Vortices and solitons in condensed matter

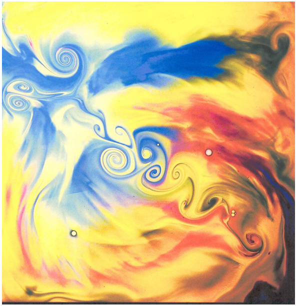
Stavros Komineas

University of Crete

Τμήμα Επιστήμης και Τεχνολογίας Υλικών, 9 Νοεμβρίου 2015

Vortices in fluids

Vortex is a part of the fluid rotating clockwise or anticlockwise in closed loops.

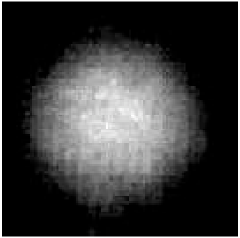


A particle crosses through the surface of a fluid at a high speed. Vortices and antivortices (oppositely circulating fluid) are created.

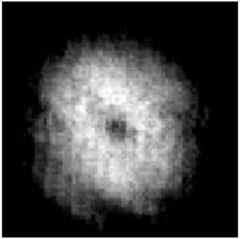
Atomic Bose-Einstein condensates

Vapours of Rb, Li, etc in temparatures $T \sim 10\,nK$

Ground state

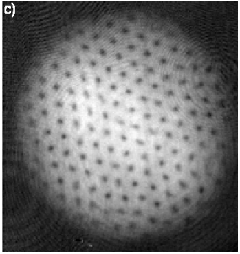


Quantized vortex



[Dalibard group]

Vortex lattice



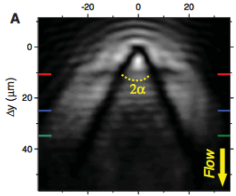
[Ketterle group]

Superfluid flow

The fluid is flowing rotating around a region with zero fluid density, without deceleration.

Bose-Einstein condensates of exciton-polaritons in semiconductors

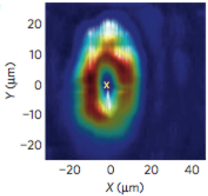
Polaritons are quasiparticles with small effective mass. Therefore they Bose-condense at higher temperatures.



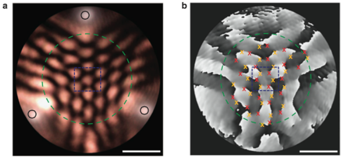
Solitons are localised density depletions of the fluid.

Picture (experiment): [Amo et al, Science 2011]

Theory on polariton solitons: [Komineas, Shipman, Venakides, PRB, Physica D, 2015]



[Sanvitto etal, Nat.Phys. 2010]

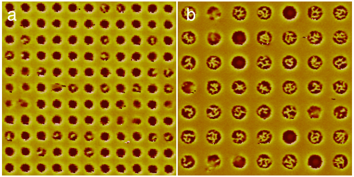


vortex-antivortex lattice

[Tosi et al, Nat. Comm. 2012]

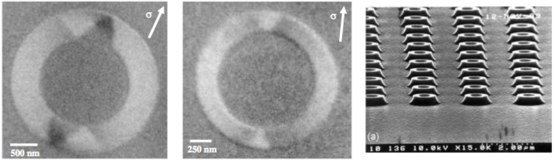
Vortices in polariton condensates.

Ferromagnetic materials (ferromagnetic elements)



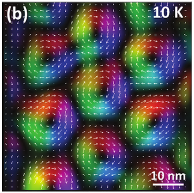
FePt dots, diameter 0.5-1 μm.

[Moutafis, Komineas et al, Phys. Rev. B 2007]



Co ring particles

[Kläui et al, J. Phys.: Condens. Matter. 2003]

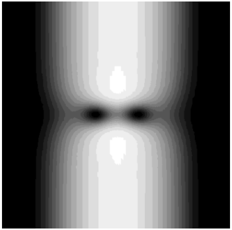


MnSi films

[Tonomura et al, NanoLett 2012]

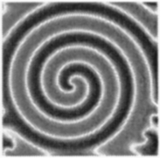
Vortex rings in fluids, superfluids, ...

Rings of air in a fluid:
they propagate along their axis.
Fuid flow goes around the ring.

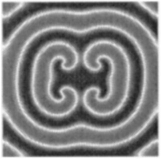


Vortex rings can be found in superfluids
(also magnetic materials, etc.)
[Komineas, Papanicolaou, PRL 2002]

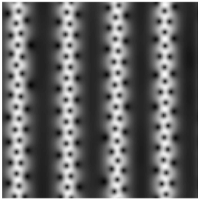
Spiral patterns in nematic liquid crystals



A spiral configuration for the director in a nematic liquid crystal.
The grey scale gives the phase of the complex order parameter.



Four-spiral pattern.



Pattern of defects.
The grey scale gives the density.
[Komineas, Zhao, Kramer, PRE 2003]

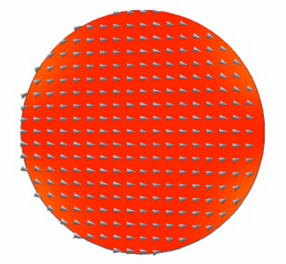
A ferromagnetic film

The magnetisation vector $\mathbf{M} = \mathbf{M}(x, y, t)$

has constant length at every point (x, y) in the film: $\mathbf{M}^2(x, y, t) = M_s^2$, where M_s is the *saturation magnetisation*.
We typically normalise $\mathbf{m} = \mathbf{M}/M_s$, thus $\mathbf{m}^2 = 1$.

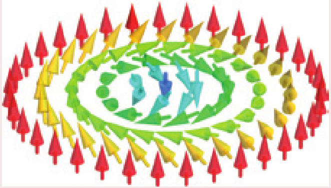
Magnetic states

Uniform



Ferromagnetic state

Non-uniform (magnetic solitons)



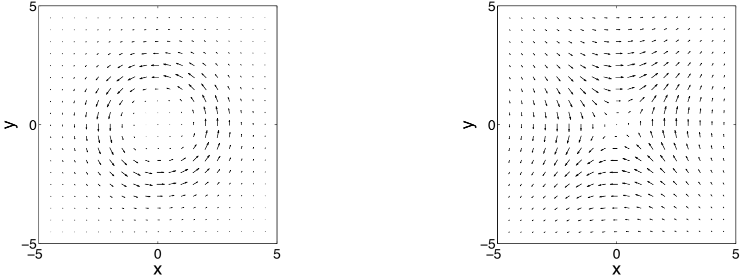
Skyrmion

The skyrmion number Q

is a topological invariant and it counts the number of times that the magnetisation vector $\mathbf{m}$ covers the sphere $\mathbf{m}^2 = 1$:

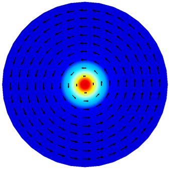
$$Q = \frac{1}{4\pi} \int q \, d^2x, \quad q = \frac{1}{2} \epsilon_{\mu\nu} \mathbf{m} \cdot (\partial_\nu \mathbf{m} \times \partial_\mu \mathbf{m})$$:topological density

Skyrmion ($Q = 1$) Antiskyrmion ($Q = -1$)



Vector plots for projection of magnetisation (m_1, m_2).

Magnetic vortices

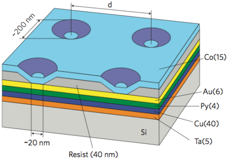


Vortices found in films and magnetic particles.
A single vortex may be the ground state in a magnetic dot.
It has $Q = \pm\frac{1}{2}$ (half skyrmion).

Spin-transfer nano-oscillators

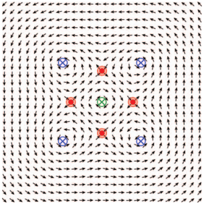
Vortex is set in motion via spin-polarised electrical current passing through the magnetic material.

Nanocontacts on film

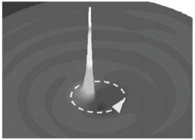


[Ruotolo et al, Nat. Nano. 2009]

Vortex configurations

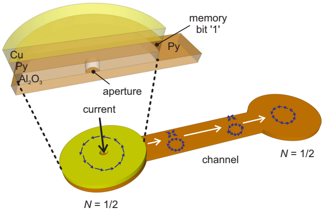


Rotating vortex



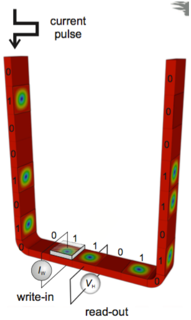
Transfer of magnetic information

Magnetic logic



[Zhang, Baker, Komineas, Hesjedal, Scientific Reports 2015]

Magnetic recording



Dynamics of magnetisation

Skyrmion or vortex motion is obtained by external magnetic field or by spin-polarised electrical current.

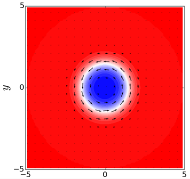
Energy of magnetisation

A typical energy functional for $\mathbf{m} = (m_1, m_2, m_3)$ is

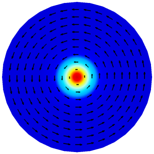
$$W = W_{\rm ex} + W_{\rm a} = \frac{1}{2}\int \partial_\mu \mathbf{m}\cdot\partial_\mu \mathbf{m}\, d^2x + \frac{\kappa}{2}\int (m_1^2 + m_2^2)\, d^2x, \quad \mu = 1, 2.$$

- The exchange energy $W_{\rm ex}$ is minimized for $\mathbf{m}$ a constant vector $\rightarrow$ ferromagnetic state.
- The anisotropy energy $W_{\rm a}$ (with $\kappa > 0$) favours the magnetisation $\mathbf{m} = (0, 0, \pm 1)$ $\rightarrow$ perpendicular to the film.

Easy-axis anisotropy $\kappa > 0$



Easy-plane anisotropy $\kappa < 0$



Antisymmetric exchange interactions: Dzyaloshinskii-Moriya (DM) materials

The exchange interaction may have a symmetric and an antisymmetric part

$$W = \frac{1}{2}\int \partial_\mu \mathbf{m}\cdot\partial_\mu \mathbf{m}\, d^2x + \frac{\kappa}{2}\int (m_1^2 + m_2^2)\, d^2x + \lambda \int \mathbf{m}\cdot(\nabla\times\mathbf{m})\, d^2x.$$

The last term (DM energy) is antisymmetric with respect to the transformation $\mathbf{r}\to -\mathbf{r}$ (due to 1st derivatives).

For antisymmetric exchange we need materials (DM materials)

- with non-centrosymmetric crystal structure,
- with spin-orbit coupling.
- DM interaction can also emerge at interfaces due to broken mirror symmetry. [Fert, Levy, 1980]

[Dzyaloshinskii, 1957, Moriya, 1960]

The Landau-Lifshitz (LL) equation (1935)

The conservative (Hamiltonian) LL equation associated with the energy is

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{f}, \qquad \mathbf{m}^2 = 1$$

$$\mathbf{f} \equiv -\frac{\delta W}{\delta \mathbf{m}} = \Delta \mathbf{m} + \kappa m_3 \hat{\mathbf{e}}_3 - 2\lambda \nabla \times \mathbf{m}.$$

- If $\mathbf{f} = \mathbf{h}_{\rm ext}$ (external magnetic field), then we have a basic result from electromagnetism: $\mathbf{m}$ precesses around $\mathbf{h}_{\rm ext}$.
- The LL eqn gives precession of $\mathbf{m}$ around the effective field $\mathbf{f}$.
- As $\mathbf{f} = \mathbf{f}(\mathbf{m})$ the LL eqn is nonlinear.

Static solutions: $\mathbf{m} \times \mathbf{f} = 0$ - one dimension (wire)

Use spherical coordinates for $\mathbf{m} = (\sin\Theta\cos\Phi, \sin\Theta\sin\Phi, \cos\Theta)$. Domain wall solutions are sought in the form $\Phi = \pi/2$ and $\mathbf{m}(x) = (0, \sin\Theta(x), \cos\Theta(x))$. The energy is

$$W = \int \left[\frac{1}{2}(\partial_x\Theta)^2 - \lambda\partial_x\Theta - \frac{\kappa}{2}\cos^2\Theta\right] dx.$$

Minimisation of energy gives a domain wall

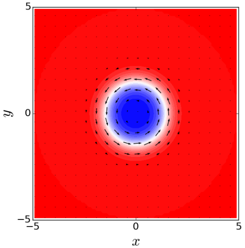
$$\tan\left(\frac{\Theta}{2}\right) = e^{\sqrt{\kappa}\,x}, \qquad W = 2\sqrt{\kappa} - \pi\lambda.$$

Spiral state

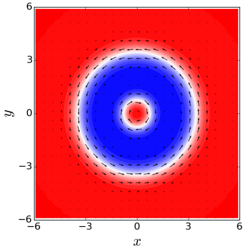
For $\kappa \to (\pi^2/4)\lambda^2$ the domain wall energy $W \to 0$. For $\kappa \geq (\pi^2/4)\lambda^2$ we have a proliferation of domain walls. A helical magnetisation configuration the "spiral state/helical state/..." is the ground state of the system.

Static solutions: $\mathbf{m} \times \mathbf{f} = 0$ - two dimensions (film)

Skyrmion ($Q = 1$)



Skyrmionium ($Q = 0$)



For axially symmetric configurations $\Theta = \Theta(\rho)$, $[m_3 = \cos\Theta]$

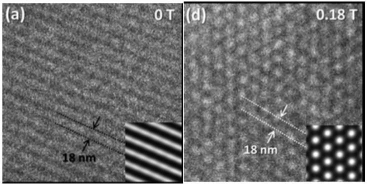
$$W_{\rm DM} = -\lambda \int \left[\partial_\rho \Theta + \frac{\cos\Theta \sin\Theta}{\rho}\right] (2\pi\rho\, d\rho)$$

indicates possibility for $W_{\rm DM} < 0$.

[Bogdanov, Yablonskii, JETP 1989 and Bogdanov, Hubert, JMMM 1994]

Periodic states and phase diagram

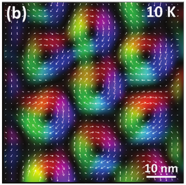
Helical state　　Skyrmion lattice



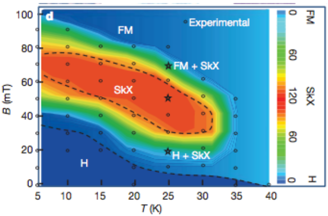
Grey scale gives the perpendicular component of $\mathbf{m}$: (m_3).

[Tonomura et al, Nanoletters 2012]

Skyrmion lattice



Vectors give (m_1, m_2).



Phase diagram for temperature (T) and external magnetic field (B). Ferromagnetic (FM), Helical (H) and Skyrmion lattice (SkX) phases.

[Yu et al, Nature 2010]

Dynamics of skyrmions

Define the moments of topological density q:

$$I_x = \int xq\,dxdy, \quad I_y = \int yq\,dxdy \quad \left(\text{or} \quad I_\mu = \int x_\mu q\, d^2x\right)$$

Prove that they are conserved $\dot{I}_\mu = 0,\ \mu = 1, 2.$

A rigid translation of spatial coordinates by a constant vector c_μ

$$x_\mu \to x_\mu + c_\mu \quad \Rightarrow \quad I_\mu \to I_\mu + 4\pi Q\, c_\mu$$

reveals difference between topological ($Q \neq 0$) and non-topological ($Q = 0$) magnetic solitons.

- For $Q \neq 0$, the (I_x, I_y) gives position of skyrmion and this is fixed.
- For $Q = 0$, skyrmions may propagate feely (solitary waves).

[Papanicolaou, Tomaras, Nucl. Phys. B 1991]

$Q = 0$ skyrmionium as traveling wave

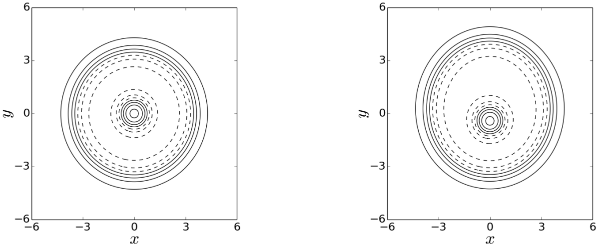
Assume propagating skyrmionium with velocity v (solitary wave). We make the traveling wave ansatz $\mathbf{m} = \mathbf{m}(x - vt, y)$, and this satisfies

$$v \frac{\partial \mathbf{m}}{\partial x} = \mathbf{m} \times \mathbf{f}.$$

[Komineas, Papanicolaou, Phys. Rev. B, 2015]

We find numerically traveling solutions for $0 \leq v < v_c \approx 0.102$

$v = 0$ $\qquad$ $v = 0.07$

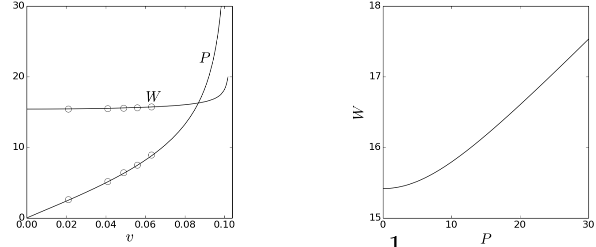


m_3 contour plots (solid: $m_3 > 0$, dashed: $m_3 < 0$)

Energy – Momentum relation

The linear momentum $\mathbf{P} = (P_x, P_y)$ is defined from

$$P_x = I_y, \quad P_y = -I_x \quad (\text{or} \quad P_\mu = \epsilon_{\mu\nu} I_\nu)$$



$$(P_x =) P = mv, \qquad W = W_0 + \frac{1}{2} m v^2, \qquad v \ll v_c$$

We may associate a mass (m) to the skyrmionium

At low momenta $W = W_0 + \frac{P^2}{2m}$ (Newtonian)

At high momenta $W \approx v_c P$ (relativistic).

Force on skyrmions

Apply an external non-homogeneous magnetic field, e.g.,

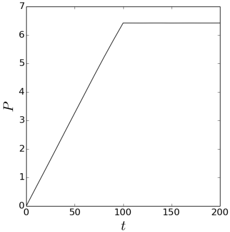
$$\mathbf{h} = (0, 0, h), \qquad h = g\,x.$$

Force and acceleration on a skyrmionium

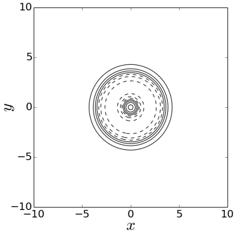
The force changes the linear momentum

$$\dot{P}_x = -\int \partial_x h(1 - m_3)\, d^2x, \quad \dot{P}_y = 0.$$

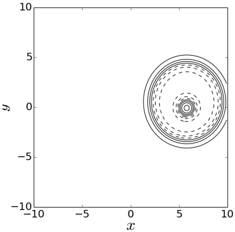
Force for $t \leq 100$



$t = 0$



$t = 160$



Skyrmion dynamics for $Q = 0$: Newtonian

Propagates freely in the absence of force. When forced, it accelerates.

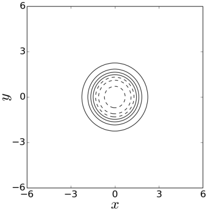
Force and Hall motion of $Q = 1$ skyrmion

We follow the skyrmion guiding center $\mathbf{R} = (R_1, R_2)$:

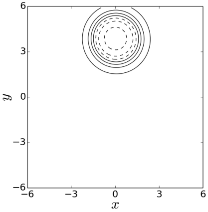
$$R_\mu = \frac{I_\mu}{4\pi Q} = \frac{1}{4\pi Q} \int x_\mu q \, d^2x.$$

We have evolution equations: $\dot{R}_x = 0, \quad \dot{R}_y = -\frac{1}{4\pi Q} \int \partial_x h \,(1 - m_3)\, d^2x.$

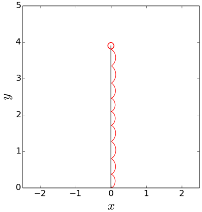
$t = 0$



$t = 30$



Trajectory



Skyrmion dynamics for $Q \neq 0$: Hall motion

It is spontaneously pinned in the absence of force.

When forced, propagates with constant velocity, perpendicular to force.

Concluding remarks

- Vortices are pervasive in nature: in fluids, superfluids, magnetic materials, etc. Localised robust excitations also include solitons and vortex rings.
- Magnetic vortices and skyrmions can be used to produce nano-oscillators, transfer information, or construct logic gates.
- Dzyaloshinskii-Moriya materials supports both topological $(Q \neq 0)$ and non-topological $(Q = 0)$ magnetic solitons. They offer the opportunity to study and exploit their unusual dynamical behaviour.
- A topological skyrmion is pinned in a ferromagnetic film. A non-topological skyrmionium may move freely as a solitary wave. It responds to forces as a Newtonian particle.

Synthetic DM materials

- [Fert group, arXiv:1502.07853]: cobalt-based multilayered thin films.
- [Kläui, Beach et al, arXiv: 1502.07376:] ultrathin transition metal