Butterfly theorem

by Paris Pamfilos

Facts as facts do not always create a spirit of reality, because reality is a spirit.

Chesterton, G.K., On the Classics p. 49

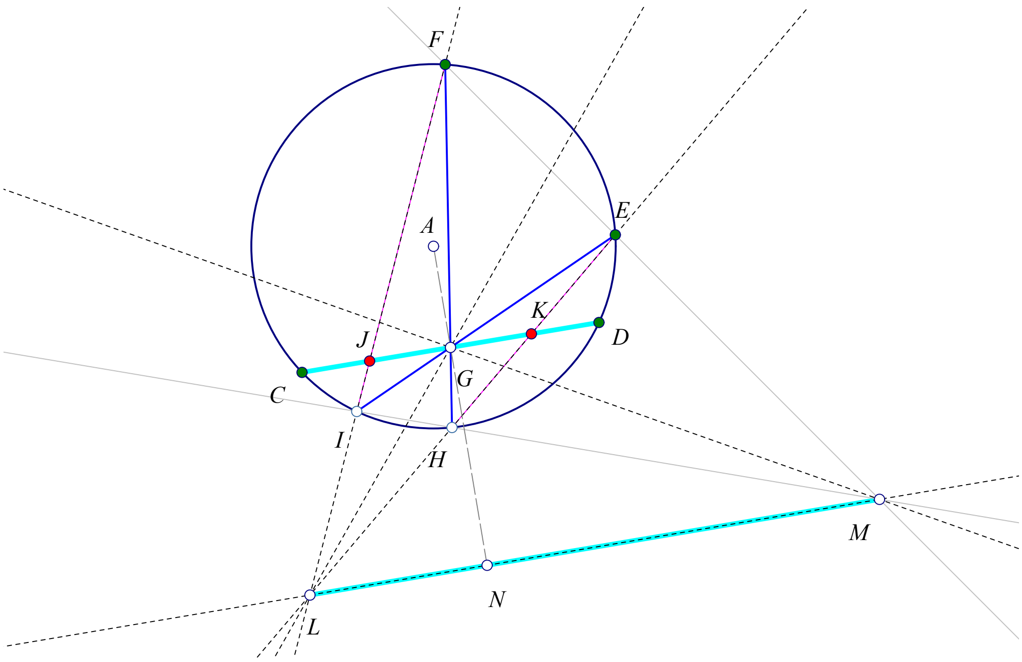

Figure 1: $GJ = GK$

Butterfly theorem

Theorem. *Consider a circle* $\kappa(O)$ *and a chord* CD *of it whose middle is* G*. Draw two other chords* $\{FH, EI\}$ *through G. Then lines* $\{FI, EH\}$ *intersect chord* CD *at points respectively* J, K*, which are symmetric with respect to* G*.*

Proof. The theorem is a consequence of the fact that line LM is the polar of G with respect to the circle. Then lines $L(F,E,G,M)$ form a harmonic pencil and every line intersecting these lines is divided harmonically by them. Thus, JK being parallel to LM is bisected by LG. □

Notice that $\{LG, MG\}$ are respectively the polars of M and L and line AGN is orthogonal to the parallel lines, hence G is the middle of the chord CD. The relevant properties of the polars are discussed in the file **Cyclic Projective**.

Information on the history of the problem and discussion of various alternative proofs can be found in the article by *Bankoff* [Ban87].

Bibliography

[Ban87] Leon Bankoff. The metamorphosis of the butterfly problem. *Mathematics Magazine*, 60:195–210, 1987.