 22

For quadrilaterals within circles, the (sum of the) opposite angles is equal to two right-angles.



Let $ABCD$ be a circle, and let $ABCD$ be a quadrilateral within it. I say that the (sum of the) opposite angles is equal to two right-angles.

Let AC and BD have been joined.

Therefore, since the three angles of any triangle are equal to two right-angles [Prop. 1.32], the three angles CAB , ABC , and BCA of triangle ABC are thus equal to two right-angles. And CAB (is) equal to BDC . For they are in the same segment $BADC$ [Prop. 3.21]. And ACB (is equal) to ADB . For they are in the same segment $ADCB$ [Prop. 3.21]. Thus, the whole of ADC is equal to BAC and ACB . Let ABC have been added to both. Thus, ABC , BAC , and ACB are equal to ABC and ADC . But, ABC , BAC , and ACB are equal to two right-angles. Thus, ABC and ADC are also equal to two right-angles. Similarly, we can show that angles BAD and DCB are also equal to two right-angles.

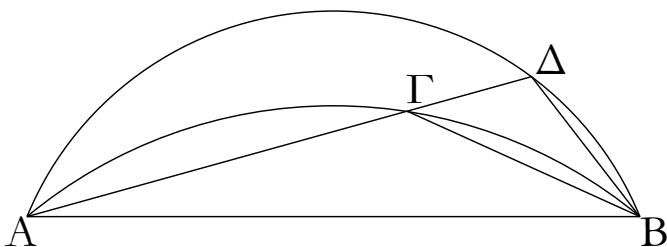
Thus, for quadrilaterals within circles, the (sum of the) opposite angles is equal to two right-angles. (Which is) the very thing it was required to show.

Proposition 23

Two similar and unequal segments of circles cannot be constructed on the same side of the same straight-line.

For, if possible, let the two similar and unequal segments of circles, ACB and ADB , have been constructed on the same side of the same straight-line AB . And let

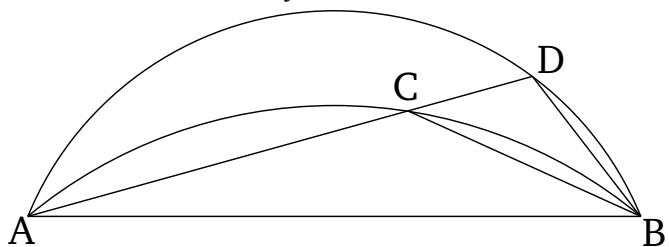
αί ΓΒ, ΔΒ.



Ἐπεὶ οὖν ὁμοίον ἐστὶ τὸ ΑΓΒ τμήμα τῶ ΑΔΒ τμήματι, ὅμοια δὲ τμήματα κύκλων ἐστὶ τὰ δεχόμενα γωνίας ἴσας, ἴση ἄρα ἐστὶν ἡ ὑπὸ ΑΓΒ γωνία τῇ ὑπὸ ΑΔΒ ἢ ἐκτὸς τῇ ἐντὸς· ὅπερ ἐστὶν ἀδύνατον.

Οὐκ ἄρα ἐπὶ τῆς αὐτῆς εὐθείας δύο τμήματα κύκλων ὅμοια καὶ ἄνισα συσταθήσεται ἐπὶ τὰ αὐτὰ μέρη· ὅπερ ἔδει δεῖξαι.

ACD have been drawn through (the segments), and let *CB* and *DB* have been joined.

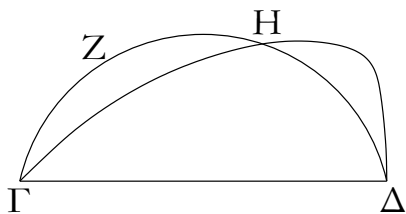
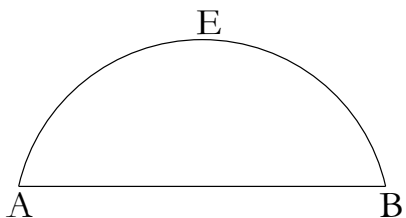


Therefore, since segment *ACB* is similar to segment *ADB*, and similar segments of circles are those accepting equal angles [Def. 3.11], angle *ACB* is thus equal to *ADB*, the external to the internal. The very thing is impossible [Prop. 1.16].

Thus, two similar and unequal segments of circles cannot be constructed on the same side of the same straight-line.

κδ'.

Τὰ ἐπὶ ἴσων εὐθειῶν ὅμοια τμήματα κύκλων ἴσα ἀλλήλοις ἐστίν.

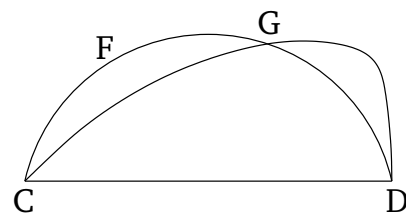
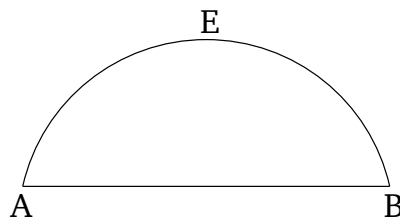


Ἐστωσαν γὰρ ἐπὶ ἴσων εὐθειῶν τῶν ΑΒ, ΓΔ ὅμοια τμήματα κύκλων τὰ ΑΕΒ, ΓΖΔ· λέγω, ὅτι ἴσον ἐστὶ τὸ ΑΕΒ τμήμα τῶ ΓΖΔ τμήματι.

Ἐφαρμοζομένου γὰρ τοῦ ΑΕΒ τμήματος ἐπὶ τὸ ΓΖΔ καὶ τιθεμένου τοῦ μὲν Α σημείου ἐπὶ τὸ Γ τῆς δὲ ΑΒ εὐθείας ἐπὶ τὴν ΓΔ, ἐφαρμόσει καὶ τὸ Β σημεῖον ἐπὶ τὸ Δ σημεῖον διὰ τὸ ἴσην εἶναι τὴν ΑΒ τῇ ΓΔ· τῆς δὲ ΑΒ ἐπὶ τὴν ΓΔ ἐφαρμολογήσεται καὶ τὸ ΑΕΒ τμήμα ἐπὶ τὸ ΓΖΔ. εἰ γὰρ ἡ ΑΒ εὐθεῖα ἐπὶ τὴν ΓΔ ἐφαρμόσει, τὸ δὲ ΑΕΒ τμήμα ἐπὶ τὸ ΓΖΔ μὴ ἐφαρμόσει, ἤτοι ἐντὸς αὐτοῦ πεσεῖται ἢ ἐκτὸς ἢ παραλλάξει, ὡς τὸ ΓΗΔ, καὶ κύκλος κύκλον τέμνει κατὰ πλείονα σημεία ἢ δύο· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα ἐφαρμοζομένης τῆς ΑΒ εὐθείας ἐπὶ τὴν ΓΔ οὐκ ἐφαρμόσει καὶ

Proposition 24

Similar segments of circles on equal straight-lines are equal to one another.



For let *AEB* and *CFD* be similar segments of circles on the equal straight-lines *AB* and *CD* (respectively). I say that segment *AEB* is equal to segment *CFD*.

For if the segment *AEB* is applied to the segment *CFD*, and point *A* is placed on (point) *C*, and the straight-line *AB* on *CD*, then point *B* will also coincide with point *D*, on account of *AB* being equal to *CD*. And if *AB* coincides with *CD* then the segment *AEB* will also coincide with *CFD*. For if the straight-line *AB* coincides with *CD*, and the segment *AEB* does not coincide with *CFD*, then it will surely either fall inside it, outside (it),[†] or it will miss like *CGD* (in the figure), and a circle (will) cut (another) circle at more than two points. The very

τὸ AEB τμήμα ἐπὶ τὸ $\Gamma Z\Delta$ · ἐφαρμόσει ἄρα, καὶ ἴσον αὐτῷ ἔσται.

Τὰ ἄρα ἐπὶ ἴσων εὐθειῶν ὅμοια τμήματα κύκλων ἴσα ἀλλήλοις ἐστίν· ὅπερ ἔδει δεῖξαι.

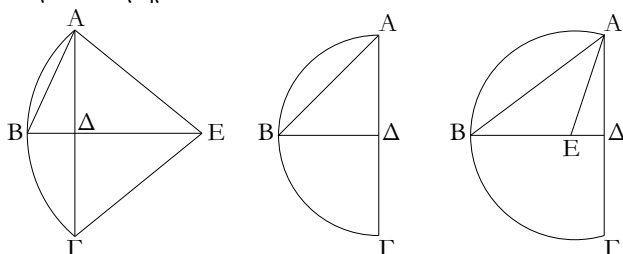
thing is impossible [Prop. 3.10]. Thus, if the straight-line AB is applied to CD , the segment AEB cannot not also coincide with CFD . Thus, it will coincide, and will be equal to it [C.N. 4].

Thus, similar segments of circles on equal straight-lines are equal to one another. (Which is) the very thing it was required to show.

† Both this possibility, and the previous one, are precluded by Prop. 3.23.

κε'.

Κύκλου τμήματος δοθέντος προσαναγράψαι τὸν κύκλον, οὐπὲρ ἐστὶ τμήμα.



Ἐστω τὸ δοθὲν τμήμα κύκλου τὸ $AB\Gamma$ · δεῖ δὴ τοῦ $AB\Gamma$ τμήματος προσαναγράψαι τὸν κύκλον, οὐπὲρ ἐστὶ τμήμα.

Τετμησθῶ γὰρ ἡ AB δίχα κατὰ τὸ D , καὶ ἤχθῳ ἀπὸ τοῦ D σημείου τῆς AB πρὸς ὀρθὰς ἡ DE , καὶ ἐπεζεύχθῳ ἡ AE · ἡ ὑπὸ ABD γωνία ἄρα τῆς ὑπὸ BAE ἴση ἢ ἐλάττω.

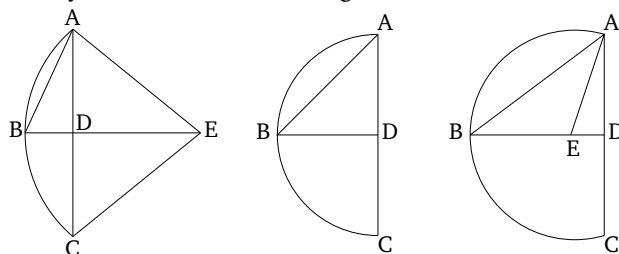
Ἐστω πρότερον μείζων, καὶ συνεστάτω πρὸς τῆς BA εὐθείας καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ A τῆς ὑπὸ ABD γωνία ἴση ἢ ὑπὸ BAE , καὶ διήχθῳ ἡ DE ἐπὶ τὸ E , καὶ ἐπεζεύχθῳ ἡ EF . ἐπεὶ οὖν ἴση ἐστὶν ἡ ὑπὸ ABE γωνία τῆς ὑπὸ BAE , ἴση ἄρα ἐστὶ καὶ ἡ EB εὐθεῖα τῆς EA . καὶ ἐπεὶ ἴση ἐστὶν ἡ AD τῆς DF , κοινὴ δὲ ἡ DE , δύο δὴ αἱ AD , DE δύο ταῖς FD , DE ἴσαι εἰσὶν ἑκατέρα ἑκατέρᾳ· καὶ γωνία ἡ ὑπὸ ADE γωνία τῆς ὑπὸ DFE ἐστὶν ἴση· ὀρθὴ γὰρ ἑκατέρα· βάσις ἄρα ἡ AE βάσει τῆς FE ἐστὶν ἴση· ἀλλὰ ἡ AE τῆς BE ἐδείχθη ἴση· καὶ ἡ BE ἄρα τῆς FE ἐστὶν ἴση· αἱ τρεῖς ἄρα αἱ AE , EB , EF ἴσαι ἀλλήλαις εἰσὶν· ὁ ἄρα κέντρον τῷ E διαστήματι δὲ ἐνὶ τῶν AE , EB , EF κύκλος γραφόμενος ἔξει καὶ διὰ τῶν λοιπῶν σημείων καὶ ἔσται προσαναγεγραμμένος· κύκλου ἄρα τμήματος δοθέντος προσαναγράφεται ὁ κύκλος· καὶ δῆλον, ὡς τὸ $AB\Gamma$ τμήμα ἐλάττω ἐστὶν ἡμικυκλίου διὰ τὸ τὸ E κέντρον ἐκτὸς αὐτοῦ τυγχάνειν.

Ὅμοίως [δὲ] καὶ ἡ ὑπὸ ABD γωνία ἴση τῆς ὑπὸ BAE , τῆς AD ἴσης γενομένης ἑκατέρα τῶν BD , DF αἱ τρεῖς αἱ AD , DB , DF ἴσαι ἀλλήλαις ἔσονται, καὶ ἔσται τὸ D κέντρον τοῦ προσαναπεληρωμένου κύκλου, καὶ δηλαδὴ ἔσται τὸ $AB\Gamma$ ἡμικύκλιον.

Ἐὰν δὲ ἡ ὑπὸ ABD ἐλάττω ἢ τῆς ὑπὸ BAE , καὶ συστησώμεθα πρὸς τῆς BA εὐθείας καὶ τῷ πρὸς αὐτῇ σημείῳ

Proposition 25

For a given segment of a circle, to complete the circle, the very one of which it is a segment.



Let ABC be the given segment of a circle. So it is required to complete the circle for segment ABC , the very one of which it is a segment.

For let AC have been cut in half at (point) D [Prop. 1.10], and let DB have been drawn from point D , at right-angles to AC [Prop. 1.11]. And let AB have been joined. Thus, angle ABD is surely either greater than, equal to, or less than (angle) BAD .

First of all, let it be greater. And let (angle) BAE , equal to angle ABD , have been constructed on the straight-line BA , at the point A on it [Prop. 1.23]. And let DB have been drawn through to E , and let EC have been joined. Therefore, since angle ABE is equal to BAE , the straight-line EB is thus also equal to EA [Prop. 1.6]. And since AD is equal to DC , and DE (is) common, the two (straight-lines) AD , DE are equal to the two (straight-lines) CD , DE , respectively. And angle ADE is equal to angle CDE . For each (is) a right-angle. Thus, the base AE is equal to the base CE [Prop. 1.4]. But, AE was shown (to be) equal to BE . Thus, BE is also equal to CE . Thus, the three (straight-lines) AE , EB , and EC are equal to one another. Thus, if a circle is drawn with center E , and radius one of AE , EB , or EC , it will also go through the remaining points (of the segment), and the (associated circle) will have been completed [Prop. 3.9]. Thus, a circle has been completed from the given segment of a circle. And (it is) clear that the segment ABC is less than a semi-circle, because the center E happens to lie outside it.

τῷ A τῇ ὑπὸ $AB\Delta$ γωνίᾳ ἴσην, ἐντὸς τοῦ $AB\Gamma$ τμήματος πεσεῖται τὸ κέντρον ἐπὶ τῆς ΔB , καὶ ἔσται δηλαδὴ τὸ $AB\Gamma$ τμήμα μείζον ἡμικυκλίου.

Κύκλου ἄρα τμήματος δοθέντος προσαναγέγραπται ὁ κύκλος· ὅπερ ἔδει ποιῆσαι.

[And], similarly, even if angle ABD is equal to BAD , (since) AD becomes equal to each of BD [Prop. 1.6] and DC , the three (straight-lines) DA , DB , and DC will be equal to one another. And point D will be the center of the completed circle. And ABC will manifestly be a semi-circle.

And if ABD is less than BAD , and we construct (angle BAE), equal to angle ABD , on the straight-line BA , at the point A on it [Prop. 1.23], then the center will fall on DB , inside the segment ABC . And segment ABC will manifestly be greater than a semi-circle.

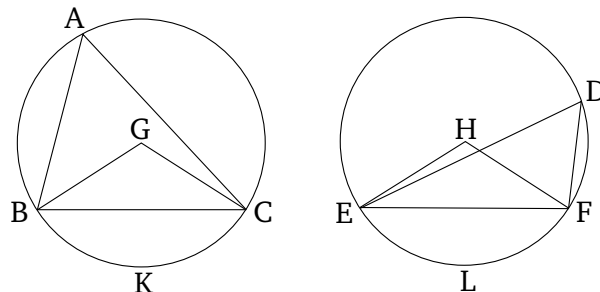
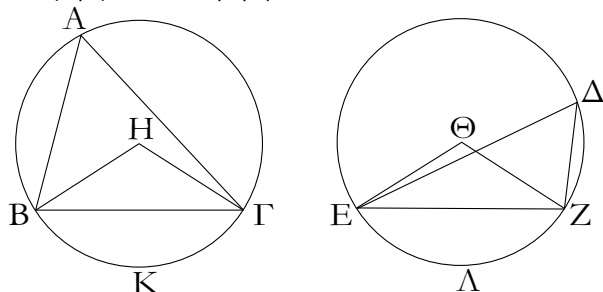
Thus, a circle has been completed from the given segment of a circle. (Which is) the very thing it was required to do.

κς'.

Proposition 26

Ἐν τοῖς ἴσοις κύκλοις αἱ ἴσαι γωνίαι ἐπὶ ἴσων περιφερειῶν βεβήκασιν, ἂν τε πρὸς τοῖς κέντροις ἂν τε πρὸς ταῖς περιφερείαις ὡσι βεβηκυῖαι.

In equal circles, equal angles stand upon equal circumferences whether they are standing at the center or at the circumference.



Ἐστωσαν ἴσοι κύκλοι οἱ $AB\Gamma$, ΔEZ καὶ ἐν αὐτοῖς ἴσαι γωνίαι ἔστωσαν πρὸς μὲν τοῖς κέντροις αἱ ὑπὸ BHG , $E\Theta Z$, πρὸς δὲ ταῖς περιφερείαις αἱ ὑπὸ BAG , $E\Delta Z$ · λέγω, ὅτι ἴση ἔστιν ἡ $BK\Gamma$ περιφέρεια τῇ ELZ περιφερείᾳ.

Let ABC and DEF be equal circles, and within them let BGC and EHF be equal angles at the center, and BAC and EDF (equal angles) at the circumference. I say that circumference BKC is equal to circumference ELF .

Ἐπεζεύχθωσαν γὰρ αἱ $B\Gamma$, EZ .

For let BC and EF have been joined.

Καὶ ἐπεὶ ἴσοι εἰσὶν οἱ $AB\Gamma$, ΔEZ κύκλοι, ἴσαι εἰσὶν αἱ ἐκ τῶν κέντρων· δύο δὴ αἱ BH , $H\Gamma$ δύο ταῖς $E\Theta$, ΘZ ἴσαι· καὶ γωνία ἡ πρὸς τῷ H γωνία τῇ πρὸς τῷ Θ ἴση· βάσεις ἄρα ἡ $B\Gamma$ βάσει τῇ EZ ἔστιν ἴση. καὶ ἐπεὶ ἴση ἔστιν ἡ πρὸς τῷ A γωνία τῇ πρὸς τῷ Δ , ὅμοιον ἄρα ἔστι τὸ BAG τμήμα τῷ $E\Delta Z$ τμήματι· καὶ εἰσὶν ἐπὶ ἴσων εὐθειῶν [τῶν $B\Gamma$, EZ]· τὰ δὲ ἐπὶ ἴσων εὐθειῶν ὅμοια τμήματα κύκλων ἴσα ἀλλήλοις ἔστιν· ἴσον ἄρα τὸ BAG τμήμα τῷ $E\Delta Z$. ἔστι δὲ καὶ ὅλος ὁ $AB\Gamma$ κύκλος ὅλῳ τῷ ΔEZ κύκλῳ ἴσος· λοιπὴ ἄρα ἡ $BK\Gamma$ περιφέρεια τῇ ELZ περιφερείᾳ ἔστιν ἴση.

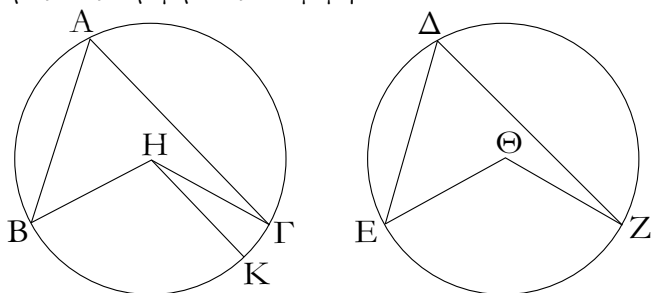
And since circles ABC and DEF are equal, their radii are equal. So the two (straight-lines) BG , GC (are) equal to the two (straight-lines) EH , HF (respectively). And the angle at G (is) equal to the angle at H . Thus, the base BC is equal to the base EF [Prop. 1.4]. And since the angle at A is equal to the (angle) at D , the segment BAC is thus similar to the segment EDF [Def. 3.11]. And they are on equal straight-lines [BC and EF]. And similar segments of circles on equal straight-lines are equal to one another [Prop. 3.24]. Thus, segment BAC is equal to (segment) EDF . And the whole circle ABC is also equal to the whole circle DEF . Thus, the remaining circumference BKC is equal to the (remaining) circumference ELF .

Ἐν ἄρα τοῖς ἴσοις κύκλοις αἱ ἴσαι γωνίαι ἐπὶ ἴσων περιφερειῶν βεβήκασιν, ἂν τε πρὸς τοῖς κέντροις ἂν τε πρὸς ταῖς περιφερείαις ὡσι βεβηκυῖαι· ὅπερ ἔδει δεῖξαι.

Thus, in equal circles, equal angles stand upon equal circumferences, whether they are standing at the center

κζ'.

Ἐν τοῖς ἴσοις κύκλοις αἱ ἐπὶ ἴσων περιφερειῶν βεβηχυῖαι γωνίαι ἴσαι ἀλλήλαις εἰσίν, ἐάν τε πρὸς τοῖς κέντροις ἐάν τε πρὸς ταῖς περιφερείαις ὡς βεβηχυῖαι.



Ἐν γὰρ ἴσοις κύκλοις τοῖς $ABΓ$, $ΔEZ$ ἐπὶ ἴσων περιφερειῶν τῶν $BΓ$, EZ πρὸς μὲν τοῖς H , $Θ$ κέντροις γωνία βεβηκέτωσαν αἱ ὑπὸ BHG , $EΘZ$, πρὸς δὲ ταῖς περιφερείαις αἱ ὑπὸ BAG , $EΔZ$ · λέγω, ὅτι ἡ μὲν ὑπὸ BHG γωνία τῇ ὑπὸ $EΘZ$ ἔστιν ἴση, ἡ δὲ ὑπὸ BAG τῇ ὑπὸ $EΔZ$ ἔστιν ἴση.

Εἰ γὰρ ἀνισός ἐστιν ἡ ὑπὸ BHG τῇ ὑπὸ $EΘZ$, μία αὐτῶν μείζων ἐστίν. ἔστω μείζων ἡ ὑπὸ BHG , καὶ συνεστάτω πρὸς τῇ BH εὐθείᾳ καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ H τῇ ὑπὸ $EΘZ$ γωνία ἴση ἡ ὑπὸ BHK · αἱ δὲ ἴσαι γωνίαι ἐπὶ ἴσων περιφερειῶν βεβήκασιν, ὅταν πρὸς τοῖς κέντροις ὡσιν· ἴση ἄρα ἡ BK περιφέρεια τῇ EZ περιφέρειᾳ. ἀλλὰ ἡ EZ τῇ $BΓ$ ἔστιν ἴση· καὶ ἡ BK ἄρα τῇ $BΓ$ ἔστιν ἴση ἢ ἐλάττων τῇ μείζονι· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα ἀνισός ἐστιν ἡ ὑπὸ BHG γωνία τῇ ὑπὸ $EΘZ$ · ἴση ἄρα. καὶ ἐστὶ τῆς μὲν ὑπὸ BHG ἡμίσεια ἢ πρὸς τῷ A , τῆς δὲ ὑπὸ $EΘZ$ ἡμίσεια ἢ πρὸς τῷ $Δ$ · ἴση ἄρα καὶ ἡ πρὸς τῷ A γωνία τῇ πρὸς τῷ $Δ$.

Ἐν ἄρα τοῖς ἴσοις κύκλοις αἱ ἐπὶ ἴσων περιφερειῶν βεβηχυῖαι γωνίαι ἴσαι ἀλλήλαις εἰσίν, ἐάν τε πρὸς τοῖς κέντροις ἐάν τε πρὸς ταῖς περιφερείαις ὡς βεβηχυῖαι· ὅπερ ἔδει δεῖξαι.

κη'.

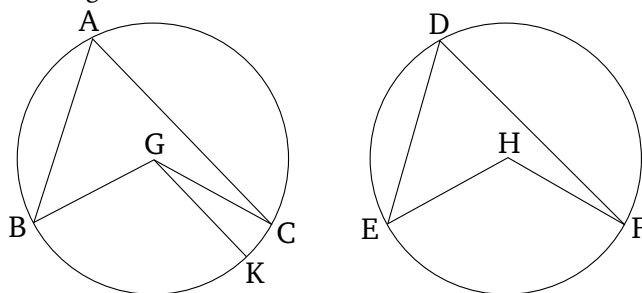
Ἐν τοῖς ἴσοις κύκλοις αἱ ἴσαι εὐθεῖαι ἴσας περιφερείας ἀφαιροῦσι τὴν μὲν μείζονα τῇ μείζονι τὴν δὲ ἐλάττονα τῇ ἐλάττωνι.

Ἐστῶσαν ἴσοι κύκλοι οἱ $ABΓ$, $ΔEZ$, καὶ ἐν τοῖς κύκλοις ἴσαι εὐθεῖαι ἔστωσαν αἱ AB , $ΔE$ τὰς μὲν AGB , AZE περιφερείας μείζονας ἀφαιροῦσαι τὰς δὲ AHB , $ΔΘE$ ἐλάττονας· λέγω, ὅτι ἡ μὲν AGB μείζων περιφέρεια ἴση ἐστὶ τῇ $ΔZE$ μείζονι περιφέρειᾳ ἢ δὲ AHB ἐλάττων περιφέρεια τῇ $ΔΘE$.

or at the circumference. (Which is) the very thing which it was required to show.

Proposition 27

In equal circles, angles standing upon equal circumferences are equal to one another, whether they are standing at the center or at the circumference.



For let the angles BGC and EHF at the centers G and H , and the (angles) BAC and EDF at the circumferences, stand upon the equal circumferences BC and EF , in the equal circles ABC and DEF (respectively). I say that angle BGC is equal to (angle) EHF , and BAC is equal to EDF .

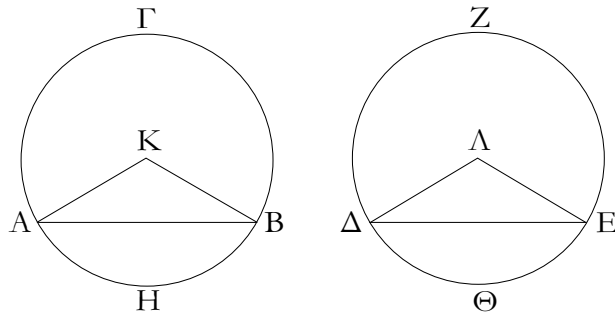
For if BGC is unequal to EHF , one of them is greater. Let BGC be greater, and let the (angle) BGK , equal to angle EHF , have been constructed on the straight-line BG , at the point G on it [Prop. 1.23]. But equal angles (in equal circles) stand upon equal circumferences, when they are at the centers [Prop. 3.26]. Thus, circumference BK (is) equal to circumference EF . But, EF is equal to BC . Thus, BK is also equal to BC , the lesser to the greater. The very thing is impossible. Thus, angle BGC is not unequal to EHF . Thus, (it is) equal. And the (angle) at A is half BGC , and the (angle) at D half EHF [Prop. 3.20]. Thus, the angle at A (is) also equal to the (angle) at D .

Thus, in equal circles, angles standing upon equal circumferences are equal to one another, whether they are standing at the center or at the circumference. (Which is) the very thing it was required to show.

Proposition 28

In equal circles, equal straight-lines cut off equal circumferences, the greater (circumference being equal) to the greater, and the lesser to the lesser.

Let ABC and DEF be equal circles, and let AB and DE be equal straight-lines in these circles, cutting off the greater circumferences ACB and DFE , and the lesser (circumferences) AGB and DHE (respectively). I say that the greater circumference ACB is equal to the greater circumference DFE , and the lesser circumfer-

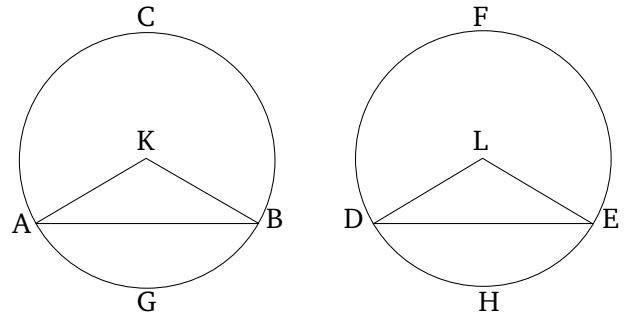


Εἰλήφθω γὰρ τὰ κέντρα τῶν κύκλων τὰ K, Λ , καὶ ἐπεζεύχθωσαν αἱ $AK, KB, \Delta\Lambda, \Lambda E$.

Καὶ ἐπεὶ ἴσοι κύκλοι εἰσὶν, ἴσαι εἰσὶ καὶ αἱ ἐκ τῶν κέντρων· δύο δὴ αἱ AK, KB δυσὶ ταῖς $\Delta\Lambda, \Lambda E$ ἴσαι εἰσὶν· καὶ βάσις ἡ AB βάσει τῆ ΔE ἴση· γωνία ἄρα ἡ ὑπὸ AKB γωνία τῆ ὑπὸ $\Delta\Lambda E$ ἴση ἐστίν. αἱ δὲ ἴσαι γωνίαι ἐπὶ ἴσων περιφερειῶν βεβήκασιν, ὅταν πρὸς τοῖς κέντροις ὦσιν· ἴση ἄρα ἡ AHB περιφέρεια τῆ $\Delta\Theta E$. ἐστὶ δὲ καὶ ὅλος ὁ $AB\Gamma$ κύκλος ὅλω τῷ $\Delta E Z$ κύκλω ἴσος· καὶ λοιπὴ ἄρα ἡ AGB περιφέρεια λοιπῆ τῆ $\Delta Z E$ περιφέρειᾳ ἴση ἐστίν.

Ἐν ἄρα τοῖς ἴσοις κύκλοις αἱ ἴσαι εὐθεῖαι ἴσας περιφερείας ἀφαιροῦσι τὴν μὲν μείζονα τῆ μείζονι τὴν δὲ ἐλάττονα τῆ ἐλάττονι· ὅπερ εἶδει δεῖξαι.

ence AGB to (the lesser) DHE .



For let the centers of the circles, K and L , have been found [Prop. 3.1], and let AK, KB, DL , and LE have been joined.

And since $(ABC$ and $DEF)$ are equal circles, their radii are also equal [Def. 3.1]. So the two (straight-lines) AK, KB are equal to the two (straight-lines) DL, LE (respectively). And the base AB (is) equal to the base DE . Thus, angle AKB is equal to angle DLE [Prop. 1.8]. And equal angles stand upon equal circumferences, when they are at the centers [Prop. 3.26]. Thus, circumference AGB (is) equal to DHE . And the whole circle ABC is also equal to the whole circle DEF . Thus, the remaining circumference ACB is also equal to the remaining circumference DFE .

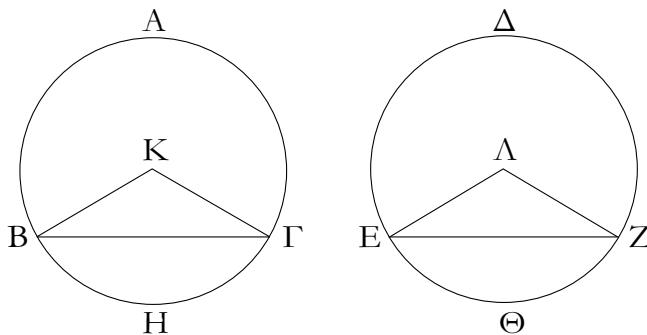
Thus, in equal circles, equal straight-lines cut off equal circumferences, the greater (circumference being equal) to the greater, and the lesser to the lesser. (Which is) the very thing it was required to show.

κθ'.

Proposition 29

Ἐν τοῖς ἴσοις κύκλοις τὰς ἴσας περιφερείας ἴσαι εὐθεῖαι ὑποτείνουσιν.

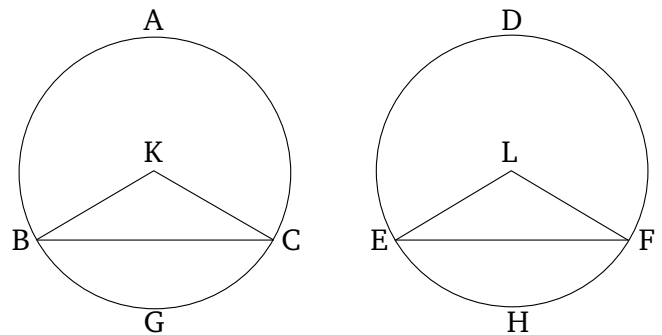
In equal circles, equal straight-lines subtend equal circumferences.



Ἐστῶσαν ἴσοι κύκλοι οἱ $AB\Gamma, \Delta E Z$, καὶ ἐν αὐτοῖς ἴσαι περιφέρεια ἀπειλήφθωσαν αἱ $BH\Gamma, E\Theta Z$, καὶ ἐπεζεύχθωσαν αἱ $B\Gamma, E Z$ εὐθεῖαι· λέγω, ὅτι ἴση ἐστὶν ἡ $B\Gamma$ τῆ $E Z$.

Εἰλήφθω γὰρ τὰ κέντρα τῶν κύκλων, καὶ ἔστω τὰ K, Λ , καὶ ἐπεζεύχθωσαν αἱ $BK, ΚΓ, E\Lambda, \Lambda Z$.

Καὶ ἐπεὶ ἴση ἐστὶν ἡ $BH\Gamma$ περιφέρεια τῆ $E\Theta Z$ περιφέρειᾳ,



Let ABC and DEF be equal circles, and within them let the equal circumferences BGC and EHF have been cut off. And let the straight-lines BC and EF have been joined. I say that BC is equal to EF .

For let the centers of the circles have been found [Prop. 3.1], and let them be (at) K and L . And let BK ,

ἴση ἐστὶ καὶ γωνία ἡ ὑπὸ ΒΚΓ τῇ ὑπὸ ΕΛΖ. καὶ ἐπεὶ ἴσοι εἰσὶν οἱ ΑΒΓ, ΔΕΖ κύκλοι, ἴσοι εἰσὶ καὶ αἱ ἐκ τῶν κέντρων· δύο δὴ αἱ ΒΚ, ΚΓ δυσὶ ταῖς ΕΛ, ΛΖ ἴσοι εἰσὶν· καὶ γωνίας ἴσας περιέχουσιν· βάσις ἄρα ἡ ΒΓ βάσει τῇ ΕΖ ἴση ἐστίν·

Ἐν ἄρα τοῖς ἴσοις κύκλοις τὰς ἴσας περιφερείας ἴσαι εὐθεΐαι ὑποτείνουσιν· ὅπερ ἔδει δεῖξαι.

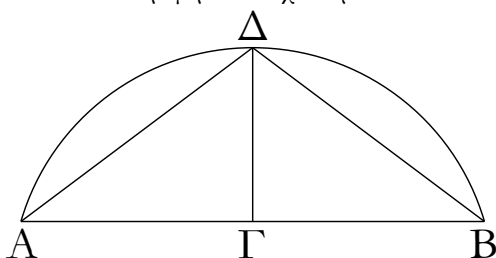
KC , EL , and LF have been joined.

And since the circumference BGC is equal to the circumference EHF , the angle BKC is also equal to (angle) ELF [Prop. 3.27]. And since the circles ABC and DEF are equal, their radii are also equal [Def. 3.1]. So the two (straight-lines) BK , KC are equal to the two (straight-lines) EL , LF (respectively). And they contain equal angles. Thus, the base BC is equal to the base EF [Prop. 1.4].

Thus, in equal circles, equal straight-lines subtend equal circumferences. (Which is) the very thing it was required to show.

λ'.

Τὴν δοθεῖσαν περιφέρειαν δίχα τεμεῖν.



Ἐστω ἡ δοθεῖσα περιφέρεια ἡ ΑΔΒ· δεῖ δὴ τὴν ΑΔΒ περιφέρειαν δίχα τεμεῖν.

Ἐπεζεύχθω ἡ ΑΒ, καὶ τετμήσθω δίχα κατὰ τὸ Γ, καὶ ἀπὸ τοῦ Γ σημείου τῇ ΑΒ εὐθείᾳ πρὸς ὀρθὰς ἦχθω ἡ ΓΔ, καὶ ἐπεζεύχθωσαν αἱ ΑΔ, ΔΒ.

Καὶ ἐπεὶ ἴση ἐστὶν ἡ ΑΓ τῇ ΓΒ, κοινὴ δὲ ἡ ΓΔ, δύο δὴ αἱ ΑΓ, ΓΔ δυσὶ ταῖς ΒΓ, ΓΔ ἴσοι εἰσὶν· καὶ γωνία ἡ ὑπὸ ΑΓΔ γωνία τῇ ὑπὸ ΒΓΔ ἴση· ὀρθὴ γὰρ ἑκατέρω· βάσις ἄρα ἡ ΑΔ βάσει τῇ ΔΒ ἴση ἐστίν· αἱ δὲ ἴσαι εὐθεΐαι ἴσας περιφερείας ἀφαιροῦσι τὴν μὲν μείζονα τῇ μείζονι τὴν δὲ ἐλάττονα τῇ ἐλάττονι· καὶ ἐστὶν ἑκατέρα τῶν ΑΔ, ΔΒ περιφερειῶν ἐλάττων ἡμικυκλίου· ἴση ἄρα ἡ ΑΔ περιφέρεια τῇ ΔΒ περιφερείᾳ.

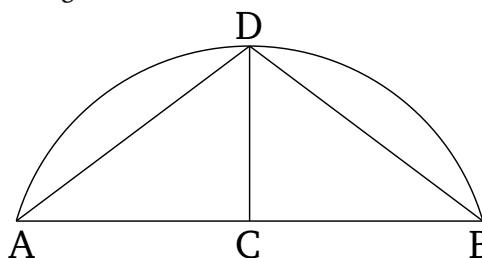
Ἡ ἄρα δοθεῖσα περιφέρεια δίχα τέτμηται κατὰ τὸ Δ σημεῖον· ὅπερ ἔδει ποιῆσαι.

λα'.

Ἐν κύκλῳ ἡ μὲν ἐν τῷ ἡμικυκλίῳ γωνία ὀρθὴ ἐστίν, ἡ δὲ ἐν τῷ μείζονι τμήματι ἐλάττων ὀρθῆς, ἡ δὲ ἐν τῷ ἐλάττονι τμήματι μείζων ὀρθῆς· καὶ ἔπι ἡ μὲν τοῦ μείζονος τμήματος γωνία μείζων ἐστὶν ὀρθῆς, ἡ δὲ τοῦ ἐλάττονος τμήματος γωνία ἐλάττων ὀρθῆς.

Proposition 30

To cut a given circumference in half.



Let ADB be the given circumference. So it is required to cut circumference ADB in half.

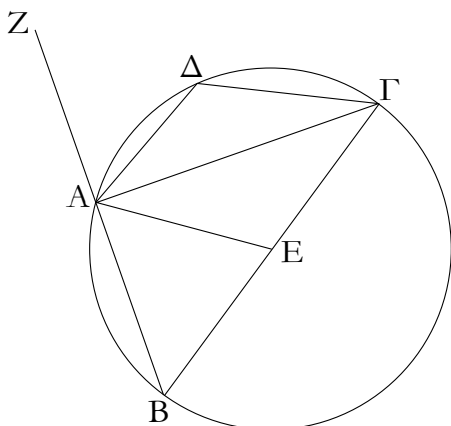
Let AB have been joined, and let it have been cut in half at (point) C [Prop. 1.10]. And let CD have been drawn from point C , at right-angles to AB [Prop. 1.11]. And let AD , and DB have been joined.

And since AC is equal to CB , and CD (is) common, the two (straight-lines) AC , CD are equal to the two (straight-lines) BC , CD (respectively). And angle ACD (is) equal to angle BCD . For (they are) each right-angles. Thus, the base AD is equal to the base DB [Prop. 1.4]. And equal straight-lines cut off equal circumferences, the greater (circumference being equal) to the greater, and the lesser to the lesser [Prop. 1.28]. And the circumferences AD and DB are each less than a semi-circle. Thus, circumference AD (is) equal to circumference DB .

Thus, the given circumference has been cut in half at point D . (Which is) the very thing it was required to do.

Proposition 31

In a circle, the angle in a semi-circle is a right-angle, and that in a greater segment (is) less than a right-angle, and that in a lesser segment (is) greater than a right-angle. And, further, the angle of a segment greater (than a semi-circle) is greater than a right-angle, and the an-



Ἐστω κύκλος ὁ ΑΒΓΔ, διάμετρος δὲ αὐτοῦ ἔστω ἡ ΒΓ, κέντρον δὲ τὸ Ε, καὶ ἐπεζεύχθωσαν αἱ ΒΑ, ΑΓ, ΑΔ, ΔΓ· λέγω, ὅτι ἡ μὲν ἐν τῷ ΒΑΓ ἡμικυκλίῳ γωνία ἢ ὑπὸ ΒΑΓ ὀρθή ἐστίν, ἡ δὲ ἐν τῷ ΑΒΓ μείζονι τοῦ ἡμικυκλίου τμήματι γωνία ἢ ὑπὸ ΑΒΓ ἐλάττων ἐστίν ὀρθῆς, ἡ δὲ ἐν τῷ ΑΔΓ ἐλάττονι τοῦ ἡμικυκλίου τμήματι γωνία ἢ ὑπὸ ΑΔΓ μείζων ἐστίν ὀρθῆς.

Ἐπεζεύχθω ἡ ΑΕ, καὶ διήχθω ἡ ΒΑ ἐπὶ τὸ Ζ.

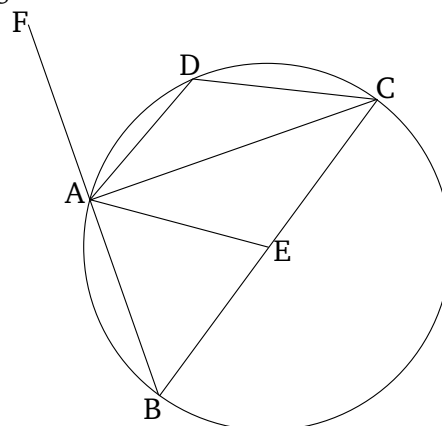
Καὶ ἐπεὶ ἴση ἐστίν ἡ ΒΕ τῇ ΕΑ, ἴση ἐστὶ καὶ γωνία ἢ ὑπὸ ΑΒΕ τῇ ὑπὸ ΒΑΕ. πάλιν, ἐπεὶ ἴση ἐστίν ἡ ΓΕ τῇ ΕΑ, ἴση ἐστὶ καὶ ἡ ὑπὸ ΑΓΕ τῇ ὑπὸ ΓΑΕ· ὅλη ἄρα ἡ ὑπὸ ΒΑΓ δυοὶ ταῖς ὑπὸ ΑΒΓ, ΑΓΒ ἴση ἐστίν. ἐστὶ δὲ καὶ ἡ ὑπὸ ΖΑΓ ἐκτὸς τοῦ ΑΒΓ τριγώνου δυοὶ ταῖς ὑπὸ ΑΒΓ, ΑΓΒ γωνίαις ἴση· ἴση ἄρα καὶ ἡ ὑπὸ ΒΑΓ γωνία τῇ ὑπὸ ΖΑΓ· ὀρθὴ ἄρα ἐκατέρω· ἡ ἄρα ἐν τῷ ΒΑΓ ἡμικυκλίῳ γωνία ἢ ὑπὸ ΒΑΓ ὀρθή ἐστίν.

Καὶ ἐπεὶ τοῦ ΑΒΓ τρίγωνου δύο γωνίαι αἱ ὑπὸ ΑΒΓ, ΒΑΓ δύο ὀρθῶν ἐλάττονές εἰσιν, ὀρθὴ δὲ ἡ ὑπὸ ΒΑΓ, ἐλάττων ἄρα ὀρθῆς ἐστίν ἡ ὑπὸ ΑΒΓ γωνία· καὶ ἐστίν ἐν τῷ ΑΒΓ μείζονι τοῦ ἡμικυκλίου τμήματι.

Καὶ ἐπεὶ ἐν κύκλῳ τετραπλευρόν ἐστὶ τὸ ΑΒΓΔ, τῶν δὲ ἐν τοῖς κύκλοις τετραπλευρῶν αἱ ἀπεναντίον γωνίαι δυοῖν ὀρθαῖς ἴσαι εἰσὶν [αἱ ἄρα ὑπὸ ΑΒΓ, ΑΔΓ γωνίαι δυοῖν ὀρθαῖς ἴσας εἰσὶν], καὶ ἐστίν ἡ ὑπὸ ΑΒΓ ἐλάττων ὀρθῆς· λοιπὴ ἄρα ἡ ὑπὸ ΑΔΓ γωνία μείζων ὀρθῆς ἐστίν· καὶ ἐστίν ἐν τῷ ΑΔΓ ἐλάττονι τοῦ ἡμικυκλίου τμήματι.

Λέγω, ὅτι καὶ ἡ μὲν τοῦ μείζονος τμήματος γωνία ἢ περιεχομένη ὑπὸ [τε] τῆς ΑΒΓ περιφερείας καὶ τῆς ΑΓ εὐθείας μείζων ἐστίν ὀρθῆς, ἡ δὲ τοῦ ἐλάττονος τμήματος γωνία ἢ περιεχομένη ὑπὸ [τε] τῆς ΑΔ[Γ] περιφερείας καὶ τῆς ΑΓ εὐθείας ἐλάττων ἐστίν ὀρθῆς. καὶ ἐστίν αὐτόθεν φανερόν. ἐπεὶ γὰρ ἡ ὑπὸ τῶν ΒΑ, ΑΓ εὐθειῶν ὀρθὴ ἐστίν, ἡ ἄρα ὑπὸ τῆς ΑΒΓ περιφερείας καὶ τῆς ΑΓ εὐθείας περιεχομένη μείζων ἐστίν ὀρθῆς. πάλιν, ἐπεὶ ἡ ὑπὸ τῶν ΑΓ, ΑΖ εὐθειῶν ὀρθὴ ἐστίν, ἡ ἄρα ὑπὸ τῆς ΓΑ εὐθείας καὶ τῆς ΑΔ[Γ] περι-

gle of a segment less (than a semi-circle) is less than a right-angle.



Let $ABCD$ be a circle, and let BC be its diameter, and E its center. And let $BA, AC, AD,$ and DC have been joined. I say that the angle BAC in the semi-circle BAC is a right-angle, and the angle ABC in the segment ABC , (which is) greater than a semi-circle, is less than a right-angle, and the angle ADC in the segment ADC , (which is) less than a semi-circle, is greater than a right-angle.

Let AE have been joined, and let BA have been drawn through to F .

And since BE is equal to EA , angle ABE is also equal to BAE [Prop. 1.5]. Again, since CE is equal to EA , ACE is also equal to CAE [Prop. 1.5]. Thus, the whole (angle) BAC is equal to the two (angles) ABC and ACB . And FAC , (which is) external to triangle ABC , is also equal to the two angles ABC and ACB [Prop. 1.32]. Thus, angle BAC (is) also equal to FAC . Thus, (they are) each right-angles. [Def. 1.10]. Thus, the angle BAC in the semi-circle BAC is a right-angle.

And since the two angles ABC and BAC of triangle ABC are less than two right-angles [Prop. 1.17], and BAC is a right-angle, angle ABC is thus less than a right-angle. And it is in segment ABC , (which is) greater than a semi-circle.

And since $ABCD$ is a quadrilateral within a circle, and for quadrilaterals within circles the (sum of the) opposite angles is equal to two right-angles [Prop. 3.22] [angles ABC and ADC are thus equal to two right-angles], and (angle) ABC is less than a right-angle. The remaining angle ADC is thus greater than a right-angle. And it is in segment ADC , (which is) less than a semi-circle.

I also say that the angle of the greater segment, (namely) that contained by the circumference ABC and the straight-line AC , is greater than a right-angle. And the angle of the lesser segment, (namely) that contained

φερείας περιεχομένη ἐλάττων ἐστὶν ὀρθῆς.

Ἐν κύκλῳ ἄρα ἡ μὲν ἐν τῷ ἡμικυκλίῳ γωνία ὀρθή ἐστίν, ἡ δὲ ἐν τῷ μείζονι τμήματι ἐλάττων ὀρθῆς, ἡ δὲ ἐν τῷ ἐλάττονι [τμήματι] μείζων ὀρθῆς· καὶ ἔπι ἡ μὲν τοῦ μείζονος τμήματος [γωνία] μείζων [ἐστὶν] ὀρθῆς, ἡ δὲ τοῦ ἐλάττονος τμήματος [γωνία] ἐλάττων ὀρθῆς· ὅπερ ἔδει δεῖξαι.

by the circumference $AD[C]$ and the straight-line AC , is less than a right-angle. And this is immediately apparent. For since the (angle contained by) the two straight-lines BA and AC is a right-angle, the (angle) contained by the circumference ABC and the straight-line AC is thus greater than a right-angle. Again, since the (angle contained by) the straight-lines AC and AF is a right-angle, the (angle) contained by the circumference $AD[C]$ and the straight-line CA is thus less than a right-angle.

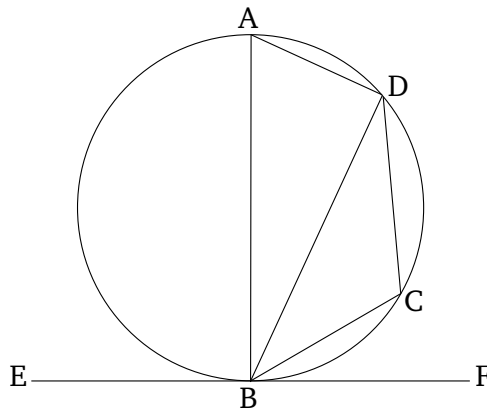
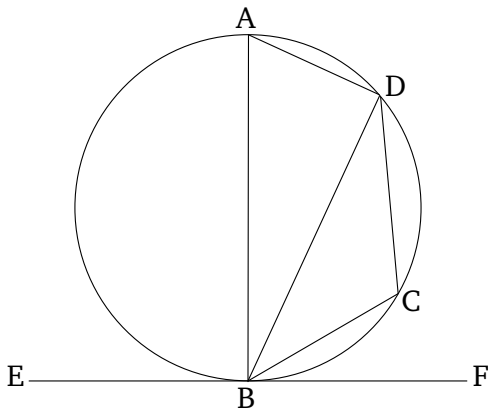
Thus, in a circle, the angle in a semi-circle is a right-angle, and that in a greater segment (is) less than a right-angle, and that in a lesser [segment] (is) greater than a right-angle. And, further, the [angle] of a segment greater (than a semi-circle) [is] greater than a right-angle, and the [angle] of a segment less (than a semi-circle) is less than a right-angle. (Which is) the very thing it was required to show.

λβ'.

Ἐὰν κύκλου ἐφάπτηται τις εὐθεῖα, ἀπὸ δὲ τῆς ἀφῆς εἰς τὸν κύκλον διαχθῆ τις εὐθεῖα τέμνουσα τὸν κύκλον, ἃς ποιῆ γωνίας πρὸς τῇ ἐφαπτομένῃ, ἴσαι ἔσονται ταῖς ἐν τοῖς ἐναλλάξ τοῦ κύκλου τμήμασι γωνίαις.

Proposition 32

If some straight-line touches a circle, and some (other) straight-line is drawn across, from the point of contact into the circle, cutting the circle (in two), then those angles the (straight-line) makes with the tangent will be equal to the angles in the alternate segments of the circle.



Κύκλου γὰρ τοῦ $ABΓΔ$ ἐφαπτέσθω τις εὐθεῖα ἡ EZ κατὰ τὸ B σημεῖον, καὶ ἀπὸ τοῦ B σημείου διήχθω τις εὐθεῖα εἰς τὸν $ABΓΔ$ κύκλον τέμνουσα αὐτὸν ἡ BD . λέγω, ὅτι ἃς ποιῆ γωνίας ἡ BD μετὰ τῆς EZ ἐφαπτομένης, ἴσας ἔσονται ταῖς ἐν τοῖς ἐναλλάξ τμήμασι τοῦ κύκλου γωνίαις, τουτέστιν, ὅτι ἡ μὲν ὑπὸ ZBD γωνία ἴση ἐστὶ τῇ ἐν τῷ $BAΔ$ τμήματι συνισταμένῃ γωνίᾳ, ἡ δὲ ὑπὸ EBD γωνία ἴση ἐστὶ τῇ ἐν τῷ $ΔΓB$ τμήματι συνισταμένῃ γωνίᾳ.

For let some straight-line EF touch the circle $ABCD$ at the point B , and let some (other) straight-line BD have been drawn from point B into the circle $ABCD$, cutting it (in two). I say that the angles BD makes with the tangent EF will be equal to the angles in the alternate segments of the circle. That is to say, that angle FBD is equal to the angle constructed in segment BAD , and angle EBD is equal to the angle constructed in segment DCB .

Ἦχθω γὰρ ἀπὸ τοῦ B τῇ EZ πρὸς ὀρθὰς ἡ BA , καὶ εἰλήφθω ἐπὶ τῆς BD περιφερείας τυχὸν σημεῖον τὸ $Γ$, καὶ ἐπεζεύχθωσαν αἱ AD , $ΔΓ$, $ΓB$.

For let BA have been drawn from B , at right-angles to EF [Prop. 1.11]. And let the point C have been taken at random on the circumference BD . And let AD , DC ,

Καὶ ἐπεὶ κύκλου τοῦ $ABΓΔ$ ἐφάπτεται τις εὐθεῖα ἡ EZ

κατὰ τὸ B, καὶ ἀπὸ τῆς ἀφῆς ἦκται τῇ ἐφαπτομένη πρὸς ὀρθὰς ἢ BA, ἐπὶ τῆς BA ἄρα τὸ κέντρον ἐστὶ τοῦ ABΓΔ κύκλου. ἢ BA ἄρα διάμετρος ἐστὶ τοῦ ABΓΔ κύκλου. ἢ ἄρα ὑπὸ AΔB γωνία ἐν ἡμικυκλίῳ οὕσα ὀρθὴ ἐστίν. λοιπαὶ ἄρα αἱ ὑπὸ BAΔ, ABΔ μιᾶ ὀρθῇ ἴσαι εἰσίν. ἐστὶ δὲ καὶ ἡ ὑπὸ ABZ ὀρθή· ἢ ἄρα ὑπὸ ABZ ἴση ἐστὶ ταῖς ὑπὸ BAΔ, ABΔ. κοινὴ ἀφῆρῆσθω ἢ ὑπὸ ABΔ· λοιπὴ ἄρα ἢ ὑπὸ ΔBZ γωνία ἴση ἐστὶ τῇ ἐν τῷ ἐναλλάξ τμήματι τοῦ κύκλου γωνία τῇ ὑπὸ BAΔ. καὶ ἐπεὶ ἐν κύκλῳ τετράπλευρόν ἐστι τὸ ABΓΔ, αἱ ἀπεναντίον αὐτοῦ γωναὶ δυσὶν ὀρθαῖς ἴσαι εἰσίν. εἰσὶ δὲ καὶ αἱ ὑπὸ ΔBZ, ΔBE δυσὶν ὀρθαῖς ἴσαι· αἱ ἄρα ὑπὸ ΔBZ, ΔBE ταῖς ὑπὸ BAΔ, BΓΔ ἴσαι εἰσίν, ὧν ἢ ὑπὸ BAΔ τῇ ὑπὸ ΔBZ ἐδείχθη ἴση· λοιπὴ ἄρα ἢ ὑπὸ ΔBE τῇ ἐν τῷ ἐναλλάξ τοῦ κύκλου τμήματι τῷ ΔΓB τῇ ὑπὸ ΔΓB γωνία ἐστὶν ἴση.

Ἐὰν ἄρα κύκλου ἐφάπτηται τις εὐθεῖα, ἀπὸ δὲ τῆς ἀφῆς εἰς τὸν κύκλον διαχθῆ τις εὐθεῖα τέμνουσα τὸν κύκλον, ἃς ποιῆ γωνίας πρὸς τῇ ἐφαπτομένη, ἴσαι ἔσονται ταῖς ἐν τοῖς ἐναλλάξ τοῦ κύκλου τμήμασι γωνίαις· ὅπερ ἔδει δεῖξαι.

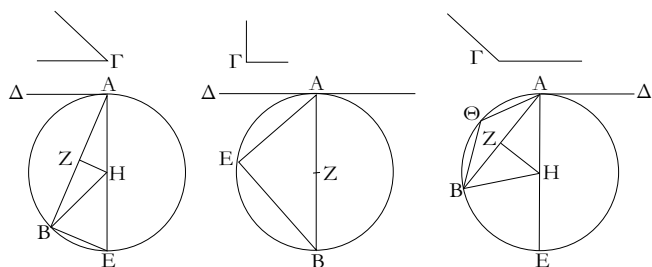
and CB have been joined.

And since some straight-line EF touches the circle $ABCD$ at point B , and BA has been drawn from the point of contact, at right-angles to the tangent, the center of circle $ABCD$ is thus on BA [Prop. 3.19]. Thus, BA is a diameter of circle $ABCD$. Thus, angle ADB , being in a semi-circle, is a right-angle [Prop. 3.31]. Thus, the remaining angles (of triangle ADB) BAD and ABD are equal to one right-angle [Prop. 1.32]. And ABF is also a right-angle. Thus, ABF is equal to BAD and ABD . Let ABD have been subtracted from both. Thus, the remaining angle DBF is equal to the angle BAD in the alternate segment of the circle. And since $ABCD$ is a quadrilateral in a circle, (the sum of) its opposite angles is equal to two right-angles [Prop. 3.22]. And DBF and DBE is also equal to two right-angles [Prop. 1.13]. Thus, DBF and DBE is equal to BAD and BCD , of which BAD was shown (to be) equal to DBF . Thus, the remaining (angle) DBE is equal to the angle DCB in the alternate segment DCB of the circle.

Thus, if some straight-line touches a circle, and some (other) straight-line is drawn across, from the point of contact into the circle, cutting the circle (in two), then those angles the (straight-line) makes with the tangent will be equal to the angles in the alternate segments of the circle. (Which is) the very thing it was required to show.

λγ'.

Ἐπὶ τῆς δοθείσης εὐθείας γράψαι τμήμα κύκλου δεχόμενον γωνίαν ἴσην τῇ δοθείσῃ γωνία εὐθυγράμμω.

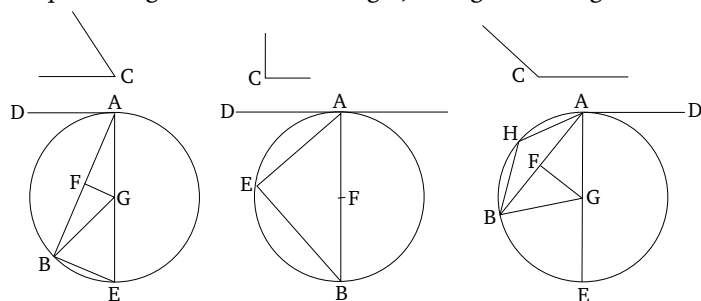


Ἐστω ἡ δοθεῖσα εὐθεῖα ἢ AB, ἢ δὲ δοθεῖσα γωνία εὐθύγραμμος ἢ πρὸς τῷ Γ· δεῖ δὴ ἐπὶ τῆς δοθείσης εὐθείας τῆς AB γράψαι τμήμα κύκλου δεχόμενον γωνίαν ἴσην τῇ πρὸς τῷ Γ.

Ἡ δὴ πρὸς τῷ Γ [γωνία] ἤτοι ὀξεῖα ἐστὶν ἢ ὀρθὴ ἢ ἀμβλεία· ἔστω πρότερον ὀξεῖα, καὶ ὡς ἐπὶ τῆς πρώτης καταγραφῆς συνεστάτω πρὸς τῇ AB εὐθείᾳ καὶ τῷ A σημείῳ τῇ πρὸς τῷ Γ γωνία ἴση ἢ ὑπὸ BAΔ· ὀξεῖα ἄρα ἐστὶ καὶ ἢ ὑπὸ BAΔ. ἤχθω τῇ ΔA πρὸς ὀρθὰς ἢ AE, καὶ τεμήσθω ἢ AB δίχα κατὰ τὸ Z, καὶ ἤχθω ἀπὸ τοῦ Z σημείου τῇ AB

Proposition 33

To draw a segment of a circle, accepting an angle equal to a given rectilinear angle, on a given straight-line.



Let AB be the given straight-line, and C the given rectilinear angle. So it is required to draw a segment of a circle, accepting an angle equal to C , on the given straight-line AB .

So the [angle] C is surely either acute, a right-angle, or obtuse. First of all, let it be acute. And, as in the first diagram (from the left), let (angle) BAD , equal to angle C , have been constructed on the straight-line AB , at the point A (on it) [Prop. 1.23]. Thus, BAD is also acute. Let AE have been drawn, at right-angles to DA [Prop. 1.11].

πρὸς ὀρθὰς ἢ ΖΗ, καὶ ἐπεζεύχθω ἢ ΗΒ.

Καὶ ἐπεὶ ἴση ἐστὶν ἢ ΑΖ τῆ ΖΒ, κοινὴ δὲ ἢ ΖΗ, δύο δὴ αἱ ΑΖ, ΖΗ δύο ταῖς ΒΖ, ΖΗ ἴσαι εἰσὶν· καὶ γωνία ἢ ὑπὸ ΑΖΗ [γωνία] τῆ ὑπὸ ΒΖΗ ἴση· βάσις ἄρα ἢ ΑΗ βάσει τῆ ΒΗ ἴση ἐστίν. ὁ ἄρα κέντρον μὲν τῶ Η διαστήματι δὲ τῶ ΗΑ κύκλος γραφόμενος ἤξει καὶ διὰ τοῦ Β. γεγράφθω καὶ ἔστω ὁ ΑΒΕ, καὶ ἐπεζεύχθω ἢ ΕΒ. ἐπεὶ οὖν ἀπ' ἄκρας τῆς ΑΕ διαμέτρου ἀπὸ τοῦ Α τῆ ΑΕ πρὸς ὀρθὰς ἐστὶν ἢ ΑΔ, ἢ ΑΔ ἄρα ἐφάπτεται τοῦ ΑΒΕ κύκλου· ἐπεὶ οὖν κύκλου τοῦ ΑΒΕ ἐφάπτεται τις εὐθεῖα ἢ ΑΔ, καὶ ἀπὸ τῆς κατὰ τὸ Α ἀφῆς εἰς τὸν ΑΒΕ κύκλον διήκται τις εὐθεῖα ἢ ΑΒ, ἢ ἄρα ὑπὸ ΔΑΒ γωνία ἴση ἐστὶ τῆ ἐν τῶ ἐναλλάξ τοῦ κύκλου τμήματι γωνία τῆ ὑπὸ ΑΕΒ. ἀλλ' ἢ ὑπὸ ΔΑΒ τῆ πρὸς τῶ Γ ἐστὶν ἴση· καὶ ἢ πρὸς τῶ Γ ἄρα γωνία ἴση ἐστὶ τῆ ὑπὸ ΑΕΒ.

Ἐπὶ τῆς δοθείσης ἄρα εὐθείας τῆς ΑΒ τμήμα κύκλου γέγραπται τὸ ΑΕΒ δεχόμενον γωνίαν τὴν ὑπὸ ΑΕΒ ἴσην τῆ δοθείση τῆ πρὸς τῶ Γ.

Ἄλλὰ δὴ ὀρθὴ ἔστω ἢ πρὸς τῶ Γ· καὶ δεόν πάλιν ἔστω ἐπὶ τῆς ΑΒ γράψαι τμήμα κύκλου δεχόμενον γωνίαν ἴσην τῆ πρὸς τῶ Γ ὀρθῆ [γωνία]. συνεστάτω [πάλιν] τῆ πρὸς τῶ Γ ὀρθῆ γωνία ἴση ἢ ὑπὸ ΒΑΔ, ὡς ἔχει ἐπὶ τῆς δευτέρας καταγραφῆς, καὶ τεμηθῆτω ἢ ΑΒ δίχα κατὰ τὸ Ζ, καὶ κέντρον τῶ Ζ, διαστήματι δὲ ὁποτέρω τῶν ΖΑ, ΖΒ, κύκλος γεγράφθω ὁ ΑΕΒ.

Ἐφάπτεται ἄρα ἢ ΑΔ εὐθεῖα τοῦ ΑΒΕ κύκλου διὰ τὸ ὀρθὴν εἶναι τὴν πρὸς τῶ Α γωνίαν. καὶ ἴση ἐστὶν ἢ ὑπὸ ΒΑΔ γωνία τῆ ἐν τῶ ΑΕΒ τμήματι· ὀρθὴ γὰρ καὶ αὐτὴ ἐν ἡμικυκλίῳ οὔσα. ἀλλὰ καὶ ἢ ὑπὸ ΒΑΔ τῆ πρὸς τῶ Γ ἴση ἐστίν. καὶ ἢ ἐν τῶ ΑΕΒ ἄρα ἴση ἐστὶ τῆ πρὸς τῶ Γ.

Γέγραπται ἄρα πάλιν ἐπὶ τῆς ΑΒ τμήμα κύκλου τὸ ΑΕΒ δεχόμενον γωνίαν ἴσην τῆ πρὸς τῶ Γ.

Ἄλλὰ δὴ ἢ πρὸς τῶ Γ ἀμβλεῖα ἔστω· καὶ συνεστάτω αὐτῆ ἴση πρὸς τῆ ΑΒ εὐθεῖα καὶ τῶ Α σημείω ἢ ὑπὸ ΒΑΔ, ὡς ἔχει ἐπὶ τῆς τρίτης καταγραφῆς, καὶ τῆ ΑΔ πρὸς ὀρθὰς ἤχθω ἢ ΑΕ, καὶ τεμηθῆτω πάλιν ἢ ΑΒ δίχα κατὰ τὸ Ζ, καὶ τῆ ΑΒ πρὸς ὀρθὰς ἤχθω ἢ ΖΗ, καὶ ἐπεζεύχθω ἢ ΗΒ.

Καὶ ἐπεὶ πάλιν ἴση ἐστὶν ἢ ΑΖ τῆ ΖΒ, καὶ κοινὴ ἢ ΖΗ, δύο δὴ αἱ ΑΖ, ΖΗ δύο ταῖς ΒΖ, ΖΗ ἴσαι εἰσὶν· καὶ γωνία ἢ ὑπὸ ΑΖΗ γωνία τῆ ὑπὸ ΒΖΗ ἴση· βάσις ἄρα ἢ ΑΗ βάσει τῆ ΒΗ ἴση ἐστίν· ὁ ἄρα κέντρον μὲν τῶ Η διαστήματι δὲ τῶ ΗΑ κύκλος γραφόμενος ἤξει καὶ διὰ τοῦ Β. ἐρχέσθω ὡς ὁ ΑΕΒ. καὶ ἐπεὶ τῆ ΑΕ διαμέτρου ἀπ' ἄκρας πρὸς ὀρθὰς ἐστὶν ἢ ΑΔ, ἢ ΑΔ ἄρα ἐφάπτεται τοῦ ΑΒΕ κύκλου. καὶ ἀπὸ τῆς κατὰ τὸ Α ἐπαφῆς διήκται ἢ ΑΒ· ἢ ἄρα ὑπὸ ΒΑΔ γωνία ἴση ἐστὶ τῆ ἐν τῶ ἐναλλάξ τοῦ κύκλου τμήματι τῶ ΑΘΒ συνισταμένη γωνία. ἀλλ' ἢ ὑπὸ ΒΑΔ γωνία τῆ πρὸς τῶ Γ ἴση ἐστίν. καὶ ἢ ἐν τῶ ΑΘΒ ἄρα τμήματι γωνία ἴση ἐστὶ τῆ πρὸς τῶ Γ.

Ἐπὶ τῆς ἄρα δοθείσης εὐθείας τῆς ΑΒ γέγραπται τμήμα κύκλου τὸ ΑΘΒ δεχόμενον γωνίαν ἴσην τῆ πρὸς τῶ Γ· ὅπερ ἔδει ποιῆσαι.

And let AB have been cut in half at F [Prop. 1.10]. And let FG have been drawn from point F , at right-angles to AB [Prop. 1.11]. And let GB have been joined.

And since AF is equal to FB , and FG (is) common, the two (straight-lines) AF , FG are equal to the two (straight-lines) BF , FG (respectively). And angle AFG (is) equal to [angle] BFG . Thus, the base AG is equal to the base BG [Prop. 1.4]. Thus, the circle drawn with center G , and radius GA , will also go through B (as well as A). Let it have been drawn, and let it be (denoted) ABE . And let EB have been joined. Therefore, since AD is at the extremity of diameter AE , (namely, point) A , at right-angles to AE , the (straight-line) AD thus touches the circle ABE [Prop. 3.16 corr.]. Therefore, since some straight-line AD touches the circle ABE , and some (other) straight-line AB has been drawn across from the point of contact A into circle ABE , angle DAB is thus equal to the angle AEB in the alternate segment of the circle [Prop. 3.32]. But, DAB is equal to C . Thus, angle C is also equal to AEB .

Thus, a segment AEB of a circle, accepting the angle AEB (which is) equal to the given (angle) C , has been drawn on the given straight-line AB .

And so let C be a right-angle. And let it again be necessary to draw a segment of a circle on AB , accepting an angle equal to the right-[angle] C . Let the (angle) BAD [again] have been constructed, equal to the right-angle C [Prop. 1.23], as in the second diagram (from the left). And let AB have been cut in half at F [Prop. 1.10]. And let the circle AEB have been drawn with center F , and radius either FA or FB .

Thus, the straight-line AD touches the circle ABE , on account of the angle at A being a right-angle [Prop. 3.16 corr.]. And angle BAD is equal to the angle in segment AEB . For (the latter angle), being in a semi-circle, is also a right-angle [Prop. 3.31]. But, BAD is also equal to C . Thus, the (angle) in (segment) AEB is also equal to C .

Thus, a segment AEB of a circle, accepting an angle equal to C , has again been drawn on AB .

And so let (angle) C be obtuse. And let (angle) BAD , equal to (C), have been constructed on the straight-line AB , at the point A (on it) [Prop. 1.23], as in the third diagram (from the left). And let AE have been drawn, at right-angles to AD [Prop. 1.11]. And let AB have again been cut in half at F [Prop. 1.10]. And let FG have been drawn, at right-angles to AB [Prop. 1.10]. And let GB have been joined.

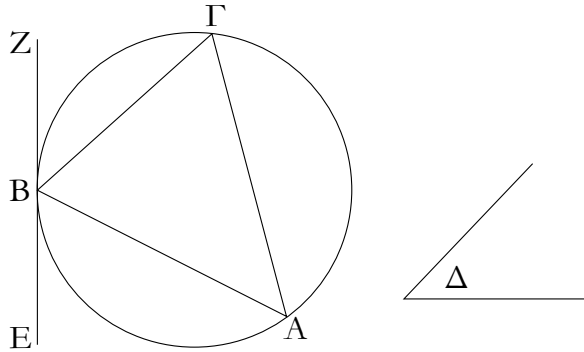
And again, since AF is equal to FB , and FG (is) common, the two (straight-lines) AF , FG are equal to the two (straight-lines) BF , FG (respectively). And angle AFG (is) equal to angle BFG . Thus, the base AG is

equal to the base BG [Prop. 1.4]. Thus, a circle of center G , and radius GA , being drawn, will also go through B (as well as A). Let it go like AEB (in the third diagram from the left). And since AD is at right-angles to the diameter AE , at its extremity, AD thus touches circle AEB [Prop. 3.16 corr.]. And AB has been drawn across (the circle) from the point of contact A . Thus, angle BAD is equal to the angle constructed in the alternate segment AHB of the circle [Prop. 3.32]. But, angle BAD is equal to C . Thus, the angle in segment AHB is also equal to C .

Thus, a segment AHB of a circle, accepting an angle equal to C , has been drawn on the given straight-line AB . (Which is) the very thing it was required to do.

λδ'.

Ἄπο τοῦ δοθέντος κύκλου τμήμα ἀφελεῖν δεχόμενον γωνίαν ἴσην τῇ δοθείσῃ γωνίᾳ εὐθύγραμμῳ.



Ἐστω ὁ δοθεὶς κύκλος ὁ $AB\Gamma$, ἡ δὲ δοθεῖσα γωνία εὐθύγραμμος ἡ πρὸς τῷ Δ . δεῖ δὴ ἀπὸ τοῦ $AB\Gamma$ κύκλου τμήμα ἀφελεῖν δεχόμενον γωνίαν ἴσην τῇ δοθείσῃ γωνίᾳ εὐθύγραμμῳ τῇ πρὸς τῷ Δ .

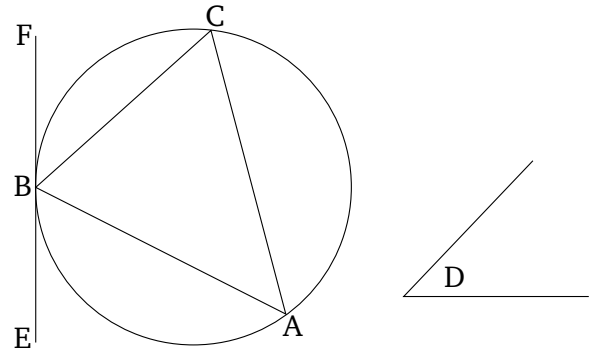
Ἦχθω τοῦ $AB\Gamma$ ἐφαπτομένη ἡ EZ κατὰ τὸ B σημεῖον, καὶ συνεστάτω πρὸς τῇ ZB εὐθείᾳ καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ B τῇ πρὸς τῷ Δ γωνίᾳ ἴση ἡ ὑπὸ $ZB\Gamma$.

Ἐπεὶ οὖν κύκλου τοῦ $AB\Gamma$ ἐφάπτεται τις εὐθεῖα ἡ EZ , καὶ ἀπὸ τῆς κατὰ τὸ B ἐπαφῆς διῆκται ἡ $B\Gamma$, ἡ ὑπὸ $ZB\Gamma$ ἄρα γωνία ἴση ἐστὶ τῇ ἐν τῷ $BA\Gamma$ ἐναλλάξ τμήματι συνισταμένῃ γωνίᾳ. ἀλλ' ἡ ὑπὸ $ZB\Gamma$ τῇ πρὸς τῷ Δ ἐστὶν ἴση· καὶ ἡ ἐν τῷ $BA\Gamma$ ἄρα τμήματι ἴση ἐστὶ τῇ πρὸς τῷ Δ [γωνίᾳ].

Ἄπο τοῦ δοθέντος ἄρα κύκλου τοῦ $AB\Gamma$ τμήμα ἀφῆρηται τὸ $BA\Gamma$ δεχόμενον γωνίαν ἴσην τῇ δοθείσῃ γωνίᾳ εὐθύγραμμῳ τῇ πρὸς τῷ Δ : ὅπερ ἔδει ποιῆσαι.

Proposition 34

To cut off a segment, accepting an angle equal to a given rectilinear angle, from a given circle.



Let ABC be the given circle, and D the given rectilinear angle. So it is required to cut off a segment, accepting an angle equal to the given rectilinear angle D , from the given circle ABC .

Let EF have been drawn touching ABC at point B .[†] And let (angle) FBC , equal to angle D , have been constructed on the straight-line FB , at the point B on it [Prop. 1.23].

Therefore, since some straight-line EF touches the circle ABC , and BC has been drawn across (the circle) from the point of contact B , angle FBC is thus equal to the angle constructed in the alternate segment BAC [Prop. 1.32]. But, FBC is equal to D . Thus, the (angle) in the segment BAC is also equal to [angle] D .

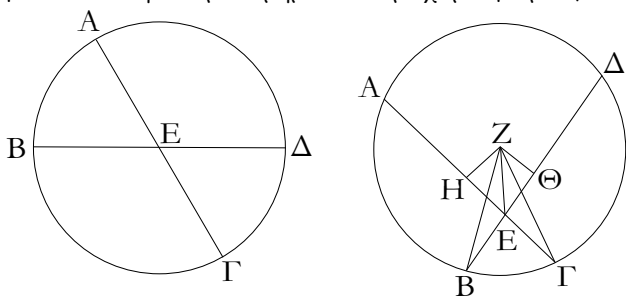
Thus, the segment BAC , accepting an angle equal to the given rectilinear angle D , has been cut off from the given circle ABC . (Which is) the very thing it was required to do.

[†] Presumably, by finding the center of ABC [Prop. 3.1], drawing a straight-line between the center and point B , and then drawing EF through

point B , at right-angles to the aforementioned straight-line [Prop. 1.11].

λε'.

Ἐάν ἐν κύκλῳ δύο εὐθεῖαι τέμνωσιν ἀλλήλας, τὸ ὑπὸ τῶν τῆς μιᾶς τμημάτων περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ὑπὸ τῶν τῆς ἐτέρας τμημάτων περιεχομένῳ ὀρθογωνίῳ.



Ἐν γὰρ κύκλῳ τῷ $ABΓΔ$ δύο εὐθεῖαι αἱ $ΑΓ$, $ΒΔ$ τεμνέτωσαν ἀλλήλας κατὰ τὸ E σημεῖον· λέγω, ὅτι τὸ ὑπὸ τῶν $ΑΕ$, $ΕΓ$ περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ὑπὸ τῶν $ΔΕ$, $ΕΒ$ περιεχομένῳ ὀρθογωνίῳ.

Εἰ μὲν οὖν αἱ $ΑΓ$, $ΒΔ$ διὰ τοῦ κέντρου εἰσὶν ὥστε τὸ E κέντρον εἶναι τοῦ $ΑΒΓΔ$ κύκλου, φανερόν, ὅτι ἴσων οὐσῶν τῶν $ΑΕ$, $ΕΓ$, $ΔΕ$, $ΕΒ$ καὶ τὸ ὑπὸ τῶν $ΑΕ$, $ΕΓ$ περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ὑπὸ τῶν $ΔΕ$, $ΕΒ$ περιεχομένῳ ὀρθογωνίῳ.

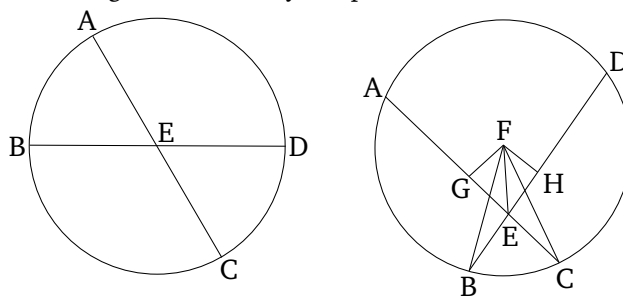
Μὴ ἔστωσαν δὴ αἱ $ΑΓ$, $ΔΒ$ διὰ τοῦ κέντρου, καὶ εἰλήφθω τὸ κέντρον τοῦ $ΑΒΓΔ$, καὶ ἔστω τὸ Z , καὶ ἀπὸ τοῦ Z ἐπὶ τὰς $ΑΓ$, $ΔΒ$ εὐθείας κάθεται ἤχθωσαν αἱ ZH , $ZΘ$, καὶ ἐπεζεύχθωσαν αἱ ZB , $ZΓ$, ZE .

Καὶ ἐπεὶ εὐθεῖά τις διὰ τοῦ κέντρου ἢ HZ εὐθειάν τινα μὴ διὰ τοῦ κέντρου τὴν $ΑΓ$ πρὸς ὀρθὰς τέμνει, καὶ δίχα αὐτὴν τέμνει· ἴση ἄρα ἢ AH τῇ $HΓ$. ἐπεὶ οὖν εὐθεῖα ἢ $ΑΓ$ τέτμηται εἰς μὲν ἴσα κατὰ τὸ H , εἰς δὲ ἄνισα κατὰ τὸ E , τὸ ἄρα ὑπὸ τῶν $ΑΕ$, $ΕΓ$ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς $ΕH$ τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς $HΓ$. [κοινὸν] προσκείσθω τὸ ἀπὸ τῆς HZ : τὸ ἄρα ὑπὸ τῶν $ΑΕ$, $ΕΓ$ μετὰ τῶν ἀπὸ τῶν HE , HZ ἴσον ἐστὶ τοῖς ἀπὸ τῶν $ΓH$, HZ . ἀλλὰ τοῖς μὲν ἀπὸ τῶν EH , HZ ἴσον ἐστὶ τὸ ἀπὸ τῆς ZE , τοῖς δὲ ἀπὸ τῶν $ΓH$, HZ ἴσον ἐστὶ τὸ ἀπὸ τῆς $ZΓ$: τὸ ἄρα ὑπὸ τῶν $ΑΕ$, $ΕΓ$ μετὰ τοῦ ἀπὸ τῆς ZE ἴσον ἐστὶ τῷ ἀπὸ τῆς $ZΓ$. ἴση δὲ ἢ $ZΓ$ τῇ ZB : τὸ ἄρα ὑπὸ τῶν $ΑΕ$, $ΕΓ$ μετὰ τοῦ ἀπὸ τῆς EZ ἴσον ἐστὶ τῷ ἀπὸ τῆς ZB . διὰ τὰ αὐτὰ δὴ καὶ τὸ ὑπὸ τῶν $ΔΕ$, $ΕΒ$ μετὰ τοῦ ἀπὸ τῆς ZE ἴσον ἐστὶ τῷ ἀπὸ τῆς ZB . ἐδείχθη δὲ καὶ τὸ ὑπὸ τῶν $ΑΕ$, $ΕΓ$ μετὰ τοῦ ἀπὸ τῆς ZE ἴσον τῷ ἀπὸ τῆς ZB : τὸ ἄρα ὑπὸ τῶν $ΑΕ$, $ΕΓ$ μετὰ τοῦ ἀπὸ τῆς ZE ἴσον ἐστὶ τῷ ὑπὸ τῶν $ΔΕ$, $ΕΒ$ μετὰ τοῦ ἀπὸ τῆς ZE . κοινὸν ἀφῆρήσθω τὸ ἀπὸ τῆς ZE : λοιπὸν ἄρα τὸ ὑπὸ τῶν $ΑΕ$, $ΕΓ$ περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ὑπὸ τῶν $ΔΕ$, $ΕΒ$ περιεχομένῳ ὀρθογωνίῳ.

Ἐάν ἄρα ἐν κύκλῳ εὐθεῖαι δύο τέμνωσιν ἀλλήλας, τὸ ὑπὸ τῶν τῆς μιᾶς τμημάτων περιεχόμενον ὀρθογώνιον ἴσον

Proposition 35

If two straight-lines in a circle cut one another then the rectangle contained by the pieces of one is equal to the rectangle contained by the pieces of the other.



For let the two straight-lines AC and BD , in the circle $ABCD$, cut one another at point E . I say that the rectangle contained by AE and EC is equal to the rectangle contained by DE and EB .

In fact, if AC and BD are through the center (as in the first diagram from the left), so that E is the center of circle $ABCD$, then (it is) clear that, AE , EC , DE , and EB being equal, the rectangle contained by AE and EC is also equal to the rectangle contained by DE and EB .

So let AC and DB not be though the center (as in the second diagram from the left), and let the center of $ABCD$ have been found [Prop. 3.1], and let it be (at) F . And let FG and FH have been drawn from F , perpendicular to the straight-lines AC and DB (respectively) [Prop. 1.12]. And let FB , FC , and FE have been joined.

And since some straight-line, GF , through the center, cuts at right-angles some (other) straight-line, AC , not through the center, then it also cuts it in half [Prop. 3.3]. Thus, AG (is) equal to GC . Therefore, since the straight-line AC is cut equally at G , and unequally at E , the rectangle contained by AE and EC plus the square on EG is thus equal to the (square) on GC [Prop. 2.5]. Let the (square) on GF have been added [to both]. Thus, the (rectangle contained) by AE and EC plus the (sum of the squares) on GE and GF is equal to the (sum of the squares) on CG and GF . But, the (square) on FE is equal to the (sum of the squares) on EG and GF [Prop. 1.47], and the (square) on FC is equal to the (sum of the squares) on CG and GF [Prop. 1.47]. Thus, the (rectangle contained) by AE and EC plus the (square) on FE is equal to the (square) on FC . And FC (is) equal to FB . Thus, the (rectangle contained) by AE and EC plus the (square) on FE is equal to the (square) on FB . So, for the same (reasons), the (rectangle contained) by DE and EB plus the (square) on FE is equal

ἐστὶ τῶ ὑπὸ τῶν τῆς ἐτέρας τμημάτων περιεχομένῳ ὀρθογώνῳ· ὅπερ ἔδει δεῖξαι.

to the (square) on FB . And the (rectangle contained) by AE and EC plus the (square) on FE was also shown (to be) equal to the (square) on FB . Thus, the (rectangle contained) by AE and EC plus the (square) on FE is equal to the (rectangle contained) by DE and EB plus the (square) on FE . Let the (square) on FE have been taken from both. Thus, the remaining rectangle contained by AE and EC is equal to the rectangle contained by DE and EB .

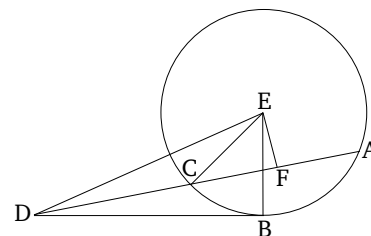
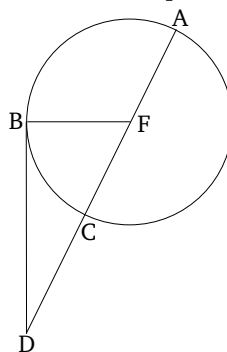
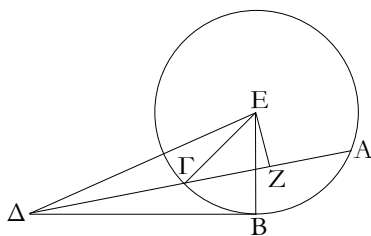
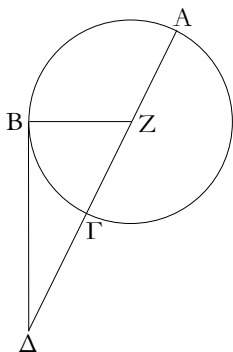
Thus, if two straight-lines in a circle cut one another then the rectangle contained by the pieces of one is equal to the rectangle contained by the pieces of the other. (Which is) the very thing it was required to show.

λζ'.

Proposition 36

Ἐὰν κύκλου ληφθῆ τι σημεῖον ἐκτός, καὶ ἀπ' αὐτοῦ πρὸς τὸν κύκλον προσπίπτωσι δύο εὐθεῖαι, καὶ ἡ μὲν αὐτῶν τέμνη τὸν κύκλον, ἡ δὲ ἐφάπτηται, ἔσται τὸ ὑπὸ ὅλης τῆς τεμνούσης καὶ τῆς ἐκτός ἀπολαμβανομένης μεταξὺ τοῦ τε σημείου καὶ τῆς κυρτῆς περιφερείας ἴσον τῶ ἀπὸ τῆς ἐφαπτομένης τετραγώνῳ.

If some point is taken outside a circle, and two straight-lines radiate from it towards the circle, and (one) of them cuts the circle, and the (other) touches (it), then the (rectangle contained) by the whole (straight-line) cutting (the circle), and the (part of it) cut off outside (the circle), between the point and the convex circumference, will be equal to the square on the tangent (line).



Κύκλου γὰρ τοῦ $ABΓ$ εἰλήφθω τι σημεῖον ἐκτός τὸ $Δ$, καὶ ἀπὸ τοῦ $Δ$ πρὸς τὸν $ABΓ$ κύκλον προσπίπτωσαν δύο εὐθεῖαι αἱ $ΔΓ[A]$, $ΔB$ · καὶ ἡ μὲν $ΔΓA$ τεμνέτω τὸν $ABΓ$ κύκλον, ἡ δὲ $BΔ$ ἐφαπτέσθω· λέγω, ὅτι τὸ ὑπὸ τῶν $AΔ$, $ΔΓ$ περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῶ ἀπὸ τῆς $ΔB$ τετραγώνῳ.

For let some point D have been taken outside circle ABC , and let two straight-lines, $DC[A]$ and DB , radiate from D towards circle ABC . And let DCA cut circle ABC , and let BD touch (it). I say that the rectangle contained by AD and DC is equal to the square on DB .

Ἡ ἄρα $[Δ]ΓA$ ἤτοι διὰ τοῦ κέντρου ἐστὶν ἢ οὐ· ἔστω πρότερον διὰ τοῦ κέντρου, καὶ ἔστω τὸ Z κέντρον τοῦ $ABΓ$ κύκλου, καὶ ἐπεζεύχθω ἡ ZB · ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ $ZBΔ$. καὶ ἐπεὶ εὐθεῖα ἡ $AΓ$ δίχα τέτμηται κατὰ τὸ Z , πρόσκειται δὲ αὐτῇ ἡ $ΓΔ$, τὸ ἄρα ὑπὸ τῶν $AΔ$, $ΔΓ$ μετὰ τοῦ ἀπὸ τῆς $ZΓ$ ἴσον ἐστὶ τῶ ἀπὸ τῆς $ZΔ$. ἴση δὲ ἡ $ZΓ$ τῇ ZB · τὸ ἄρα ὑπὸ τῶν $AΔ$, $ΔΓ$ μετὰ τοῦ ἀπὸ τῆς ZB ἴσον ἐστὶ τῶ ἀπὸ τῆς $ZΔ$. τῶ δὲ ἀπὸ τῆς $ZΔ$ ἴσα ἐστὶ τὰ ἀπὸ τῶν ZB , $BΔ$ · τὸ ἄρα ὑπὸ τῶν $AΔ$, $ΔΓ$ μετὰ τοῦ ἀπὸ τῆς ZB ἴσον ἐστὶ τοῖς ἀπὸ τῶν ZB , $BΔ$. κοινὸν ἀφηρήσθω τὸ ἀπὸ τῆς ZB · λοιπὸν ἄρα τὸ ὑπὸ τῶν $AΔ$, $ΔΓ$ ἴσον ἐστὶ τῶ ἀπὸ τῆς $ΔB$

$[D]CA$ is surely either through the center, or not. Let it first of all be through the center, and let F be the center of circle ABC , and let FB have been joined. Thus, (angle) FBD is a right-angle [Prop. 3.18]. And since straight-line AC is cut in half at F , let CD have been added to it. Thus, the (rectangle contained) by AD and DC plus the (square) on FC is equal to the (square) on FD [Prop. 2.6]. And FC (is) equal to FB . Thus, the (rectangle contained) by AD and DC plus the (square) on FB is equal to the (square) on FD . And the (square) on FD is equal to the (sum of the squares) on FB and BD [Prop. 1.47]. Thus, the (rectangle contained) by AD

ἐφαπτομένης.

Ἀλλὰ δὴ ἡ $\Delta\Gamma\Lambda$ μὴ ἔστω διὰ τοῦ κέντρου τοῦ $AB\Gamma$ κύκλου, καὶ εἰλήφθω τὸ κέντρον τὸ E , καὶ ἀπὸ τοῦ E ἐπὶ τὴν AG κάθετος ἤχθω ἡ EZ , καὶ ἐπεξεύχθωσαν αἱ EB , EG , ED . ὀρθὴ ἄρα ἐστὶν ἡ ὑπὸ $EB\Delta$. καὶ ἐπεὶ εὐθείαι τις διὰ τοῦ κέντρου ἡ EZ εὐθειάν τινα μὴ διὰ τοῦ κέντρου τὴν AG πρὸς ὀρθὰς τέμνει, καὶ δίχα αὐτὴν τέμνει· ἡ AZ ἄρα τῆς $Z\Gamma$ ἐστὶν ἴση. καὶ ἐπεὶ εὐθεῖα ἡ AG τέτμηται δίχα κατὰ τὸ Z σημεῖον, πρόσκειται δὲ αὐτῇ ἡ $\Gamma\Delta$, τὸ ἄρα ὑπὸ τῶν $A\Delta$, $\Delta\Gamma$ μετὰ τοῦ ἀπὸ τῆς $Z\Gamma$ ἴσον ἐστὶ τῷ ἀπὸ τῆς $Z\Delta$. κοινὸν προσκείσθω τὸ ἀπὸ τῆς ZE · τὸ ἄρα ὑπὸ τῶν $A\Delta$, $\Delta\Gamma$ μετὰ τῶν ἀπὸ τῶν ΓZ , ZE ἴσον ἐστὶ τοῖς ἀπὸ τῶν $Z\Delta$, ZE . τοῖς δὲ ἀπὸ τῶν ΓZ , ZE ἴσον ἐστὶ τὸ ἀπὸ τῆς EG . ὀρθὴ γὰρ [ἐστὶν] ἡ ὑπὸ $EZ\Gamma$ [γωνία]· τοῖς δὲ ἀπὸ τῶν ΔZ , ZE ἴσον ἐστὶ τὸ ἀπὸ τῆς ED . τὸ ἄρα ὑπὸ τῶν $A\Delta$, $\Delta\Gamma$ μετὰ τοῦ ἀπὸ τῆς EG ἴσον ἐστὶ τῷ ἀπὸ τῆς ED . ἴση δὲ ἡ EG τῇ EB · τὸ ἄρα ὑπὸ τῶν $A\Delta$, $\Delta\Gamma$ μετὰ τοῦ ἀπὸ τῆς EB ἴσον ἐστὶ τῷ ἀπὸ τῆς ED . τῷ δὲ ἀπὸ τῆς ED ἴσα ἐστὶ τὰ ἀπὸ τῶν EB , $B\Delta$. ὀρθὴ γὰρ ἡ ὑπὸ $EB\Delta$ γωνία· τὸ ἄρα ὑπὸ τῶν $A\Delta$, $\Delta\Gamma$ μετὰ τοῦ ἀπὸ τῆς EB ἴσον ἐστὶ τοῖς ἀπὸ τῶν EB , $B\Delta$. κοινὸν ἀφηρήσθω τὸ ἀπὸ τῆς EB · λοιπὸν ἄρα τὸ ὑπὸ τῶν $A\Delta$, $\Delta\Gamma$ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΔB .

Ἐὰν ἄρα κύκλου ληφθῇ τι σημεῖον ἐκτός, καὶ ἀπ' αὐτοῦ πρὸς τὸν κύκλον προσπίπτωσι δύο εὐθεῖαι, καὶ ἡ μὲν αὐτῶν τέμνη τὸν κύκλον, ἡ δὲ ἐφάπτηται, ἔσται τὸ ὑπὸ ὅλης τῆς τεμνούσης καὶ τῆς ἐκτός ἀπολαμβανομένης μεταξὺ τοῦ τε σημείου καὶ τῆς κυρτῆς περιφερείας ἴσον τῷ ἀπὸ τῆς ἐφαπτομένης τετραγώνῳ· ὅπερ ἔδει δεῖξαι.

λζ'.

Ἐὰν κύκλου ληφθῇ τι σημεῖον ἐκτός, ἀπὸ δὲ τοῦ σημείου πρὸς τὸν κύκλον προσπίπτωσι δύο εὐθεῖαι, καὶ ἡ μὲν αὐτῶν τέμνη τὸν κύκλον, ἡ δὲ προσπίπτῃ, ἡ δὲ τὸ ὑπὸ [τῆς] ὅλης τῆς τεμνούσης καὶ τῆς ἐκτός ἀπολαμβα-

and DC plus the (square) on FB is equal to the (sum of the squares) on FB and BD . Let the (square) on FB have been subtracted from both. Thus, the remaining (rectangle contained) by AD and DC is equal to the (square) on the tangent DB .

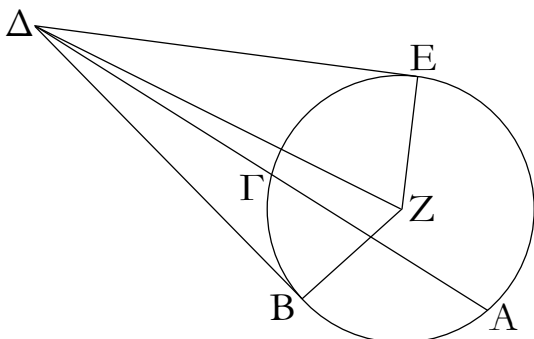
And so let DCA not be through the center of circle ABC , and let the center E have been found, and let EF have been drawn from E , perpendicular to AC [Prop. 1.12]. And let EB , EC , and ED have been joined. (Angle) EBD (is) thus a right-angle [Prop. 3.18]. And since some straight-line, EF , through the center, cuts some (other) straight-line, AC , not through the center, at right-angles, it also cuts it in half [Prop. 3.3]. Thus, AF is equal to FC . And since the straight-line AC is cut in half at point F , let CD have been added to it. Thus, the (rectangle contained) by AD and DC plus the (square) on FC is equal to the (square) on FD [Prop. 2.6]. Let the (square) on FE have been added to both. Thus, the (rectangle contained) by AD and DC plus the (sum of the squares) on CF and FE is equal to the (sum of the squares) on FD and FE . But the (square) on EC is equal to the (sum of the squares) on CF and FE . For [angle] EFC [is] a right-angle [Prop. 1.47]. And the (square) on ED is equal to the (sum of the squares) on DF and FE [Prop. 1.47]. Thus, the (rectangle contained) by AD and DC plus the (square) on EC is equal to the (square) on ED . And EC (is) equal to EB . Thus, the (rectangle contained) by AD and DC plus the (square) on EB is equal to the (square) on ED . And the (sum of the squares) on EB and BD is equal to the (square) on ED . For EBD (is) a right-angle [Prop. 1.47]. Thus, the (rectangle contained) by AD and DC plus the (square) on EB is equal to the (sum of the squares) on EB and BD . Let the (square) on EB have been subtracted from both. Thus, the remaining (rectangle contained) by AD and DC is equal to the (square) on BD .

Thus, if some point is taken outside a circle, and two straight-lines radiate from it towards the circle, and (one) of them cuts the circle, and (the other) touches (it), then the (rectangle contained) by the whole (straight-line) cutting (the circle), and the (part of it) cut off outside (the circle), between the point and the convex circumference, will be equal to the square on the tangent (line). (Which is) the very thing it was required to show.

Proposition 37

If some point is taken outside a circle, and two straight-lines radiate from the point towards the circle, and one of them cuts the circle, and the (other) meets (it), and the (rectangle contained) by the whole (straight-

νομένης μεταξύ τοῦ τε σημείου καὶ τῆς κυρτῆς περιφερείας ἴσον τῶ ἀπὸ τῆς προσπιπτούσης, ἢ προσπίπτουσα ἐφάπεται τοῦ κύκλου.

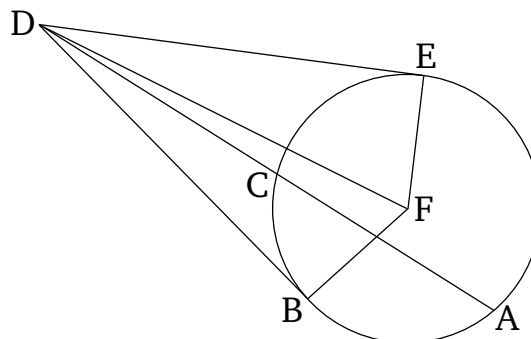


Κύκλου γὰρ τοῦ $ABΓ$ εἰλήφθω τι σημεῖον ἐκτός τὸ Δ , καὶ ἀπὸ τοῦ Δ πρὸς τὸν $ABΓ$ κύκλον προσπιπέτωσαν δύο εὐθεῖαι αἱ $\Delta ΓΑ$, $\Delta Β$, καὶ ἡ μὲν $\Delta ΓΑ$ τεμνέτω τὸν κύκλον, ἢ δὲ $\Delta Β$ προσπιπέτω, ἔστω δὲ τὸ ὑπὸ τῶν $A\Delta$, $\Delta Γ$ ἴσον τῶ ἀπὸ τῆς $\Delta Β$. λέγω, ὅτι ἡ $\Delta Β$ ἐφάπτεται τοῦ $ABΓ$ κύκλου.

Ἦχθω γὰρ τοῦ $ABΓ$ ἐφαπτομένη ἡ $\Delta Ε$, καὶ εἰλήφθω τὸ κέντρον τοῦ $ABΓ$ κύκλου, καὶ ἔστω τὸ Z , καὶ ἐπεζεύχθωσαν αἱ $ZΕ$, $ZΒ$, $Z\Delta$. ἡ ἄρα ὑπὸ $ZΕ\Delta$ ὀρθὴ ἐστίν. καὶ ἐπεὶ ἡ $\Delta Ε$ ἐφάπτεται τοῦ $ABΓ$ κύκλου, τέμνει δὲ ἡ $\Delta ΓΑ$, τὸ ἄρα ὑπὸ τῶν $A\Delta$, $\Delta Γ$ ἴσον ἐστὶ τῶ ἀπὸ τῆς $\Delta Ε$. ἦν δὲ καὶ τὸ ὑπὸ τῶν $A\Delta$, $\Delta Γ$ ἴσον τῶ ἀπὸ τῆς $\Delta Β$. τὸ ἄρα ἀπὸ τῆς $\Delta Ε$ ἴσον ἐστὶ τῶ ἀπὸ τῆς $\Delta Β$. ἴση ἄρα ἡ $\Delta Ε$ τῇ $\Delta Β$. ἐστὶ δὲ καὶ ἡ $ZΕ$ τῇ $ZΒ$ ἴση· δύο δὲ αἱ $\Delta Ε$, $EΖ$ δύο ταῖς $\Delta Β$, $BΖ$ ἴσαι εἰσίν· καὶ βάσις αὐτῶν κοινὴ ἡ $Z\Delta$. γωνία ἄρα ἡ ὑπὸ $\Delta ΕΖ$ γωνία τῇ ὑπὸ $\Delta ΒΖ$ ἐστὶν ἴση. ὀρθὴ δὲ ἡ ὑπὸ $\Delta ΕΖ$ ὀρθὴ ἄρα καὶ ἡ ὑπὸ $\Delta ΒΖ$. καὶ ἐστὶν ἡ $ZΒ$ ἐκβαλλομένη διάμετρος· ἡ δὲ τῇ διαμέτρῳ τοῦ κύκλου πρὸς ὀρθὰς ἀπ' ἄκρας ἀγομένη ἐφάπτεται τοῦ κύκλου· ἡ $\Delta Β$ ἄρα ἐφάπτεται τοῦ $ABΓ$ κύκλου. ὁμοίως δὲ δειχθήσεται, ἂν τὸ κέντρον ἐπὶ τῆς $AΓ$ τυγχάνη.

Ἐὰν ἄρα κύκλου ληφθῇ τι σημεῖον ἐκτός, ἀπὸ δὲ τοῦ σημείου πρὸς τὸν κύκλον προσπίπτωσι δύο εὐθεῖαι, καὶ ἡ μὲν αὐτῶν τέμνη τὸν κύκλον, ἢ δὲ προσπίπτῃ, ἢ δὲ τὸ ὑπὸ ὅλης τῆς τεμνούσης καὶ τῆς ἐκτός ἀπολαμβανομένης μεταξύ τοῦ τε σημείου καὶ τῆς κυρτῆς περιφερείας ἴσον τῶ ἀπὸ τῆς προσπιπτούσης, ἢ προσπίπτουσα ἐφάπεται τοῦ κύκλου· ὅπερ ἔδει δεῖξαι.

line) cutting (the circle), and the (part of it) cut off outside (the circle), between the point and the convex circumference, is equal to the (square) on the (straight-line) meeting (the circle), then the (straight-line) meeting (the circle) will touch the circle.



For let some point D have been taken outside circle ABC , and let two straight-lines, DCA and DB , radiate from D towards circle ABC , and let DCA cut the circle, and let DB meet (the circle). And let the (rectangle contained) by AD and DC be equal to the (square) on DB . I say that DB touches circle ABC .

For let DE have been drawn touching ABC [Prop. 3.17], and let the center of the circle ABC have been found, and let it be (at) F . And let FE , FB , and FD have been joined. (Angle) FED is thus a right-angle [Prop. 3.18]. And since DE touches circle ABC , and DCA cuts (it), the (rectangle contained) by AD and DC is thus equal to the (square) on DE [Prop. 3.36]. And the (rectangle contained) by AD and DC was also equal to the (square) on DB . Thus, the (square) on DE is equal to the (square) on DB . Thus, DE (is) equal to DB . And FE is also equal to FB . So the two (straight-lines) DE , EF are equal to the two (straight-lines) DB , BF (respectively). And their base, FD , is common. Thus, angle DEF is equal to angle DBF [Prop. 1.8]. And DEF (is) a right-angle. Thus, DBF (is) also a right-angle. And FB produced is a diameter, And a (straight-line) drawn at right-angles to a diameter of a circle, at its extremity, touches the circle [Prop. 3.16 corr.]. Thus, DB touches circle ABC . Similarly, (the same thing) can be shown, even if the center happens to be on AC .

Thus, if some point is taken outside a circle, and two straight-lines radiate from the point towards the circle, and one of them cuts the circle, and the (other) meets (it), and the (rectangle contained) by the whole (straight-line) cutting (the circle), and the (part of it) cut off outside (the circle), between the point and the convex circumference, is equal to the (square) on the (straight-line) meeting (the circle), then the (straight-line) meeting (the circle) will touch the circle. (Which is) the very thing it

was required to show.