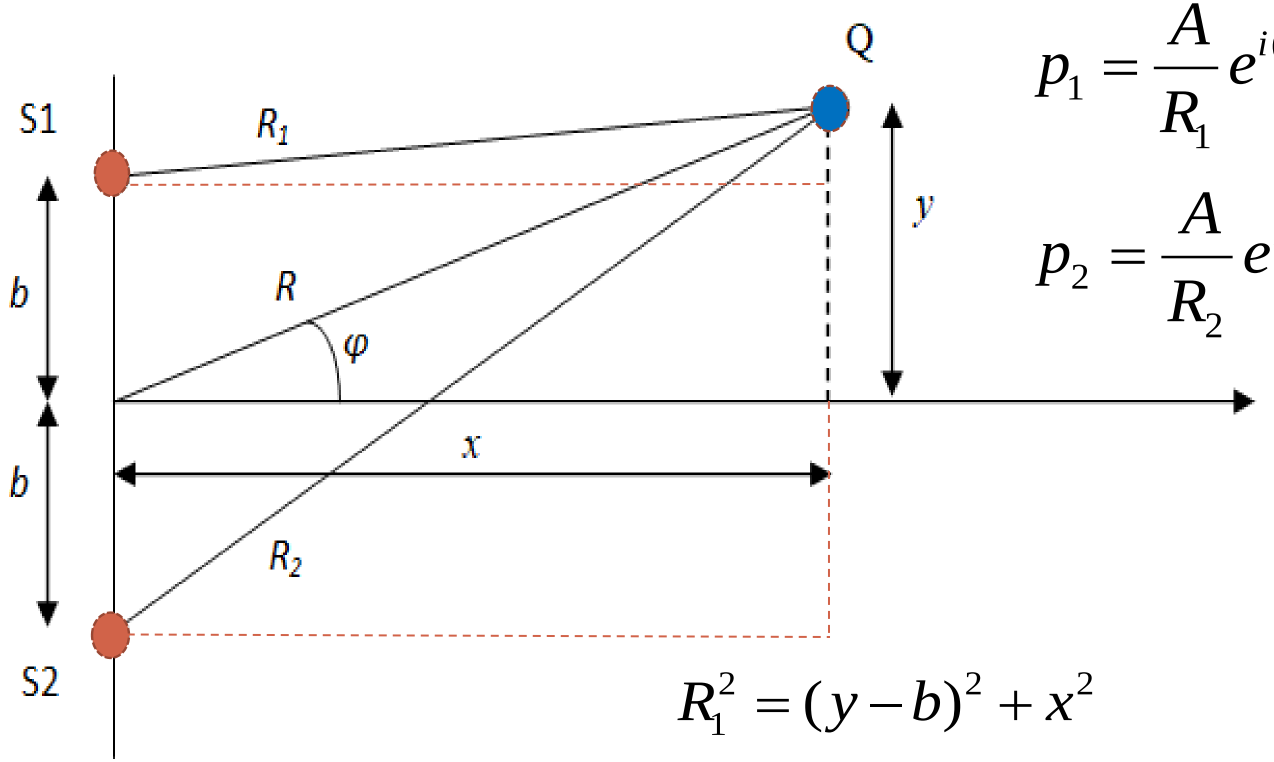
Σύνθεση σημειακών ακουστικών πηγών

Εισαγωγή στην Ακουστική Ωκεανογραφία

Δύο πηγές εκπέμπουν σε φάση



$$p_1 = \frac{A}{R_1} e^{i(\omega t - kR_1)}$$

$$p_2 = \frac{A}{R_2} e^{i(\omega t - kR_2)}$$

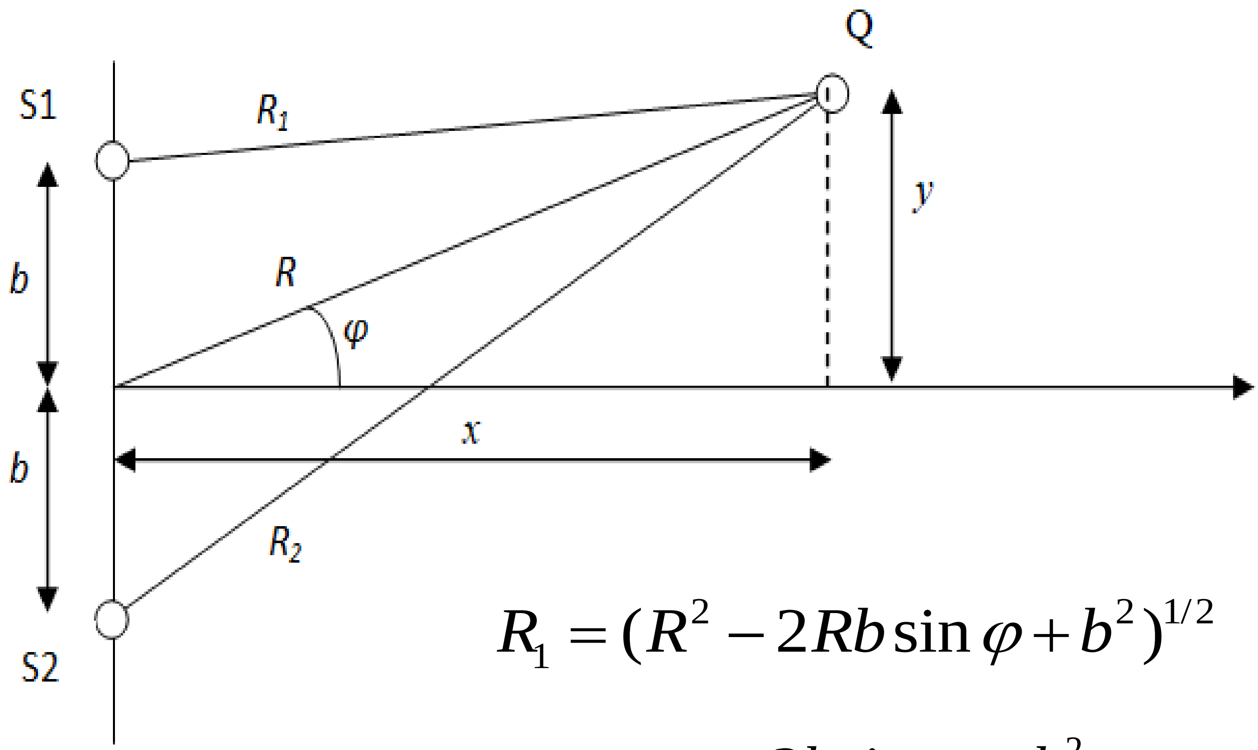
$$R_1^2 = (y-b)^2 + x^2$$

$$R_2^2 = (y+b)^2 + x^2$$

$$b \ll R$$

$$R_1^2 = (R\sin\varphi - b)^2 + R^2\cos^2\varphi = R^2 - 2Rb\sin\varphi + b^2$$

Δύο πηγές εκπέμπουν σε φάση



$$R_1 = (R^2 - 2Rb\sin\varphi + b^2)^{1/2}$$

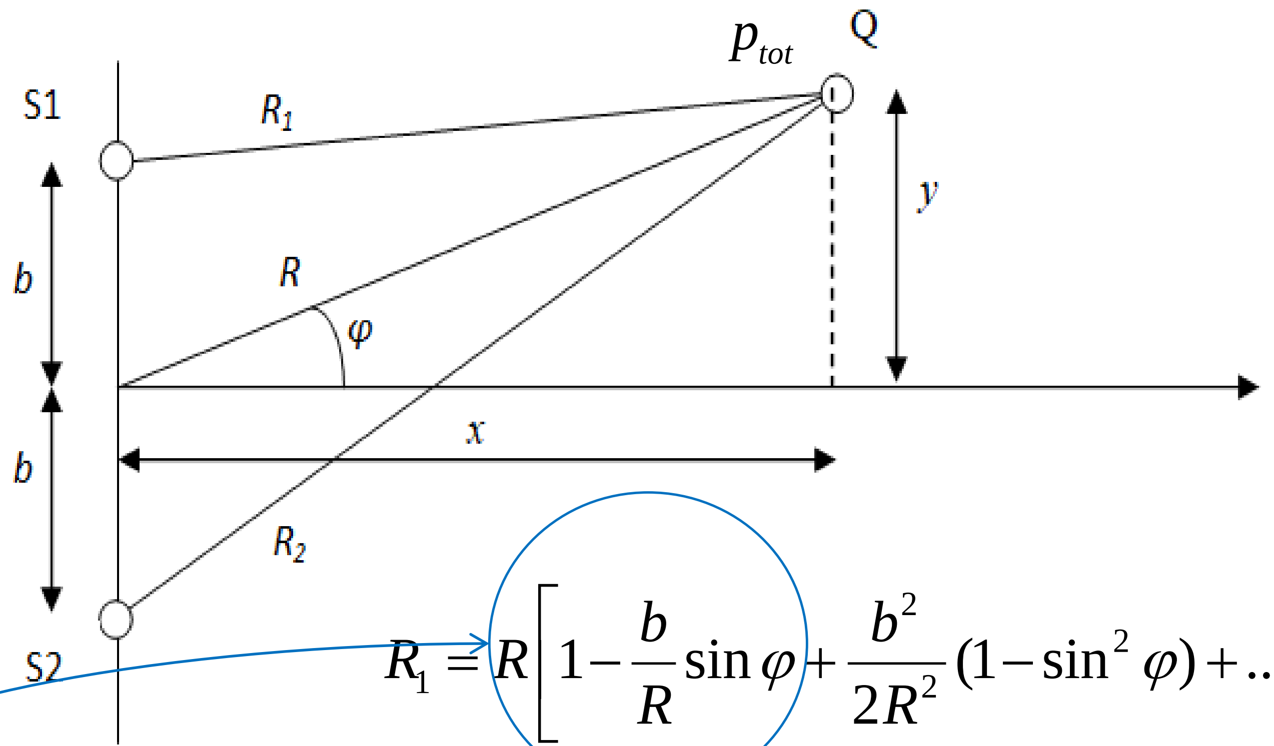
$$R_1 = R(1 - \frac{2b\sin\varphi}{R} + \frac{b^2}{R^2})^{1/2}$$

$b \ll R$

$$(1+q)^{1/2} = 1 + \frac{q}{2} - \frac{q^2}{8} + \frac{q^3}{16} + ...$$

$$q = (b^2 / R^2 - 2b\sin\varphi / R)$$

Δύο πηγές εκπέμπουν σε φάση



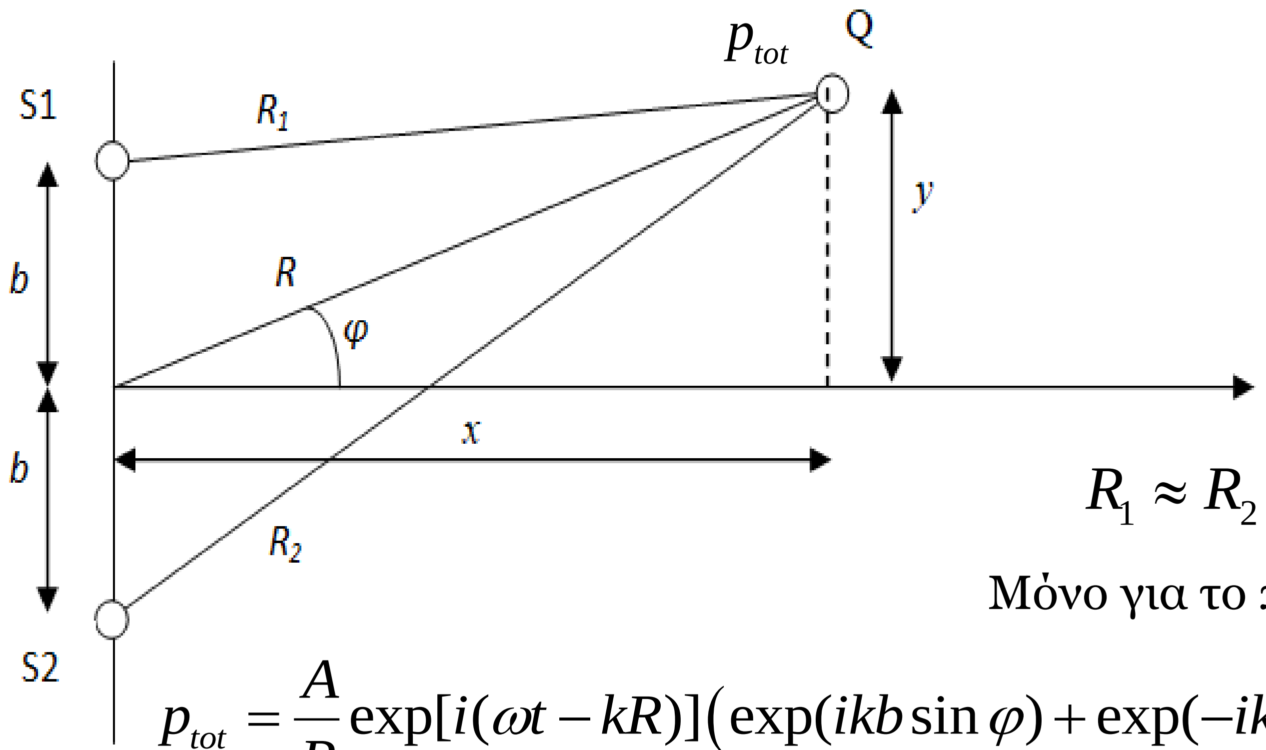
$$R_1 = R\left[1 - \frac{b}{R}\sin\varphi + \frac{b^2}{2R^2}(1-\sin^2\varphi) + \ldots\right]$$

$b \ll R$

$$R_1 \approx R - b\sin\varphi \qquad R_2 \approx R + b\sin\varphi$$

$$p_{tot} = \frac{A}{R_1}\exp[i(\omega t - kR_1)] + \frac{A}{R_2}\exp[i(\omega t - kR_2)]$$

Δύο πηγές εκπέμπουν σε φάση



$$R_1 \approx R_2$$

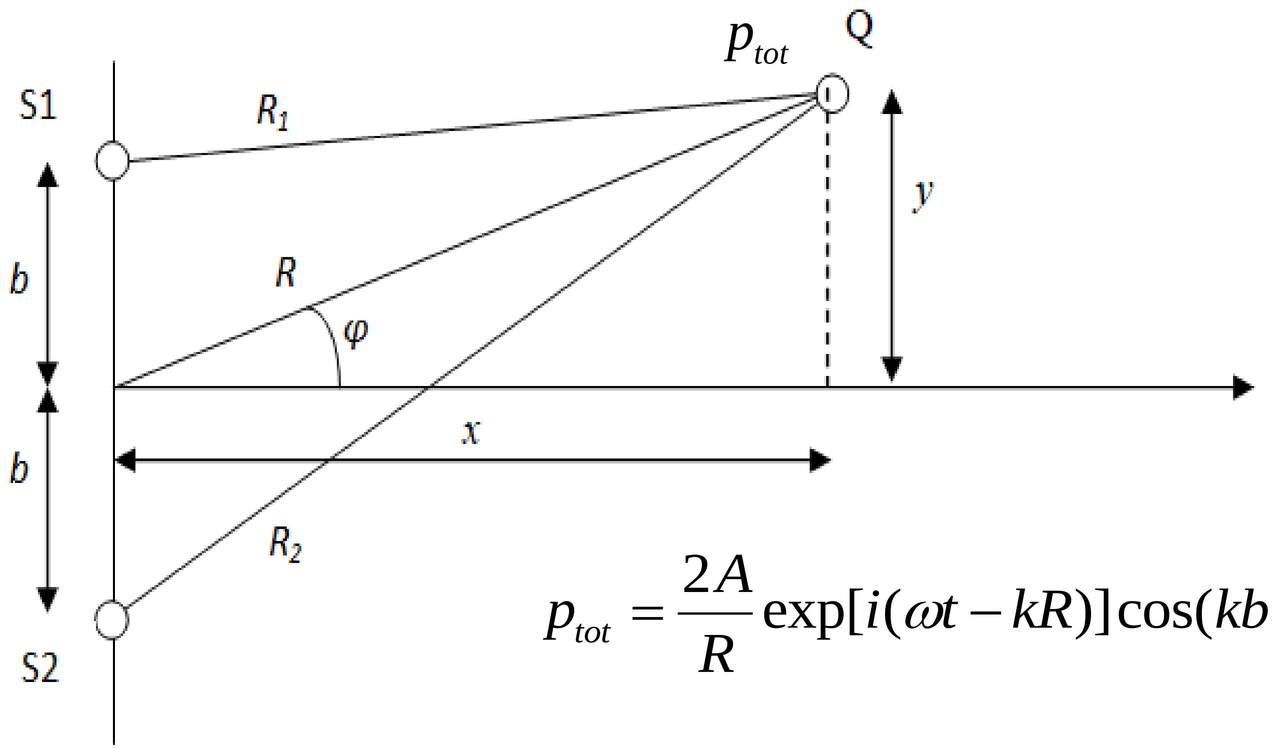
Μόνο για το πλάτος

$$p_{tot} = \frac{A}{R} \exp[i(\omega t - kR)]\left(\exp(ikb\sin\varphi) + \exp(-ikb\sin\varphi)\right)$$

$$b \ll R$$

$$p_{tot} = \frac{2A}{R} \exp[i(\omega t - kR)]\cos(kb\sin\varphi)$$

Δύο πηγές εκπέμπουν σε φάση



$$p_{tot} = \frac{2A}{R}\exp[i(\omega t - kR)]\cos(kb\sin\varphi)$$

$$R_2 - R_1 = 2b\sin\varphi$$

Μέτρο p_{tot} μέγιστο για $kb\sin\varphi = 0, \pi, 2\pi, .., n\pi$

$$k(R_2 - R_1) = 0, ..., 2n\pi$$

Εναλλακτικός τρόπος υπολογισμού

$$p_{tot} = \frac{A}{R}\{\exp[i(\omega t - kR_1)] + \exp[i(\omega t - kR_2)]\}$$

$$= \frac{A}{R}\exp(i\omega t)[\exp(-ikR_1) + \exp(-ikR_2)]$$

$$\left|p_{tot}\right|^2 = \left(\frac{A}{R}\right)^2 [\exp(-ikR_1) + \exp(-ikR_2)][\exp(ikR_1) + \exp(ikR_2)]$$

$$= \left(\frac{A}{R}\right)^2 \{2 + \exp[ik(R_1 - R_2)] + \exp[-ik(R_1 - R_2)]\}$$

$$\left|p_{tot}\right|^2 = 2\left(\frac{A}{R}\right)^2 [1 + \cos k(R_1 - R_2)]$$

Μέγιστο για

$$k(R_2 - R_1) = 0, \ldots\ldots, 2n\pi$$

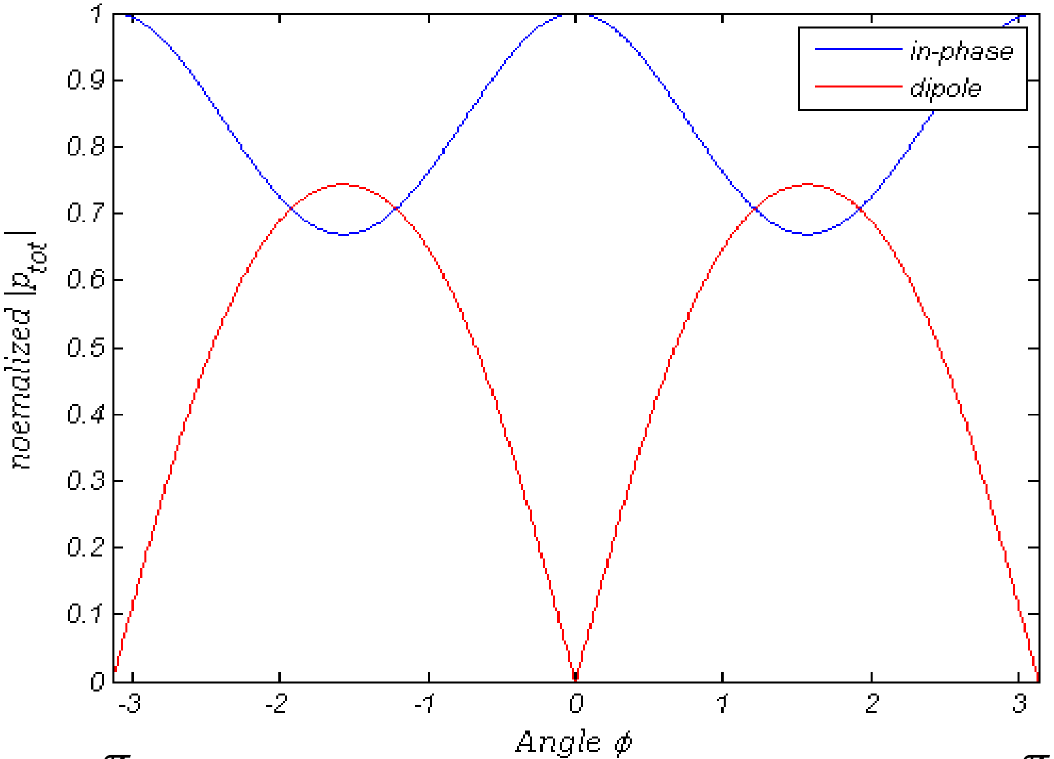
Οι πηγές εκπέμπουν σε φάση

$$p_{tot} = \frac{2A}{R}\exp[i(\omega t - kR)]\cos(kb\sin\varphi)$$

Πηγές εκπέμπουν σε διαφορά φάσης 180^{o} (Δίπολο)

$$p_{d,tot} = \frac{2A}{R}\exp[i(\omega t - kR)]\sin(kb\sin\varphi)$$

b=0,2 μ, f=1000 Hz, c=1500 m/sec.

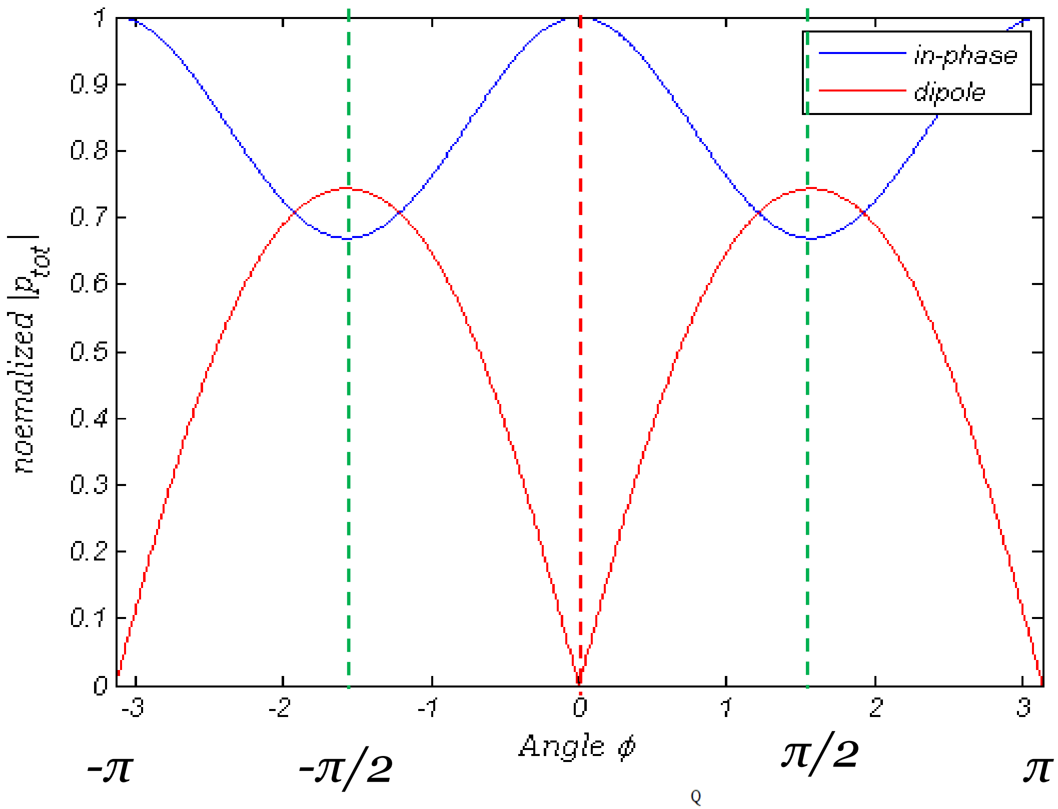


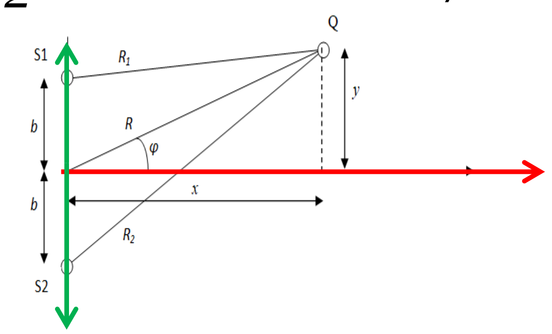
$-\pi$ π

$$p_{tot} = \frac{2A}{R}\exp[i(\omega t - kR)]\cos(kb\sin\varphi)$$

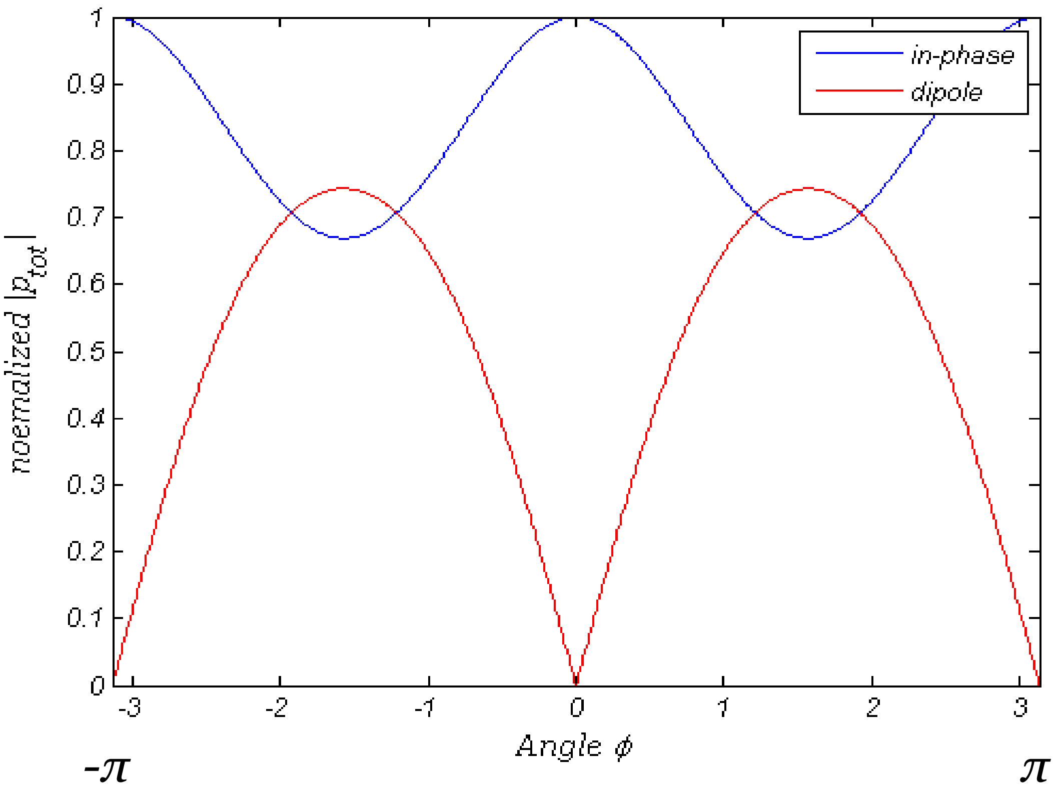
$$p_{d,tot} = \frac{2A}{R}\exp[i(\omega t - kR)]\sin(kb\sin\varphi)$$

b=0,2 μ, f=1000 Hz, c=1500 m/sec.



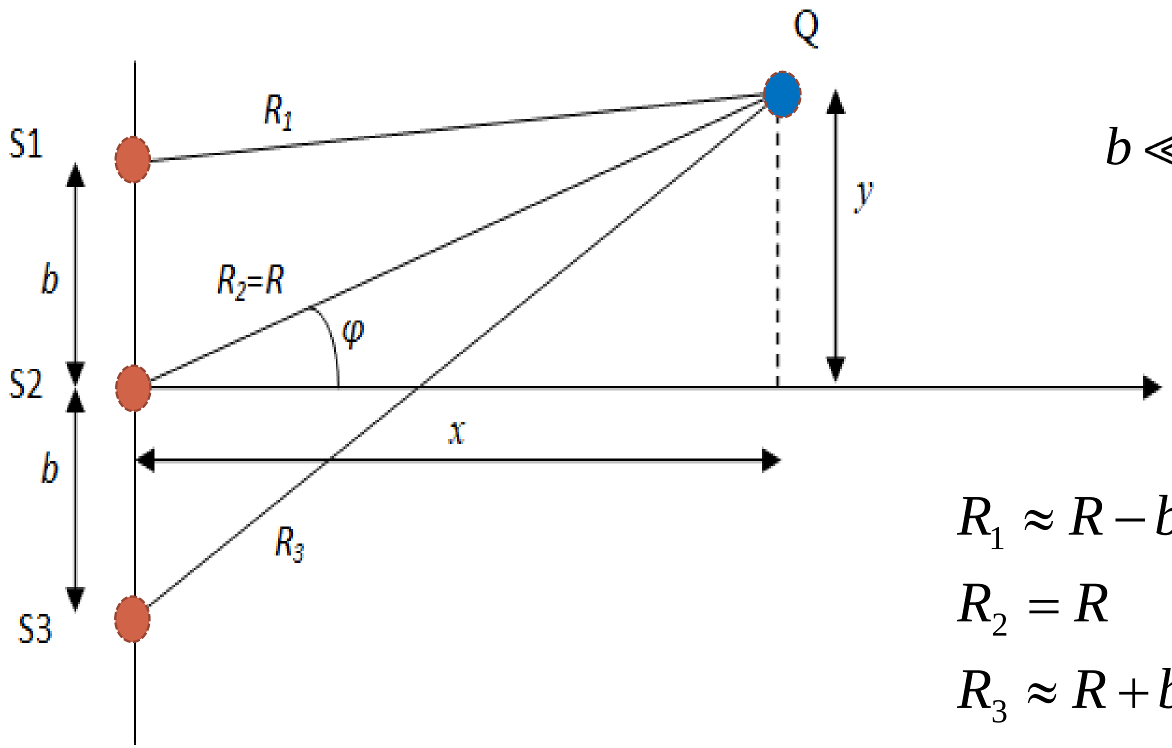


b=0,2 μ, f=1000 Hz, c=1500 m/sec.



$$p_{tot} \equiv C \exp[i(\omega t - kR)]$$

$$C_{tot} = \frac{2A}{R}\cos(kb\sin\varphi) \qquad C_{d,tot} = \frac{2A}{R}\sin(kb\sin\varphi)$$



$b \ll R$

$R_1 \approx R - b\sin\phi$

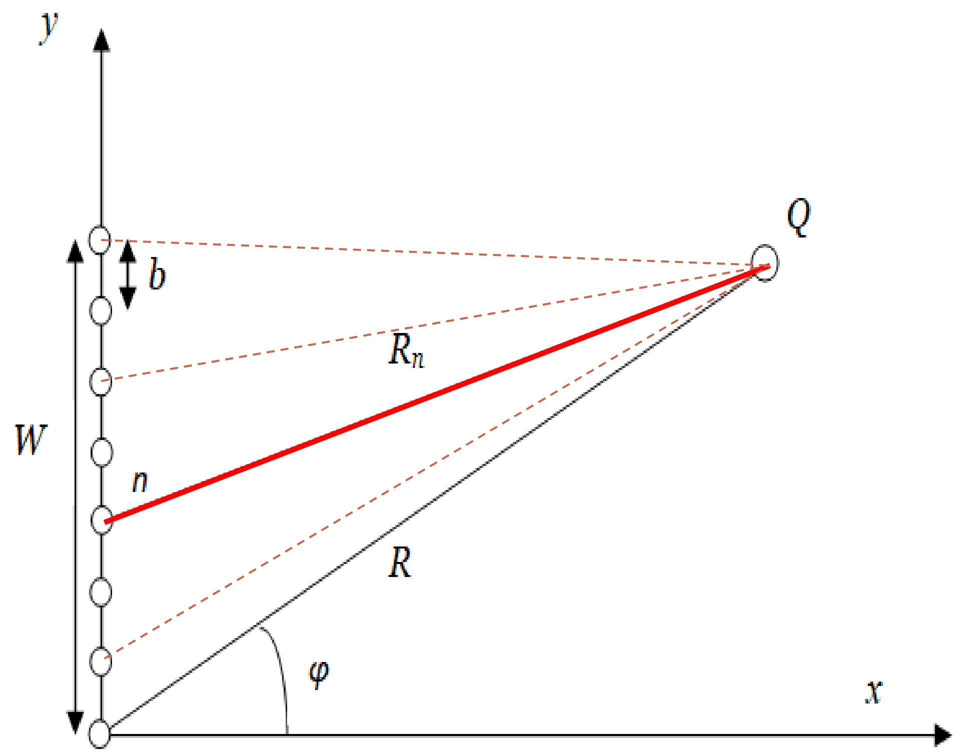
$R_2 = R$

$R_3 \approx R + b\sin\phi$

$$p_{tot} = \frac{A}{R_1}\exp[i(\omega t - kR + kbsi\varphi)] + \frac{A}{R}\exp[i(\omega t - kR)] + \frac{A}{R_3}\exp[i(\omega t - kR - kb\sin\varphi)]$$

$$p_{tot} = \frac{A}{R}\exp[i(\omega t - kR)][1 + 2\cos(kb\sin\varphi)]$$

$$C_3 = \frac{A}{R}[1 + 2\cos(kb\sin\varphi)]$$

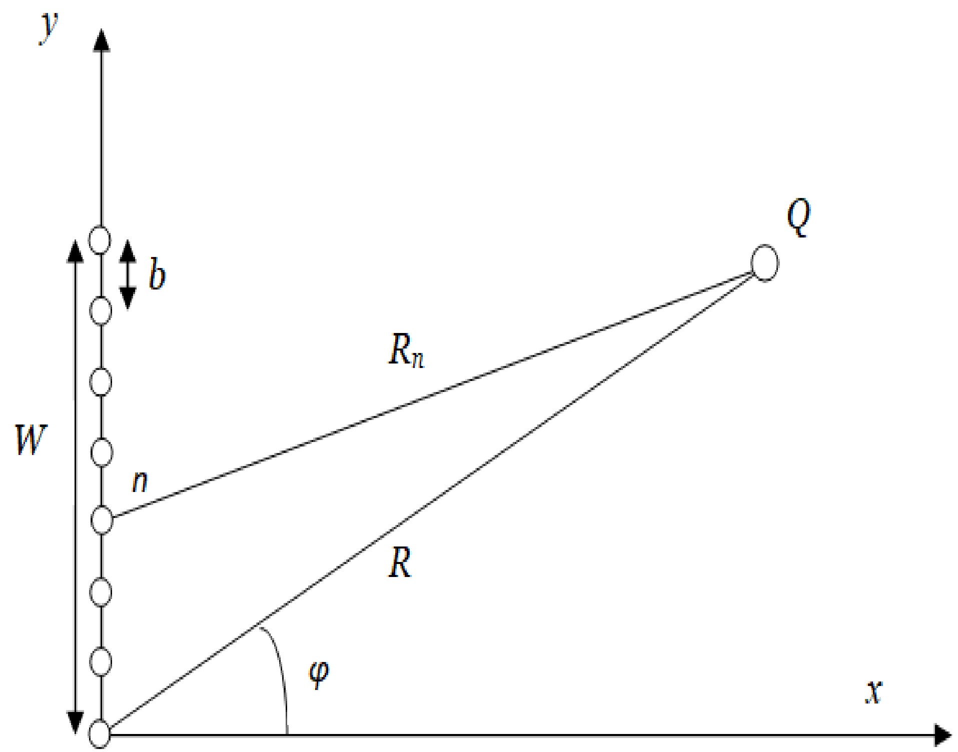


$$b = \frac{W}{N-1}$$

$$W \ll R$$

$$p_n = \frac{A}{R_n}\exp[i(\omega t - k(R - nb\sin\varphi))] = \frac{A}{R_n}\exp[i(\omega t - kR + \frac{nkW\sin\varphi}{N-1})]$$

$$p_{tot} = \frac{A}{R}\exp[i(\omega t - kR)]\sum_{n=0}^{N-1}\exp\left(\frac{inkW\sin\varphi}{N-1}\right)$$



$$b = \frac{W}{N-1}$$

$$W \ll R$$

$$p_{tot} = \frac{A}{R}\exp[i(\omega t - kR)]\sum_{n=0}^{N-1}\exp\left(\frac{inkW\sin\varphi}{N-1}\right)$$

$$C_W = \frac{A}{R}\sum_{n=0}^{N-1}\exp\left(\frac{inkW\sin\varphi}{N-1}\right)$$

$$p_{tot} \equiv C_w \exp[i(\omega t - kR)]$$

$$\sum_{n=0}^{N-1} r^n = \frac{1-r^N}{1-r}$$

$$r = \exp\left(\frac{ikW\sin\varphi}{N-1}\right)$$

$$C_W = \frac{A}{R}\left\{\frac{1-\exp\left(\frac{iNkW\sin\varphi}{N-1}\right)}{1-\exp\left(\frac{ikW\sin\varphi}{N-1}\right)}\right\}$$

$$1-e^{ix} = -e^{ix/2}(e^{ix/2}-e^{-ix/2}) = -2ie^{ix/2}\sin\frac{x}{2}$$

$$C_W = N\frac{A}{R}\left\{\frac{\exp\left(iNk(W/2)\frac{\sin\varphi}{N-1}\right)}{\exp\left(ik(W/2)\frac{\sin\varphi}{N-1}\right)}\right\}\frac{\sin\left(Nk(W/2)\frac{\sin\varphi}{N-1}\right)}{N\sin\left(k(W/2)\frac{\sin\varphi}{N-1}\right)}$$

Για N πολύ μεγάλο $\quad C_W = N\frac{A}{R}\exp\left(\frac{ikW\sin\varphi}{2}\right)\frac{\sin\left(Nk(W/2)\frac{\sin\varphi}{N-1}\right)}{N\sin\left(k(W/2)\frac{\sin\varphi}{N-1}\right)}$

$$p_{tot} \equiv C_w \exp[i(\omega t - kR)]$$

$$C_W = N\frac{A}{R}\exp\left(\frac{ikW\sin\varphi}{2}\right)\frac{\sin\left(Nk(W/2)\frac{\sin\varphi}{N-1}\right)}{N\sin\left(k(W/2)\frac{\sin\varphi}{N-1}\right)}$$

$$p_{tot} = N\frac{A}{R}\exp\left[i\left(\omega t - kR + \left(\frac{kW\sin\varphi}{2}\right)\right)\right]\frac{\sin\left(Nk(W/2)\frac{\sin\varphi}{N-1}\right)}{N\sin\left(k(W/2)\frac{\sin\varphi}{N-1}\right)}$$

$$\left|p_{tot}\right| = N\frac{A}{R\sqrt{2}}\frac{\sin\left(Nk(W/2)\frac{\sin\varphi}{N-1}\right)}{N\sin\left(k(W/2)\frac{\sin\varphi}{N-1}\right)}$$

$$\left|p_{tot}\right| = N \frac{A}{R\sqrt{2}} D$$

$$\left|p_{tot}\right| = N \frac{A}{R\sqrt{2}} \frac{\sin\left(Nk(W/2) \dfrac{\sin\varphi}{N-1} \right)}{N \sin\left(k(W/2) \dfrac{\sin\varphi}{N-1} \right)}$$

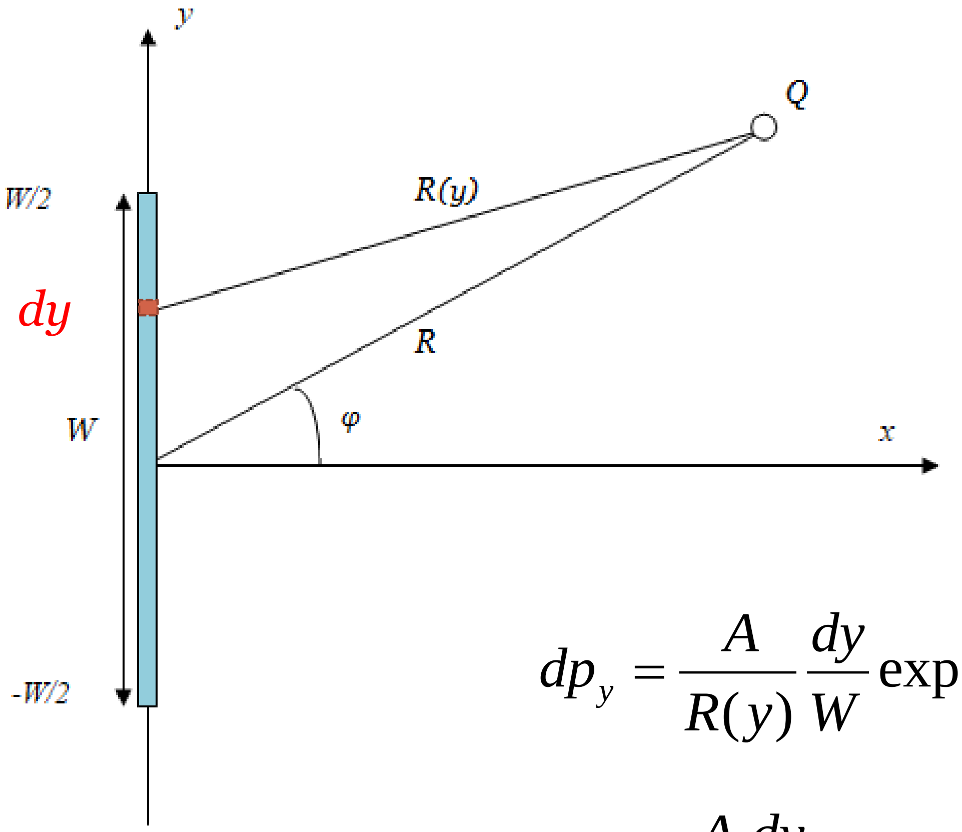
$$D = \frac{\sin\left(\dfrac{N}{N-1} \dfrac{kW}{2} \sin\varphi \right)}{N \sin\left(\dfrac{1}{N-1} \dfrac{kW}{2} \sin\varphi \right)}$$

$$D = \frac{\sin\left(\frac{N}{N-1}\frac{kW}{2}\sin\varphi\right)}{N\sin\left(\frac{1}{N-1}\frac{kW}{2}\sin\varphi\right)}$$

Για $N >>$

$$D = \frac{\sin\frac{kW\sin\phi}{2}}{\frac{kW\sin\phi}{2}}$$

Συντελεστής κατευθυντότητας ή απόκριση κατευθυντότητας (directional response)



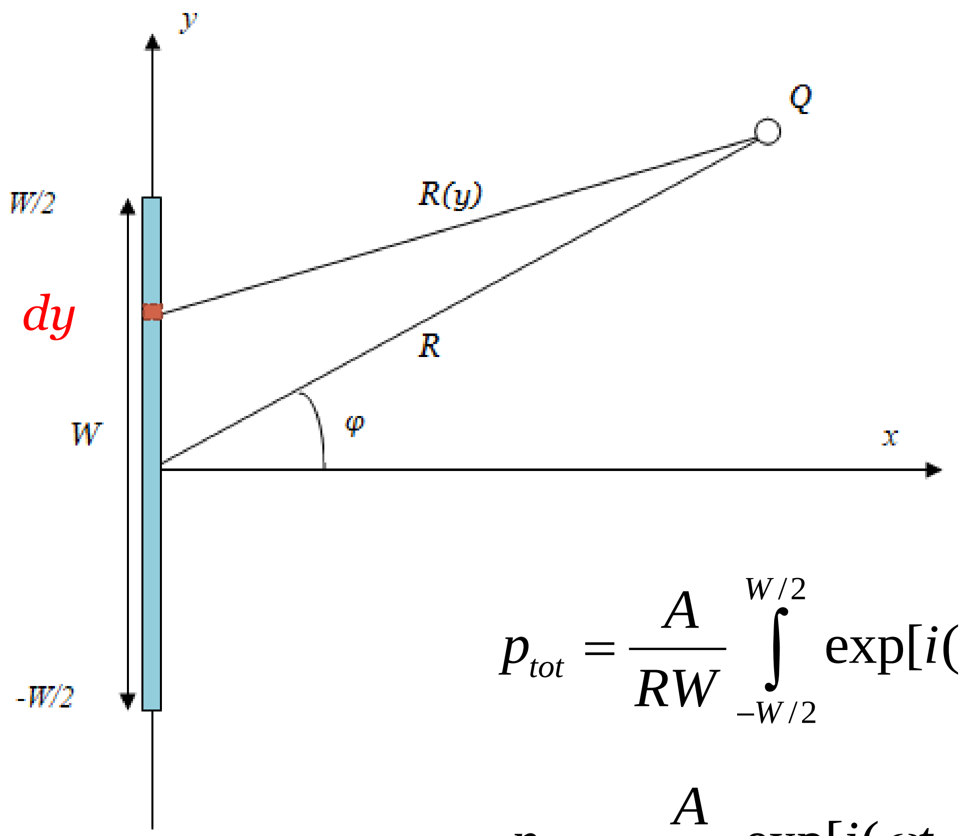
$$W \ll R$$

$$R(y) = R - y\sin\varphi$$

$$dp_y = \frac{A}{R(y)}\frac{dy}{W}\exp[i(\omega t - kR(y))]$$

$$dp_y = \frac{A}{R}\frac{dy}{W}\exp[i(\omega t - kR + ky\sin\varphi)]$$

$$p_{tot} = \frac{A}{RW}\int_{-W/2}^{W/2}\exp[i(\omega t - kR + ky\sin\varphi)]dy$$

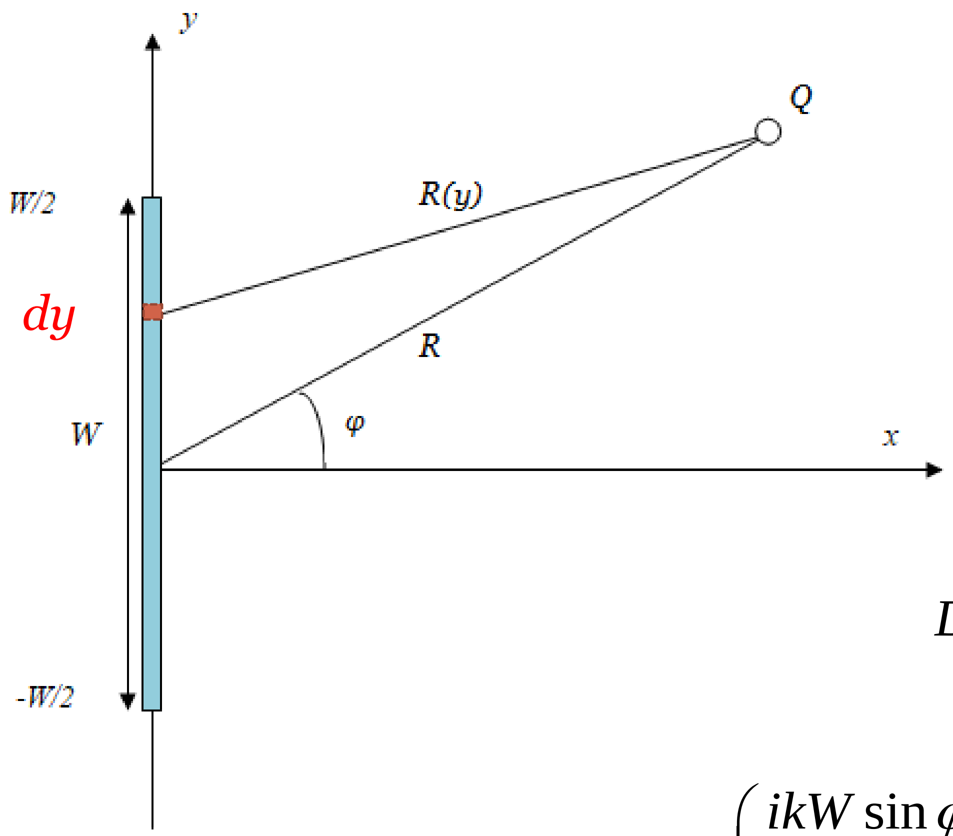


$$R(y) = R - y\sin\varphi$$

$$p_{tot} = \frac{A}{RW}\int_{-W/2}^{W/2} \exp[i(\omega t - kR + ky\sin\varphi)]dy$$

$$p_{tot} = \frac{A}{RW}\exp[i(\omega t - kR)]\int_{-W/2}^{W/2} \exp(iky\sin\varphi)dy$$

$$D = \frac{1}{W}\int_{-W/2}^{W/2} \exp(iky\sin\varphi)dy \qquad p_{tot} = \frac{A}{R}D\exp[i(\omega t - kR)]$$

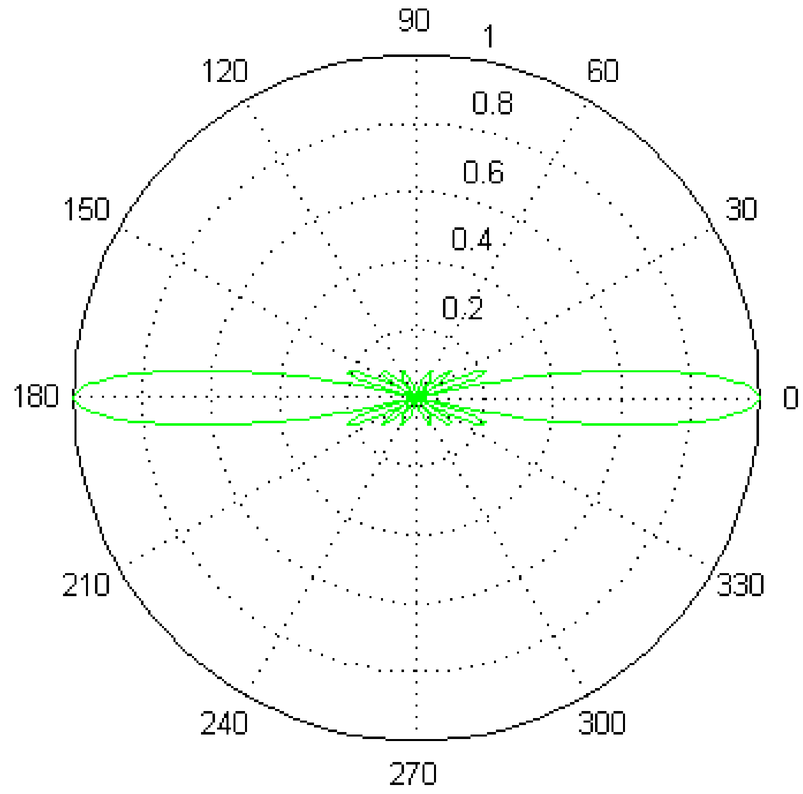


$$R(y) = R - y\sin\varphi$$

$$D = \frac{1}{W}\left.\frac{\exp(iky\sin\phi)}{ik\sin\phi}\right]_{-W/2}^{W/2}$$

$$D = \frac{\exp\left(\frac{ikW\sin\varphi}{2}\right) - \exp\left(\frac{-ikW\sin\varphi}{2}\right)}{ikW\sin\varphi}$$

$$D = \frac{\sin\frac{kW\sin\phi}{2}}{\frac{kW\sin\phi}{2}}$$



Ένα τυπικό πολικό διάγραμμα εκπομπής πηγής που μοντελοποιείται από συνεχή κατανομή σημειακών πηγών μήκους 4λ=6 m.
Η συχνότητα είναι 1000 Hz και η ταχύτητα διάδοσης του ήχου 1500 m/sec.