

EUCLID'S ELEMENTS OF GEOMETRY

The Greek text of J.L. Heiberg (1883–1885)

from *Euclidis Elementa, edidit et Latine interpretatus est I.L. Heiberg, in aedibus
B.G. Teubneri, 1883–1885*

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Introduction

Euclid's *Elements* is by far the most famous mathematical work of classical antiquity, and also has the distinction of being the world's oldest continuously used mathematical textbook. Little is known about the author, beyond the fact that he lived in Alexandria around 300 BCE. The main subjects of the work are geometry, proportion, and number theory.

Most of the theorems appearing in the *Elements* were not discovered by Euclid himself, but were the work of earlier Greek mathematicians such as Pythagoras (and his school), Hippocrates of Chios, Theaetetus of Athens, and Eudoxus of Cnidos. However, Euclid is generally credited with arranging these theorems in a logical manner, so as to demonstrate (admittedly, not always with the rigour demanded by modern mathematics) that they necessarily follow from five simple axioms. Euclid is also credited with devising a number of particularly ingenious proofs of previously discovered theorems: *e.g.*, Theorem 48 in Book 1.

The geometrical constructions employed in the *Elements* are restricted to those which can be achieved using a straight-rule and a compass. Furthermore, empirical proofs by means of measurement are strictly forbidden: *i.e.*, any comparison of two magnitudes is restricted to saying that the magnitudes are either equal, or that one is greater than the other.

The *Elements* consists of thirteen books. Book 1 outlines the fundamental propositions of plane geometry, including the three cases in which triangles are congruent, various theorems involving parallel lines, the theorem regarding the sum of the angles in a triangle, and the Pythagorean theorem. Book 2 is commonly said to deal with “geometric algebra”, since most of the theorems contained within it have simple algebraic interpretations. Book 3 investigates circles and their properties, and includes theorems on tangents and inscribed angles. Book 4 is concerned with regular polygons inscribed in, and circumscribed around, circles. Book 5 develops the arithmetic theory of proportion. Book 6 applies the theory of proportion to plane geometry, and contains theorems on similar figures. Book 7 deals with elementary number theory: *e.g.*, prime numbers, greatest common denominators, *etc.* Book 8 is concerned with geometric series. Book 9 contains various applications of results in the previous two books, and includes theorems on the infinitude of prime numbers, as well as the sum of a geometric series. Book 10 attempts to classify incommensurable (*i.e.*, irrational) magnitudes using the so-called “method of exhaustion”, an ancient precursor to integration. Book 11 deals with the fundamental propositions of three-dimensional geometry. Book 12 calculates the relative volumes of cones, pyramids, cylinders, and spheres using the method of exhaustion. Finally, Book 13 investigates the five so-called Platonic solids.

This edition of Euclid's *Elements* presents the definitive Greek text—*i.e.*, that edited by J.L. Heiberg (1883–1885)—accompanied by a modern English translation, as well as a Greek-English lexicon. Neither the spurious books 14 and 15, nor the extensive scholia which have been added to the *Elements* over the centuries, are included. The aim of the translation is to make the mathematical argument as clear and unambiguous as possible, whilst still adhering closely to the meaning of the original Greek. Text within square parenthesis (in both Greek and English) indicates material identified by Heiberg as being later interpolations to the original text (some particularly obvious or unhelpful interpolations have been omitted altogether). Text within round parenthesis (in English) indicates material which is implied, but not actually present, in the Greek text.

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