**Table of Fourier Transforms**

| | $f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\widehat{f}(\omega)e^{ix\omega}\,d\omega$ | $\widehat{f}(\omega)=\mathcal{F}(f)(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-i\omega x}\,dx$ |
|---|---|---|
| **1.** | $\begin{cases}1 & \text{if } \lvert x\rvert<a\\ 0 & \text{if } \lvert x\rvert>a\end{cases}$ | $\sqrt{\frac{2}{\pi}}\frac{\sin a\omega}{\omega}$ |
| **2.** | $\begin{cases}1 & \text{if } a<x<b\\ 0 & \text{otherwise}\end{cases}$ | $\frac{i\,(e^{-i\,b\omega}-e^{i\,a\omega})}{\sqrt{2\,\pi}\omega}$ |
| **3.** | $\begin{cases}1-\frac{\lvert x\rvert}{a} & \text{if } \lvert x\rvert<a\\ 0 & \text{if } \lvert x\rvert>a\end{cases}\quad a>0$ | $2\sqrt{\frac{2}{\pi}}\frac{\sin^2(\frac{a\,\omega}{2})}{a\,\omega^2}$ |
| **4.** | $\begin{cases}x & \text{if } \lvert x\rvert<a\\ 0 & \text{if } \lvert x\rvert>a\end{cases}\quad a>0$ | $i\sqrt{\frac{2}{\pi}}\frac{a\omega\cos(a\omega)-\sin(a\omega)}{\omega^2}$ |
| **5.** | $\begin{cases}\sin x & \text{if } \lvert x\rvert<\pi\\ 0 & \text{if } \lvert x\rvert>\pi\end{cases}$ | $i\sqrt{\frac{2}{\pi}}\frac{\sin(\pi\omega)}{\omega^2-1}$ |
| **6.** | $\begin{cases}\sin(ax) & \text{if } \lvert x\rvert<b\\ 0 & \text{if } \lvert x\rvert>b\end{cases}\quad a,b>0$ | $i\sqrt{\frac{2}{\pi}}\frac{\omega\cos(b\omega)\sin(ab)-a\cos(ab)\sin(b\omega)}{\omega^2-a^2}$ |
| **7.** | $\frac{1}{a^2+x^2},\ a>0$ | $\sqrt{\frac{\pi}{2}}\frac{e^{-a\lvert\omega\rvert}}{a}$ |
| **8.** | $\frac{x}{a^2+x^2},\ a>0$ | $-i\sqrt{\frac{\pi}{2}}\operatorname{sgn}\omega\, e^{-a\lvert\omega\rvert}$ |
| **9.** | $\sqrt{\frac{2}{\pi}}\frac{a}{1+a^2x^2},\ a>0$ | $e^{-\frac{\lvert\omega\rvert}{a}}$ |
| **10.** | $\frac{\sin ax}{x},\ a>0$ | $\begin{cases}\sqrt{\frac{\pi}{2}} & \text{if } \lvert\omega\rvert<a\\ \frac{1}{2}\sqrt{\frac{\pi}{2}} & \text{if } \lvert\omega\rvert=a\\ 0 & \text{if } \lvert\omega\rvert>a\end{cases}$ |
| **11.** | $\frac{4}{\sqrt{2\pi}}\frac{\sin^2(\frac{1}{2}ax)}{ax^2},\ a>0$ | $\begin{cases}1-\frac{\lvert\omega\rvert}{a} & \text{if } \lvert\omega\rvert<a\\ 0 & \text{if } \lvert\omega\rvert>a\end{cases}$ |
| **12.** | $\frac{4}{\sqrt{2\pi}}\frac{\sin^2(ax)-\sin^2(\frac{1}{2}ax)}{ax^2},\ a>0$ | $\begin{cases}1 & \text{if } \lvert x\rvert<a\\ (-x+2a)/a & \text{if } a<x<2a\\ (x+2a)/a & \text{if } a<x<2a\\ 0 & \text{if } \lvert x\rvert>2a\end{cases}$ |
| **13.** | $e^{-a\lvert x\rvert},\ a>0$ | $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+\omega^2}$ |
| **14.** | $\begin{cases}e^{-ax} & \text{if } x>0\\ 0 & \text{if } x<0\end{cases},\ a>0$ | $\frac{1}{\sqrt{2\pi}}\frac{1}{a+i\omega}$ |
| **15.** | $\begin{cases}0 & \text{if } x>0\\ e^{ax} & \text{if } x<0\end{cases},\ a>0$ | $\frac{1}{\sqrt{2\pi}}\frac{1}{a-i\omega}$ |
| **16.** | $\lvert x\rvert^n e^{-a\lvert x\rvert},\ a>0,\ n>0$ | $\frac{\Gamma(n+1)}{\sqrt{2\pi}}\left(\frac{1}{(a-i\omega)^{1+n}}+\frac{1}{(a+i\omega)^{1+n}}\right)$ |

**Table of Fourier Transforms (continued)**

| $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{f}(\omega) e^{ix\omega}\, d\omega$ | $\widehat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$ |
|---|---|
| **17.** $e^{-\frac{a}{2}x^2}$, $a > 0$ | $\frac{1}{\sqrt{a}} e^{-\frac{\omega^2}{2a}}$ |
| **18.** $e^{-ax^2}$, $a > 0$ | $\frac{1}{\sqrt{2a}} e^{-\frac{\omega^2}{4a}}$ |
| **19.** $xe^{-\frac{a}{2}x^2}$, $a > 0$ | $\frac{-i\omega}{a^{3/2}} e^{-\frac{\omega^2}{2a}}$ |
| **20.** $x^2 e^{-\frac{a}{2}x^2}$, $a > 0$ | $\frac{a-\omega^2}{a^{5/2}} e^{-\frac{\omega^2}{2a}}$ |
| **21.** $x^3 e^{-\frac{a}{2}x^2}$, $a > 0$ | $\frac{-i\omega(3a-\omega^2)}{a^{7/2}} e^{-\frac{\omega^2}{2a}}$ |
| **22.** $e^{-\frac{x^2}{2}} H_n(x)$, $H_n$, $n$th Hermite polynomial | $(-1)^n i^n e^{\frac{-\omega^2}{2}} H_n(\omega)$ |
| **23.** $J_0(x)$, Bessel function of order 0 | $\begin{cases} \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{1-\omega^2}} & \text{if } \lvert\omega\rvert < 1 \\ 0 & \text{if } \lvert\omega\rvert > 1 \end{cases}$ |
| **24.** $J_n(x)$, Bessel function of order $n \geq 0$ | $\begin{cases} \sqrt{\frac{2}{\pi}} \frac{(-i)^n}{\sqrt{1-\omega^2}} T_n(\omega) & \text{if } \lvert\omega\rvert < 1 \\ 0 & \text{if } \lvert\omega\rvert > 1 \end{cases}$ $T_n$, Chebyshev polynomial of degree $n$. |

**Special Transforms**

| | |
|---|---|
| **25.** $\mathcal{F}(\delta_0(x))(\omega) = \frac{1}{\sqrt{2\pi}}$ | **27.** $\mathcal{F}(\sqrt{\frac{2}{\pi}} \frac{1}{x})(\omega) = -i \operatorname{sgn} \omega$ |
| **26** $\mathcal{F}(\delta_0(x-a))(\omega) = \frac{1}{\sqrt{2\pi}} e^{-ia\omega}$ | **28.** $\mathcal{F}(e^{iax})(\omega) = \sqrt{2\pi}\delta_0(\omega - a)$ |

**Operational Properties**

| | |
|---|---|
| **29.** $\mathcal{F}(af + bg)(\omega) = a\mathcal{F}(f) + b\mathcal{F}(g)$ | **36.** $\mathcal{F}(f\,g)(\omega) = \mathcal{F}(f) * \mathcal{F}(g)(\omega)$ |
| **30.** $\mathcal{F}(f')(\omega) = i\omega\mathcal{F}(f)(\omega)$ | **37.** $\mathcal{F}(f(x-a))(\omega) = e^{-ia\omega}\mathcal{F}(f)(\omega)$ |
| **31.** $\mathcal{F}(f'')(\omega) = -\omega^2\mathcal{F}(f)(\omega)$ | **38.** $\mathcal{F}(e^{iax}f(x))(\omega) = \mathcal{F}(f)(\omega - a)$ |
| **32.** $\mathcal{F}(f^{(n)})(\omega) = (i\omega)^n\mathcal{F}(f)(\omega)$ | **39.** $\mathcal{F}(\cos(ax)f(x))(\omega) = \frac{\mathcal{F}(f)(\omega-a)+\mathcal{F}(f)(\omega+a)}{2}$ |
| **33.** $\mathcal{F}(xf(x))(\omega) = i\frac{d}{d\omega}\mathcal{F}(f)(\omega)$ | **40.** $\mathcal{F}(\sin(ax)f(x))(\omega) = \frac{\mathcal{F}(f)(\omega-a)-\mathcal{F}(f)(\omega+a)}{2i}$ |
| **34.** $\mathcal{F}(x^n f(x))(\omega) = i^n\frac{d^n}{d\omega^n}\mathcal{F}(f)(\omega)$ | **41.** $\mathcal{F}(f(ax))(\omega) = \frac{1}{\lvert a\rvert}\mathcal{F}(f)(\frac{\omega}{a})$, $a \neq 0$ |
| **35.** $\mathcal{F}(f * g)(\omega) = \mathcal{F}(f)(\omega)\mathcal{F}(g)(\omega)$ | **42.** $f(x) = \mathcal{F}(\widehat{f})(-x)$, $\mathcal{F}(\mathcal{F}(f)) = f(-x)$ |