Περίπτωση ρευστών στρωμάτων

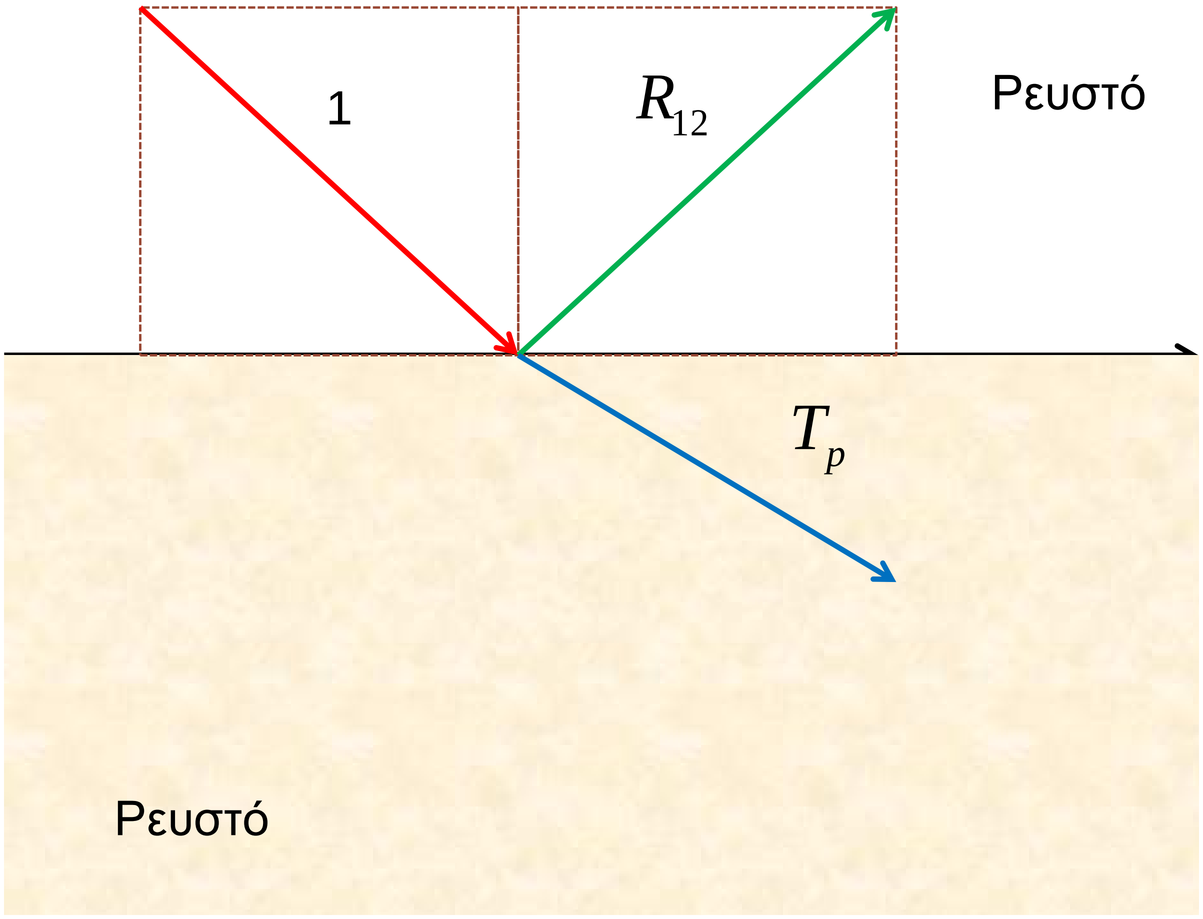
Εισαγωγή στην Ακουστική Ωκεανογραφία

Περίπτωση Ρευστού Πυθμένα

$$\Phi_{1\iota} = e^{i(k_x x + k_{z1} z - \omega t)}$$

$$\Phi_{1r} = R_{12} e^{i(k_x x - k_{z1} z - \omega t)}$$

$$\Phi_{2} = T_p e^{i(k_x x + k_{z2} z - \omega t)}$$


1
R_{12}
Ρευστό
T_p
Ρευστό

$$\sigma_{1,zz} = \sigma_{2,zz}$$

$$d_{1z} = d_{2z}$$

$$\sigma_{zz} = \lambda \nabla^2 \Phi, \quad \vec{d} = \nabla \Phi$$

$$\lambda = \rho c_p^2 \qquad \nabla^2 \Phi = \frac{1}{c_p^2} \frac{\partial^2 \Phi}{\partial t^2}$$

$$\rho_1 \frac{\partial^2 \Phi_1}{\partial t^2} = \rho_2 \frac{\partial^2 \Phi_2}{\partial t^2}$$

$$\frac{\partial \Phi_1}{\partial z} = \frac{\partial \Phi_2}{\partial z}$$

$$\Phi_1 = \Phi_{1i} + \Phi_{1r} \qquad \Phi_2$$

$$\rho_1\omega^2\{e^{i(k_x x + k_{z1}z - \omega t)} + R_{12}e^{i(k_x x - k_{z1}z - \omega t)}\} = \rho_2\omega^2 T_p e^{i(k_x x + k_{z2}z - \omega t)}$$

$$ik_{z1}\{e^{i(k_x x + k_{z1}z - \omega t)} - R_{12}e^{i(k_x x - k_{z1}z - \omega t)}\} = ik_{z2}T_p e^{i(k_x x + k_{z2}z - \omega t)}$$

$$z = 0 \quad \Rightarrow \quad \rho_1(1 + R_{12}) = \rho_2 \mathrm{T}_p$$

$$k_{z1}(1 - R_{12}) = k_{z2}T_p$$

$$R_{12} = \frac{k_{z1}\rho_2 - k_{z2}\rho_1}{k_{z1}\rho_2 + k_{z2}\rho_1}$$

$$T_p = \frac{2k_{z1}\rho_1}{k_{z1}\rho_2 + k_{z2}\rho_1}$$

$$k_{z1} = k_1 \cos\theta_1 = \frac{\omega}{c_1}\cos\theta_1$$

$$k_{z2} = k_2 \cos\theta_2 = \frac{\omega}{c_2}\cos\theta_2$$

$$R_{12} = \frac{\rho_2 c_2 \cos\theta_1 - \rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}$$

$$\sin\theta_2 = \frac{c_2}{c_1}\sin\theta_1$$

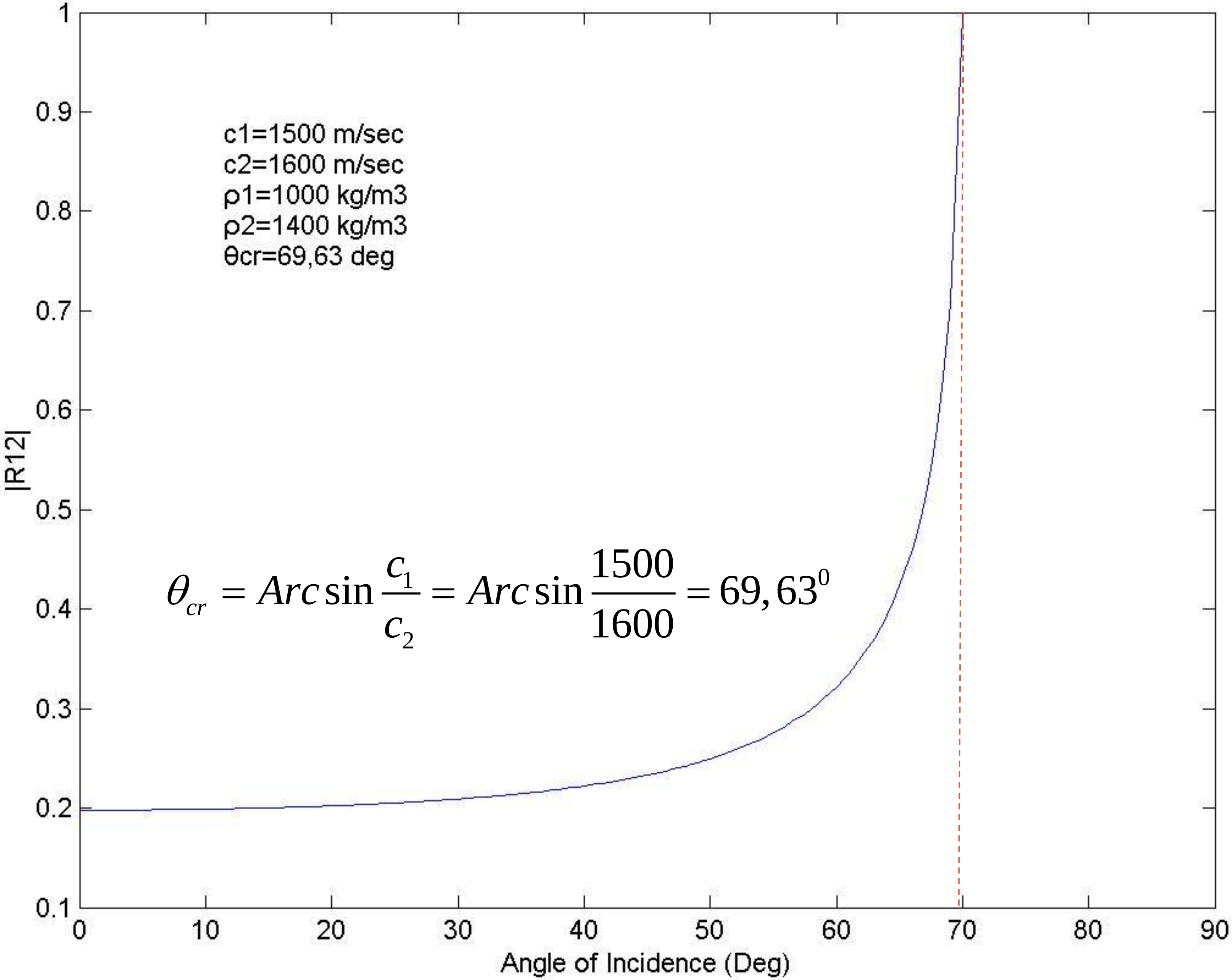
$$\cos\theta_2 = \sqrt{1-(\frac{c_2}{c_1})^2 \sin^2\theta_1}$$

$$\theta_{cr} = Arc\sin\frac{c_1}{c_2}$$ Κρίσιμη γωνία

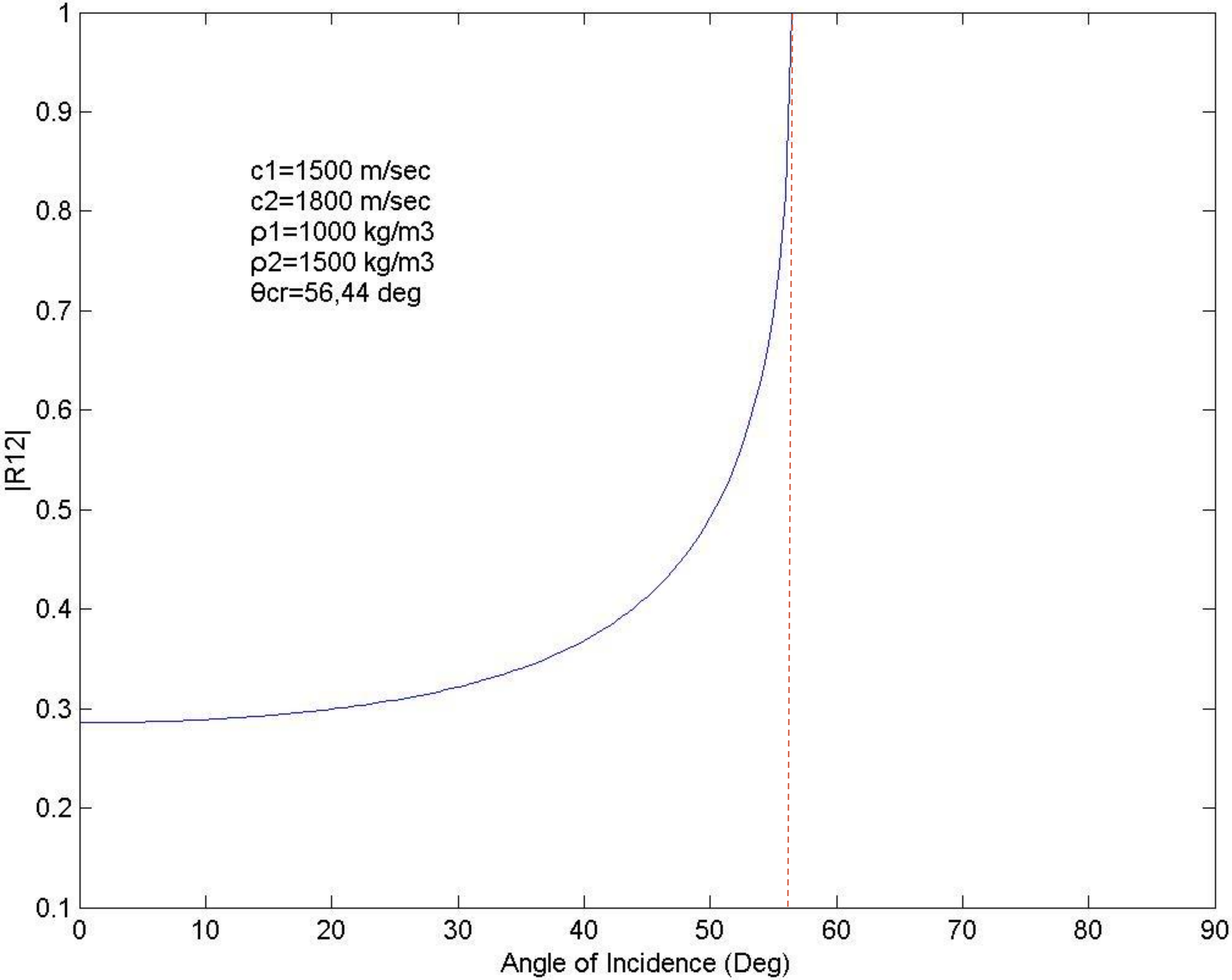
Για $\theta > \theta_{cr}$

$$R_{12} = \frac{k_{z1}\rho_2 - ig_2\rho_1}{k_{z1}\rho_2 + ig_2\rho_1}$$

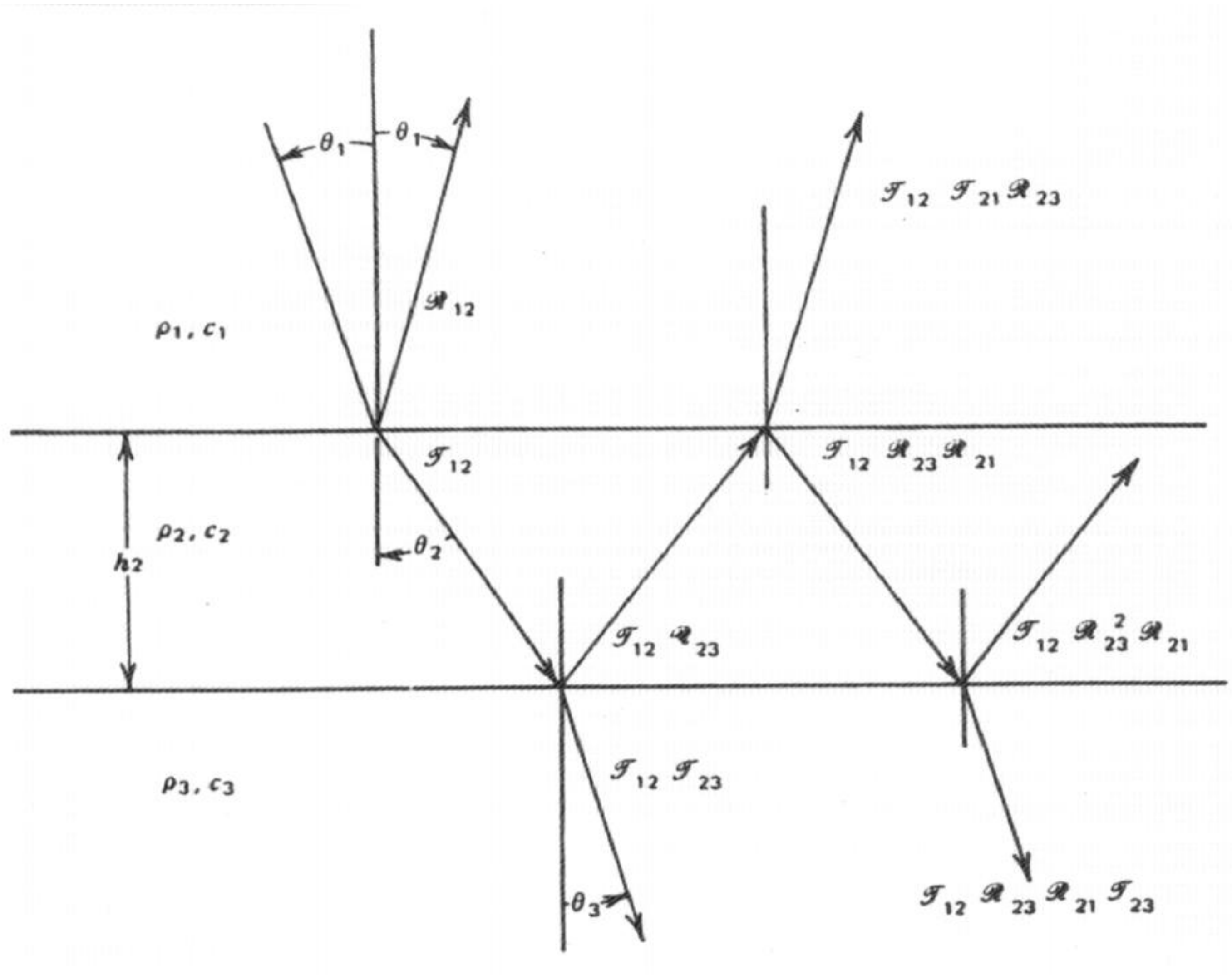
$$R_{12} = -e^{i2\chi} \qquad |R_{12}| = 1 \qquad \chi = Arc\tan(\frac{\rho_2}{\rho_1}\frac{k_{z1}}{g_2})$$



$$\theta_{cr} = Arc\sin\frac{c_1}{c_2} = Arc\sin\frac{1500}{1600} = 69,63^0$$



c1=1500 m/sec
c2=1800 m/sec
ρ1=1000 kg/m3
ρ2=1500 kg/m3
θcr=56,44 deg
|R12|
Angle of Incidence (Deg)
0
10
20
30
40
50
60
70
80
90
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

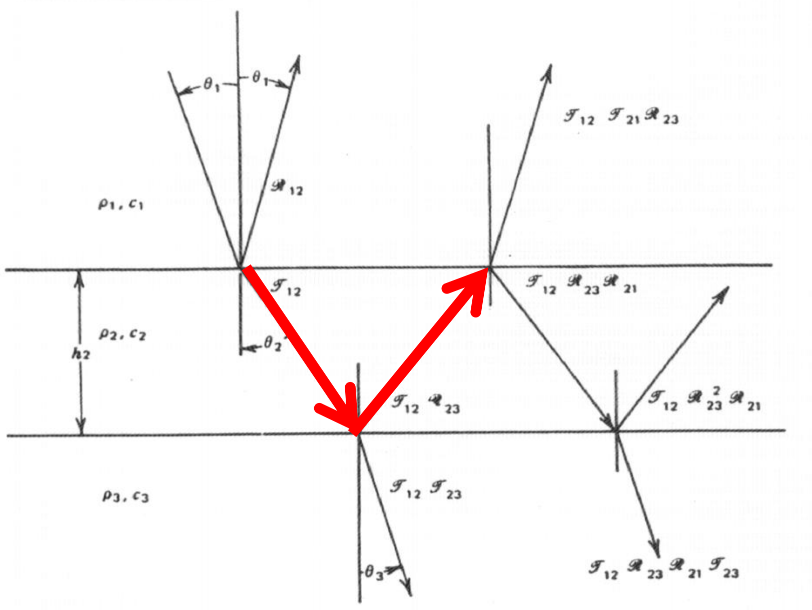

θ1
θ1
𝒯12 𝒯21 ℛ23
ℛ12
ρ1, c1
𝒯12
𝒯12 ℛ23 ℛ21
ρ2, c2
θ2
h2
𝒯12 ℛ23
𝒯12 ℛ23 2 ℛ21
ρ3, c3
𝒯12 𝒯23
θ3
𝒯12 ℛ23 ℛ21 𝒯23

$$R_{12} = \frac{\rho_2 c_2 \cos\theta_1 - \rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}$$

$$R_{23} = \frac{\rho_3 c_3 \cos\theta_2 - \rho_2 c_2 \cos\theta_3}{\rho_3 c_3 \cos\theta_2 + \rho_2 c_2 \cos\theta_3}$$

$$T_{12} = \frac{2\rho_1 c_2 \cos\theta_1}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}$$

$$T_{23} = \frac{2\rho_2 c_3 \cos\theta_2}{\rho_3 c_3 \cos\theta_2 + \rho_2 c_2 \cos\theta_3}$$



$$2k_2h_2\cos\theta_2 = 2k_{z2}h_2$$

$$R_{12} = -R_{21}$$

$$T_{12}T_{21} = 1 - R_{12}{}^2$$

$$R_{13} = R_{12} + T_{12}T_{21}R_{23}\exp(2i\phi_2) + T_{12}T_{21}R_{23}{}^2R_{21}\exp(4i\phi_2) + \ldots$$

$$\phi_2 = k_2h_2\cos\theta_2$$

$$S = \sum_{n=0}^{\infty} r^n = (1-r)^{-1} \qquad |r| < 1$$

$$R_{13} = R_{12} + T_{12} T_{21} R_{23} \exp(2i\phi_2) \sum_{n=0}^{\infty} [R_{23} R_{21} \exp(2i\phi_2)]^n$$

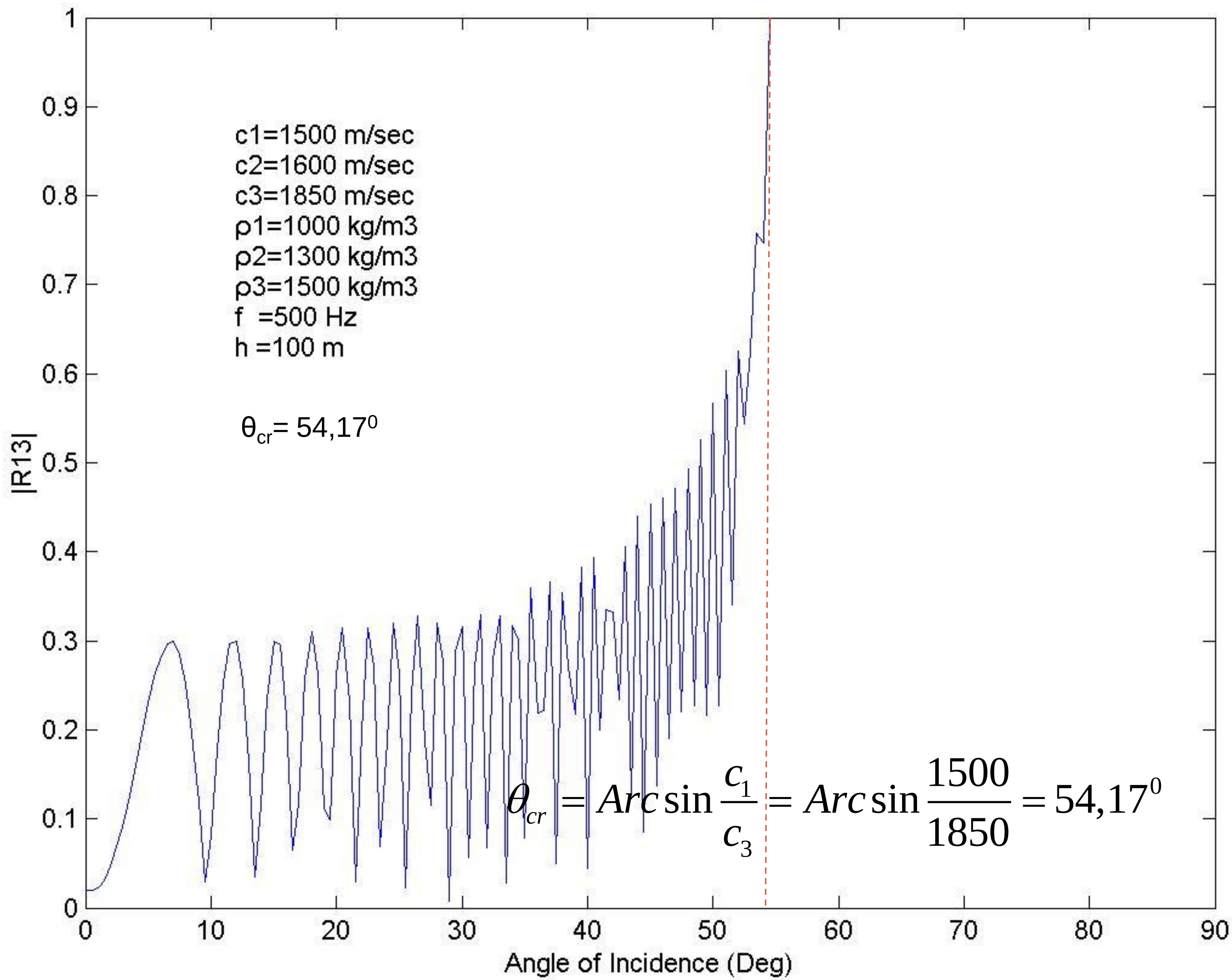
$$R_{13} = \frac{R_{12} + R_{23} \exp(2i\phi_2)}{1 + R_{12} R_{23} \exp(2i\phi_2)}$$

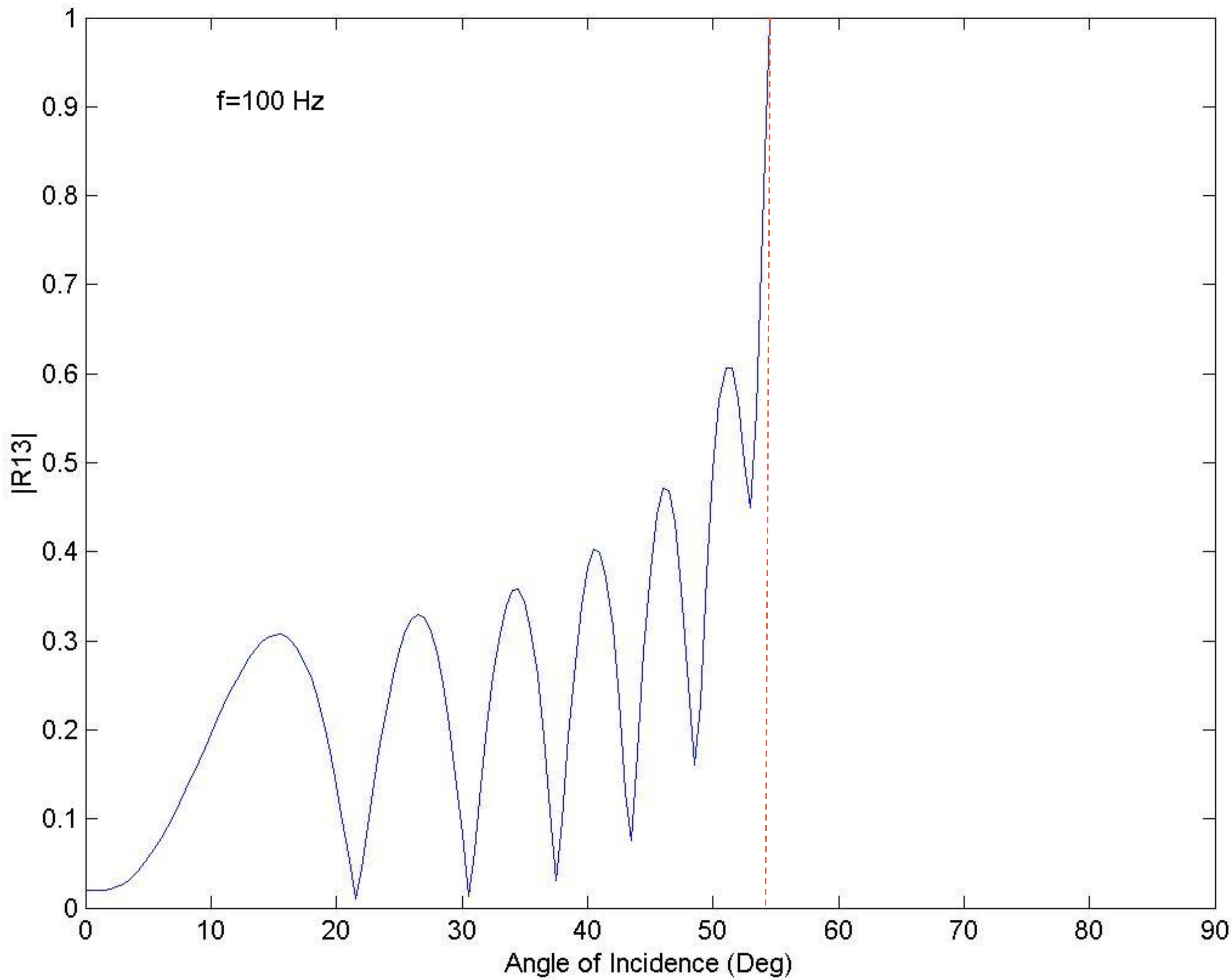
$$T_{13} = \frac{T_{12} T_{23} \exp(i\phi_2)}{1 + R_{12} R_{23} \exp(2i\phi_2)}$$

$$R_{(n-2)n} = \frac{R_{(n-2)(n-1)} + R_{(n-1)n} \exp(2i\phi_{n-1})}{1 + R_{(n-2)(n-1)} R_{(n-1)n} \exp(2i\phi_{n-1})}$$

$$R_{(n-3)n} = \frac{R_{(n-3)(n-2)} + R_{(n-2)n} \exp(2i\phi_{n-2})}{1 + R_{(n-3)(n-2)} R_{(n-2)n} \exp(2i\phi_{n-2})}$$

$$R_{1n} = \frac{R_{12} + R_{2n} \exp(2i\phi_2)}{1 + R_{12} R_{2n} \exp(2i\phi_2)}$$


c1=1500 m/sec
c2=1600 m/sec
c3=1850 m/sec
ρ1=1000 kg/m3
ρ2=1300 kg/m3
ρ3=1500 kg/m3
f =500 Hz
h =100 m
$\theta_{cr}= 54,17^0$
$\theta_{cr} = Arc\sin\frac{c_1}{c_3} = Arc\sin\frac{1500}{1850} = 54,17^0$
|R13|
Angle of Incidence (Deg)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10
20
30
40
50
60
70
80
90


f=100 Hz
|R13|
Angle of Incidence (Deg)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70
80
90