

SOME OPEN PROBLEMS ON MULTIPLE ERGODIC AVERAGES

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1. EXTENDED BIBLIOGRAPHY ORGANIZED BY TOPIC

Below we give a rather extensive bibliography with material that is directly related to the problems discussed before, organized by topic. We caution the reader that this is *not* a comprehensive list of articles in ergodic Ramsey theory, and in fact articles on several important topics are missing from this list. For instance, the reader will find very few articles related to actions of non-commuting measure preserving transformations, the richness of return times in various multiple recurrence results, and no articles related to problems in topological dynamics and applications in partition Ramsey theory. Luckily, there are several excellent places to look for such topics, for instance, the survey articles of V. Bergelson [24, 27, 28] cover a lot of related material and contain an extensive bibliography up to 2006.

Linear sequences: [2, 3, 5, 6, 7, 9, 10, 14, 15, 16, 17, 18, 25, 56, 33, 36, 37, 59, 60, 63, 64, 65, 68, 69, 73, 75, 76, 84, 91, 97, 98, 99, 111, 112, 113, 114, 115, 119, 123, 139, 151, 154, 155, 162, 173, 178, 181, 184, 186, 192, 193, 195, 196].

Polynomial sequences: [11, 12, 13, 22, 29, 30, 34, 35, 39, 40, 41, 43, 44, 52, 53, 54, 55, 58, 61, 62, 70, 74, 77, 79, 82, 83, 85, 87, 101, 108, 116, 124, 137, 142, 150, 156, 158, 168, 185].

Other sequences (smooth, random, prime numbers, generalized polynomials): [30, 31, 38, 42, 47, 50, 51, 72, 79, 80, 81, 88, 90, 104, 105, 125, 136, 148, 160, 161, 183, 189, 191].

Equidistribution on nilmanifolds and other nil-stuff: [8, 57, 78, 106, 107, 117, 118, 120, 121, 122, 138, 140, 141, 143, 144, 145, 146, 147, 148, 152, 153, 157, 163, 164, 165, 166, 174, 175, 194].

Books and survey articles on related topics: [1, 4, 8, 23, 24, 26, 27, 28, 66, 67, 71, 89, 92, 93, 94, 95, 96, 102, 110, 126, 130, 131, 132, 133, 135, 157, 159, 167, 169, 170, 172, 179, 180, 182, 187, 188].

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