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MATCHING MODAL ARRIVALS IN SHALLOW WATER FOR TOMOGRAPHIC INVERSIONS

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The paper is referred to the problem of estimating water and/or bottom parameters in shallow water, using a single receiver and measurements of the acoustic signal due to a known source in the time domain. As it is very difficult in shallow water to identify ray arrivals, our analysis is based on the assumption that alternative observables and specifically modal arrivals are identified. The success of modal arrival identification is also the criterion for the estimation of a solution to the inverse problem. In particular, mode identification is based on an optimisation scheme according to which we seek among a wide search space, the environment which provides the least L_2 norm on the travel time differences between the modes of the measured and the replica signal corresponding to a candidate environment. To this end, an identification routine based on the calculation of the group velocities of the candidate environment is utilised. As the search algorithm is based on the minimisation of the travel time differences only, the results of the inversion procedure is in fact an environment considered very close to the actual one but not necessarily the actual environment itself. This estimation is however particularly useful when a linear inversion scheme or a detailed local search for fine-tuning the results is to be applied as the starting point of both algorithms should be an environment close to the actual one. Results of this inversion approach with a simulated shallow water benchmark environment are presented and discussed.

1. INTRODUCTION

The paper presents an acoustical inversion technique for the environmental parameters estimation in shallow water based on the identification of the modal structure of a broad-band signal measured at a single hydrophone. The motivation for the development of the method presented here is that if a wide search space is the only a-priori information on the parameters to be recovered, linear methods based on modal travel time information [1] fail, unless a background environment close to the actual one is defined. On the other hand ray methods in shallow water are not easy to be applied, due to the complicated ray structure of the field, which renders the problem of ray identification in such an environment very difficult. Still, broad-band matched-field methods using the whole arrival pattern shape as their observable, have the disadvantage that as they have to handle noisy data in the real world, the actual amplitude of the signal at a single receiver is never a reliable information.

An alternative would be to look for observables not affected too much by noise. Modal arrivals seem to be the solution to this problem as their location in the time history of the signal is rather stable and if their amplitudes are above a certain threshold, they can in principle be identified. Of course source-receiver separation has to be enough so that the modal arrivals are well-resolved [2], but on the other hand not too far if bottom properties are to be recovered.

The inversion scheme proposed here, combines an optimisation procedure based on the minimisation of an appropriate cost function, and a modal identification procedure, which is necessary in order to define the cost function. An additive feature of the method is that it can be viewed as a "background" environment provider for the application of a linear inversion scheme rather than a solver to the inverse problem. The mode identification approach is described briefly in Section 2 and the basic features of the optimisation routine are given in Section 3.

Although the method can be applied for multidimensional inversions including water and bottom parameters, inversion results presented here are referred to test case of a benchmark exercise [3], for which we have chosen to invert only for the sediment sound velocity profile. This parameter is considered to be the most important for sediment classification and as several works have shown, the one that affects most the acoustic field in water.

2. THE MODAL IDENTIFICATION APPROACH

The identification of the peaks of the measured signal that correspond to modal arrivals is done according to [2] on the basis of a "background environment" which is considered known. In order that the method is valid, this environment should be close to the actual one. The arrival times of the modal peaks of the background environment are simply defined using calculations of the group velocity. The method is briefly described as following:

The absolute value of the arrival pattern of a narrow band signal contains I peaks characterized as local maxima of the field :

$$\tilde{t}_i :\rightarrow \frac{\partial p(r,z;z_0;t)}{\partial t} = 0 \quad i = 1,....,I \tag{1}$$

where p is the sound pressure measured at source r and depth z.

Some of these peaks are characterized as modal arrivals in the sense that they correspond to the modal packets. For these peaks $\tilde{t}_i = t_j, \quad j = 1,....J$, where t_j is the travel time of the j th mode and J is the highest order of propagating modes.

For the peaks in the background environment denoted by $\tilde{t}_i^{\,0}$ we have similarly :

$$\tilde{t}_i^{\,0} :\rightarrow \frac{\partial p^0(r,z;z_0;t)}{\partial t} = 0 \quad i = 1,....,I^0 \tag{2}$$

where I^0 is not necessarily equal to I.

The peaks corresponding to the modal packets are the peaks for which $\tilde{t}_i^{\,0} = t_j^0 \quad j = 1,...N$, where N is the highest order of propagating modes in the background environment. Note that *J* may be different from *N.* Thus, by setting *M = min{J,N}* we will restrict our analysis to *M* modes.

Letting $\delta t_{ij} = \tilde{t}_i - t_j^0$ to be the difference between the arrival times of the peaks of the actual signal and the modal arrivals of the reference environment and assuming that the modal arrivals of the actual field appear in locations close to those of the reference one, we search among all possible pairs, for the δt_{ij} giving the minimum difference. By repeated application of this procedure and imposing a threshold for the amplitude of the identified peaks (low amplitude peaks may be due to noise and therefore should be excluded from the identification), we come up with a set of identified peaks.

Note that if the background environment is not close to the actual one, the identification approach will probably lead to a small number of identified arrivals, which may also correspond to large travel time differences. This apparent deficiency of the algorithm is exploited in our optimisation scheme, as we are searching for the background environment giving the least travel time differences expressed in terms of an L_2 norm. This will become clear in the next section. Of course additional criteria may also be imposed that would restrict the possible solutions appearing in the final analysis. However this is a matter of future research.

3. THE SEARCH ALGORITHM AND THE OBJECTIVE FUNCTION

We define a cost function as following:

$$P(\delta t_i) = \sqrt{\tfrac{1}{\Lambda} \sum_{l=1}^{\Lambda} \hat{\delta} t_l^{\,2}} \qquad (3)$$

Here, Λ is the number of identified modes and $\hat{\delta} t_l$ is the time difference between the modal arrival of the actual signal and the predicted arrival time of the corresponding mode of the background environment. Note that the time difference is defined for the identified modes only and the index *l* does not correspond to the order of some mode.

In theory, when the background environment and the actual one coincide, the cost function takes its minimum value which is 0. If a search space on the parameters to be recovered is given, the cost function may be used in connection with a search algorithm to provide the set of parameters that minimize the cost function. This is actually the principle of any "matching" algorithm. In our case, this concept reflects a "matching" of the modal peaks. Of course due to the non-linear nature of the problem, one would expect that the cost function might exhibit more than one local minima, and therefore one should be very careful when applying an inversion procedure and should use a search method which avoids trapping to a local minimum. A Genetic Algorithm appropriately developed, would in principle be a good solution to this problem [4] especially if the optimization is performed over a multi-dimensional space. If however we restrict our search space to two dimensions (two unknowns) a simpler method utilizing an even discretization grid for the search space would provide the values of the cost function for the whole search space. Thus, by looking at the local minima and exploiting any additional a-priori information on the parameters to be recovered one would in principle be able to choose among them the most probable one. An alternative scenario would be to apply local search algorithms around all the predicted local minima as an attempt to fine-tune the "most probable" solutions. In this connection, note that the vector of the time differences appearing in (3) is the input (data) vector $\boldsymbol{\delta t_i}$ for a linear inversion scheme based on modal travel time differences which can be used for the fine tuning of the results [1].

In our case we have chosen to apply a simple grid on the search space (being two-dimensional as it will be explained below) without fine tuning and restrict our inversion to sediment properties only.

4. INVERSION RESULTS

4.1 The "actual" environment

The environment considered here is chosen among them of the test cases, which were presented, at the Geoacoustic Inversion Workshop held in Vancouver on 1997 [3]. It is the WAa case. The environment is described as shallow water with downward-refracting velocity profile in water. We have chosen to apply our method for the recovery of the sediment compressional velocity only, considering the substrate properties as known. The "measurements" were simulated by placing a single receiver at the depth of 50 m. and range 1220 m from the source and using the MODE1 normal-mode programme to calculate the pressure field for the whole bandwidth considered in the benchmark exercise (25 to 199 Hz). The signal in the time domain is thereafter calculated by inverse Fourier Transform. [2]. No noise has been added. It should be noted however, that as previously stated, noise is expected to have minimum effect on the identification of the modal peaks. Noise may change the amplitude of the peaks of the signal, and more peaks may appear, but the location of the modal arrivals is not changed.

Table 1 below presents all the known and unknown parameters of the environment, together with the search bounds for the sediment compressional velocities.

The magnitude of the signal simulating the measurement appears in Fig.1 where, the modal peaks have been identified following the procedure described in Section 2 above.

Parameter	True values	Upper search bound	Lower Search bound
Water depth D (m)	115.3		
Sound speed at the surface $c_w(0)$ (m/s)	1480		
Sound speed at the bottom $c_w(D)$ (m/s)	1460		
Source depth (m)	26.4		
Sediment thickness, h(m)	27.1		
Sediment sound speed, $c_{sed}(D)$ (m/s)	1516.2	1450	1550
Sediment sound speed, $c_{sed}(D+h)$ (m/s)	1573.2	1500	1700
Sediment density, ρ (g/cm^3)	1.538		
Subbotom sound .speed c_{hsp} (m/s)	1751.26		
Subbottom density, ρ (g/cm^3)	1.852		

Table 1: Environmental parameters of the test environment.

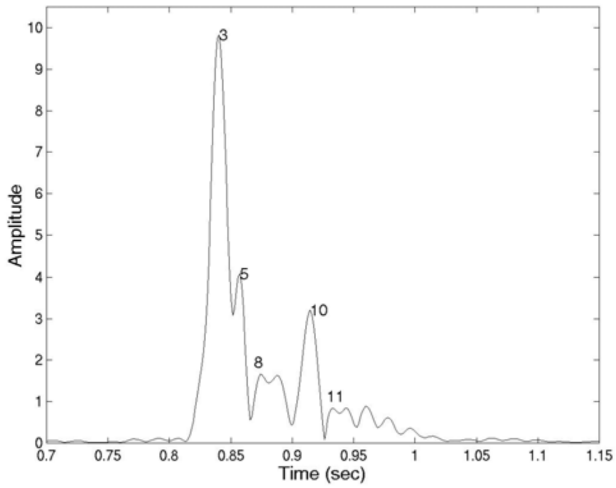

Fig.1: The magnitude of the arrival pattern of the simulated measurement.

4.2 Results of mode-identification

The two dimensional search-space was divided into 100 equally spaced cells and we have considered the sound velocities in each one of the corresponding grid points. Each pair of values for the upper and lower part of the sediment defines the "background" environment upon which the identification is based. For each one of the corresponding environments, only the group velocities for the central frequency of 112 Hz was computed and the results were used as input to the identification scheme.

For most of the pairs, the identification algorithm provided 4-6 modal arrivals, which were in most of the cases not correct, in the sense that they didn't correspond to modes 3,5,8,10,11 as in the "actual" signal. Calculating the cost function for each one of the corresponding cases, we were able to construct Fig. 2, which presents a smoothed contour of the cost function.

The analysis of the cost function values shows that there is a local minimum at the values corresponding to $c_{sed}(D)$ = 1505 m/s and to $c_{sed}(D+h)$ = 1530 m/s, which are relatively close to the actual ones especially in what concerns the value at the upper part of the sediment layer. These values are considered appropriate to define the "background" environment for the application of a linear inversion scheme based on travel time differences. It should be noted that the identified modes for this "background" environment are 3, 6, 7, 8, 10, 11. It is very important to observe that the higher order modes 8,10,11, which are the most important for bottom parameter estimation, are well identified.

5. CONCLUSIONS

The objective of the matching procedure presented in this paper was to provide a good estimate of the environmental parameters in shallow water, using the information on the modal structure of an acoustic signal recorded at a single hydrophone. The procedure is based on an identification scheme for the association of the peaks of the actual signal with modal arrivals.

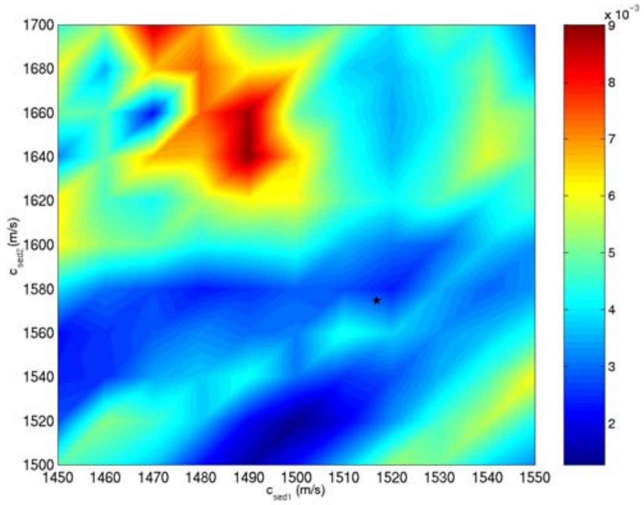

Fig. 2: The cost function calculated in the whole search-space. The actual value is marked with a star.

To this end, a set of background environments from a pre-defined search-space is used. Among them, the one corresponding to the minimum of an appropriate cost function, representing the L_2 norm of the travel time differences between the identified modes and the corresponding peaks of the actual environment is chosen as the best estimate .

Using as a test case a benchmark environment for which sediment parameters were to be recovered and a gaussian broad-band source at low frequencies so that penetration in the bottom is ensured, it was shown that the proposed scheme is indeed capable of providing good estimates of the actual parameters, which in turn could be used as the initial data for a local search algorithm. Note that no claim has been made that the results of this "matching" algorithm are the actual parameters to be recovered. This could not be the case conceptually as the aim of the matching process was to compare "background" environments rather than actual ones. Of course this does not exclude cases in which the "background" environment thus defined is very close to the actual one.

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