Homework #2

Due date: November 16, 2006

Notes:

1. Please write your name on the homework you are going to hand in.
2. Homeworks are to be solved and written individually. Any form of copying or plagiarism is prohibited.
3. This homework is to be handed in the latest by the beginning of the class on November 16th, that is by 13:15. Late homework will not be accepted.
4. In case you have any questions send email to the class mailing list:
   em201-list@tem.uoc.gr

Problem 1 [20 points] A relation over a set that is reflexive and symmetric is called a *compatibility relation*.
Let R_1 and R_2 two compatibility relations over a set A.

(a) [10 points] Is $R_1 \cap R_2$ a compatibility relation?

(b) [10 points] Is $R_1 \cup R_2$ a compatibility relation?

Problem 2 [20 points] Is the transitive closure of an antisymmetric relation always antisymmetric?

Problem 3 [20 points] Consider the set $A = \{a, b, c, d, e, f, g, h\}$, and the relation R over A, defined by the table below. The rows indicate the first element of the ordered pairs in R, whereas columns indicate the second element of the ordered pairs in R (as a result, $(a, c) \in R$, but $(c, a) \notin R$).

	a	b	c	d	e	f	g	h
a	✓		✓			✓	✓	
b		✓	✓	✓	✓	✓	✓	✓
c			✓			✓	✓	
d				✓		✓	✓	✓
e					✓			✓
f						✓		
g							✓	
h								✓

(a) [5 points] Prove that R is a partial order.

(b) [5 points] Construct the Hasse diagram of R.

(c) [2 points] Which are the minimums and maximums of R?

(d) [3 points] Is R a lattice?

(e) [2 points] What is the length of the longest chain in R?

(f) [3 points] If μ is the length of the longest chain in R, find a partition of R into μ anti-chains.

Problem 4 [20 points] Among the integers 1–200, 101 of them are chosen arbitrarily. Show that among the chosen integers there exist two, such that one divides the other.

Hint: Let $A = \{1, 2, 3, 4, \ldots, 199, 200\}$ and let C the subset of the elements of A that we choose arbitrarily. Consider the sets A_i, $1 \leq i \leq 100$, that are defined as follows:

$$A_i = \{2^{k_i}\,(2i-1)\,|\,k_i \geq 0\;\; 2^{k_i}(2i-1) \leq 200\},$$

i.e., k_i takes all values between 0 and $\lfloor \log_2(\frac{200}{2i-1}) \rfloor$.
Prove that the set of sets $\mathcal{A}$, where $\mathcal{A} = \{A_1, A_2, \ldots, A_{100}\}$, forms a partition of A. Then define a function f from C to $\mathcal{A}$, to be used along with the pigeonhole principle, in order to give you the wanted result.

Problem 5 [20 points] Show that among $n + 1$ arbitrarily chosen integers there exist two, the difference of which is divisible by n.

Total points: 100