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ON CAVITATION IN ELASTODYNAMICS

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Abstract. Motivated by the works of Ball (1982) and Pericak-Spector and Spector (1988), we investigate singular solutions of the compressible nonlinear elastodynamics equations. These singular solutions contain discontinuities in the displacement field and can be seen as describing fracture or cavitation. We explore a definition of singular solution via approximating sequences of smooth functions. We use these approximating sequences to investigate the energy of such solutions, taking into account the energy needed to open a crack or hole. In particular, we find that the existence of singular solutions and the finiteness of their energy is strongly related to the behavior of the stress response function for infinite stretching, i.e. the material has to display a sufficient amount of softening. In this note we detail our findings in one space dimension.

1. Introduction.

In the seminal work of Ball [1] compressible, nonlinear elasticity has been used as a model for fracture and cavitation in elastic materials such as rubber. For a study on the relation between the macroscopic behavior of an elastic solid and its energy functional, see [5]. Ball constructed singular solutions to variational problems, which display discontinuities in the displacement field at the origin. These solutions which are radially symmetric can be seen as describing cavitation. Ball computed their energy and compared it to the energy of trivial solutions with homogeneous strain. The upshot of his study is that for sufficiently large prescribed deformations on the boundary the solution displaying cavity is energetically favorable. Based on his findings Spector and coworkers [7, 8, 6] studied the dynamic case getting similar results. One important feature of the singular solutions constructed in these works is that the normal Cauchy stress vanishes on the boundary of the cavity such that all the integrals needed to define weak solutions and energies exist. However, these constructions do not account for any energy needed for crack and cavity creation. Therefore, we propose a new solution concept which accounts for this energy. It takes into account the layer structure of the solution at the onset of fracture by considering sequences of smooth functions which approximately solve the equations and approach the discontinuous solutions.

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We call this type of solution as singular limiting induced from continuum solution (in short *slic* solution).

We consider the equations of compressible, nonlinear elastodynamics, searching for a displacement field which satisfies

$$\mathbf{y}_{tt} = \operatorname{div} \tau(\nabla \mathbf{y}) \,, \text{ in } \mathbb{R}^d \times (0,T), \; t > 0 \tag{1}$$

for some finite time $T > 0$, where $\tau : \mathbb{R}_+^{d\times d} \to \mathbb{R}_+^{d\times d}$ is the stress response function, which is related to an energy density function $W : \mathbb{R}_+^{d\times d} \to \mathbb{R}$ via $\tau(F) = \frac{\partial W}{\partial F}(F)$.

We are interested in radially symmetric deformations

$$\mathbf{y}(\mathbf{x}, t) = w(|\mathbf{x}|, t)\frac{\mathbf{x}}{|\mathbf{x}|} \tag{2}$$

with prescribed homogeneous tensile deformations in the far field, i.e.

$$\mathbf{y}(x, t) = \lambda \mathbf{x} \quad \text{ for } |\mathbf{x}| > rt \tag{3}$$

for $r, \lambda > 0$. In practice we are interested in the case $\lambda >> 1$. We focus on solutions of (1) having the form (2) and satisfying (3), which have a discontinuity at the origin. Let us note that using the velocity $\mathbf{v} := \mathbf{y}_t$ and the deformation gradient $\mathbf{u} := \nabla \mathbf{y}$ we may rewrite (1) as the following system of conservation laws:

$$\begin{aligned} \mathbf{u}_t &= \nabla \mathbf{v} \\ \mathbf{v}_t &= \operatorname{div}(\tau(\mathbf{u})). \end{aligned} \tag{4}$$

The outline of this contribution is as follows: In Section 2 we describe the basic ideas and the main technical considerations in the one dimensional case. We determine their energy dissipation in Section 3. We would like to mention that basically the same results hold in several space dimensions, but the technical and notational complexity is considerably higher. We refer to [4] for the details.

2. **A special 1-dimensional solution including fracture.**

2.1. **A special ansatz function.** We consider the one dimensional version of (1), i.e.

$$y_{tt} = \tau(y_x)_x \,, \quad x \in \mathbb{R}, \; t > 0 \tag{5}$$

describing the longitudinal or shearing motion of an one dimensional elastic bar. We impose a homogeneous deformation far away, i.e.

$$y(x, t) = \lambda \, x \,, \quad \text{ for } |x| > rt, \tag{6}$$

for some r sufficiently large. For large values of λ the bar under consideration will break and after breaking the continuum hypothesis which is crucial in deriving (5) fails. Therefore, (5) will not be an appropriate description of the physical situation. However, we expect that there is an intermediate range of values of λ from a range of loading where the model is valid to a range where it looses validity. In some situations it may arguably be possible to give a meaning to (5) in and past this intermediate regime. In particular, we think that (5) is a reasonable description at the onset of fracture which is the situation we want to explore.

We assume that the energy density W and the stress function $\tau = W'$ satisfy

$$W \in C^3((0,\infty), \mathbb{R}), \; \tau'(u) > 0, \; \tau''(u) < 0 \quad \forall u > 0 \tag{H1}$$

$$\lim_{u \searrow 0} \tau(u) = -\infty, \quad \lim_{u \searrow 0} W(u) = \infty. \tag{H2}$$

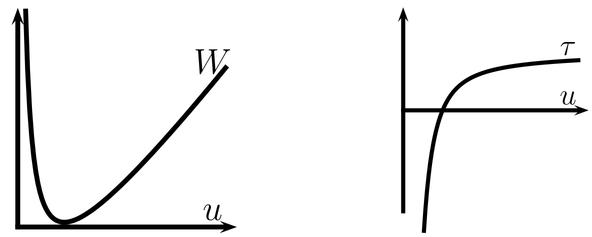

FIGURE 1. Sketch of W and τ satisfying (H1) and (H2).

Given the first condition of (H1) the one dimensional version of (4) has the wave speeds $\lambda_{1,2}(u) = \pm\sqrt{\tau'(u)}$ and is therefore hyperbolic. The second part of (H1) is related to softening elastic response of the material and is a crucial requirement of our analysis. The second hypothesis (H2) is placed to prevent a finite volume from being compressed down to zero. However, as we focus on tensile deformations this condition will not have any significant impact.

Smooth solutions of (5) satisfy the conservation law for energy

$$\left(\frac{1}{2}|y_t|^2 + W(y_x)\right)_t - (\tau(y_x)y_t)_x = 0. \tag{7}$$

This means that any change in mechanical (stored and kinetic) energy in a certain interval is due to the work performed on the boundary of said interval, as can be seen from the integral form

$$\frac{d}{dt}\int_a^b \frac{1}{2}|y_t|^2 + W(y_x)\,dx = \tau(y_x)y_t|_a^b\,. \tag{8}$$

In particular, (8) shows that if the deformation is homogeneous outside a certain interval $(-r, r)$, i.e. $y(x,t) = \lambda x$ for $|x| > r$, the energy inside $(-r, r)$ is conserved.

We focus on solutions in which stress and strain only depend on the self-similar variable $\xi = \frac{x}{t}$ which is achieved by the ansatz $y(x,t) = tY(\frac{x}{t})$. One easily verifies that y satisfies (5) if Y satisfies

$$\xi^2 Y''(\xi) = \tau(Y'(\xi))'. \tag{9}$$

Introducing the velocity $V(\xi) := \xi Y'(\xi) - Y(\xi)$ and the strain $U(\xi) = Y'(\xi)$ in the self-similar variable we can rewrite (9) as

$$\begin{aligned} -\xi U'(\xi) &= V'(\xi) \\ -\xi V'(\xi) &= \tau(U(\xi))'. \end{aligned} \tag{10}$$

We will test a class of self-similar solutions

$$Y(\xi) = \begin{cases} \lambda\xi & \xi < -\sigma \\ -Y(0) + \alpha\xi & -\sigma < \xi < 0 \\ Y(0) + \alpha\xi & 0 < \xi < \sigma \\ \lambda\xi & \sigma < \xi \end{cases} \tag{11}$$

where α, λ, σ and $Y(0)$ are positive parameters that satisfy $\lambda > \alpha$, and

$$\begin{aligned} \lambda\sigma &= Y(0) + \alpha\sigma, \\ \sigma &= \sqrt{\frac{\tau(\lambda) - \tau(\alpha)}{\lambda - \alpha}}. \end{aligned} \tag{12}$$

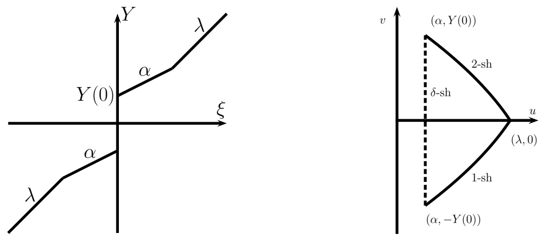

FIGURE 2. Sketches of Y and the Riemann-diagram of the solution

For this deformation the velocity and strain distributions are given by

$$\begin{aligned} U(\xi) &= 2Y(0)\delta_{\xi=0} + \alpha\chi_{|\xi|<\sigma} + \lambda\chi_{|\xi|>\sigma}, \\ V(\xi) &= Y(0)\chi_{0<\xi<\sigma} - Y(0)\chi_{0<-\xi<\sigma}. \end{aligned} \tag{13}$$

Due to the discontinuity of Y (corresponding to a delta distribution in U) it is not straightforward how such a function can be interpreted as a solution of (5) or (10). Before we give a definition of solution based on regularizations of the prospective solutions we like to make some remarks.

Remark 1. 1. The solution ansatz (11) is inspired by the dynamic cavitating solutions in three dimensions introduced in [7]. The main difference is that the solutions in [7] do not contain a delta measure in the strain.

2. For a given $\lambda > 0$ there is a one parameter family of functions (11) satisfying (12).
3. The ansatz (11) has singularities at $\xi = \pm\sigma$ and at $\xi = 0$. The former are shocks. Due to the symmetry the Rankine-Hugoniot conditions at both singularities amount to
$$\begin{aligned} -\sigma(\lambda - \alpha) &= -Y(0) \\ -\sigma(-Y(0)) &= \tau(\lambda) - \tau(\alpha) \end{aligned}$$
and are equivalent to (12). The shock at $\xi = \sigma$ belongs to the second characteristic family and the Lax shock admissibility criterion reads
$$\lambda_2(U-) = \sqrt{\tau'(\alpha)} > \sigma = \sqrt{\frac{\tau(\lambda) - \tau(\alpha)}{\lambda - \alpha}} > \sqrt{\tau'(\lambda)} = \lambda_2(U+).$$
Thus, it is satisfied provided $\lambda > \alpha$. The same holds for the singularity at $\xi = -\sigma$.
4. At positive times the solution Y has a discontinuity of size $2tY(0)$ at the origin, which is associated to a crack whose boundary is moving according to
$$y(0\pm, t) = \pm tY(0).$$
In contrast $(12)_1$ ensures the continuity of Y outside the origin.
5. For $t \searrow 0$ we have $y(x,t) \to \lambda x$ and $v(x,t) \to 0$. Therefore, the initial data is a configuration with homogeneous deformation which is at rest. The problem (5),(6) with these initial data obviously admits the trivial solution $y(x,t) = \lambda x$.
6. In view of (12) a straightforward calculation using $-\xi\partial_\xi\delta_{\xi=0} = \delta_{\xi=0}$ in $\mathcal{D}'$ shows
$$-\xi u' = v'$$

in $\mathcal{D}'$. Hence, (11) satisfies $(10)_1$. Thus, we will focus on giving a meaning to $\tau(u)$ and $(10)_2$ at the origin. This problem is strongly related to the concept of delta shocks, see [3, Sec. 7.5] and references therein. A main difference of our approach compared to those usually employed for delta shocks is that we exploit the structure of (5) to define solutions and the definition has a natural physical interpretation.

7. In one space dimension compressible elasticity coincides with the p-system in fluid dynamics. The solution of the p-system describing vacuum via a δ-measure given in [3, Sec. 7.5] can easily be handled in our framework [4].

2.2. **Slic-solutions.** Our next step is to give a meaning to the equation around $x = 0$ by introducing the notion of *singular limiting induced from continuum solution* (in short *slic*-solution). Roughly speaking a discontinuous function is called a slic-solution provided it can be obtained as the limit of a sequence of smooth functions that are approximate solutions of the problem.

Definition 2.1. Let $y \in L^1_{loc}(\mathbb{R} \times (-\infty, \infty))$, let ϕ be a mollifier: $\phi \in C_c^\infty(\mathbb{R})$, $\phi \geq 0$, $\operatorname{supp}\phi = B_1$ (the ball of radius 1), $\int \phi = 1$. Consider $\phi_n = n\phi\big(nx\big)$ and define

$$y^n(t,x) = \phi_n \star y = \int_{-\infty}^{\infty} \phi_n(x-z)y(t,z)dz.$$

Then y is called a slic-solution of (5) provided

$$\int_{\mathbb{R}}\int_{\mathbb{R}} y^n \psi_{tt} + \tau(y^n_x)\psi_x \, dt\, dx \to 0$$

for any $\psi \in C_c^2(\mathbb{R} \times (-\infty, \infty))$.

Remark 2. 1. The definition of slic-solution is in fact independent of the particular choice of mollifier ϕ.

2. In general a convolution in both space and time may be necessary. In the present context the tested solution is self-similar and it suffices to regularize in space to smooth it.
3. The definition is in fact local and it could be stated for a solution $y \in L^1_{loc}(\mathcal{O})$ with $\mathcal{O}$ an open set in space-time. Moreover, it can be adapted in the obvious way to account for initial conditions (see [4] for further details).
4. A straightforward calculation shows that any standard $W^{1,\infty}$ weak solution of (5), which satisfies the correct initial and boundary data, is also a slic-solution.

Proposition 1. *Let y defined in (11) satisfy (12) and be extended to $t < 0$ by $y(x,t) = \lambda x$, then it is a slic-solution of (5) if and only if*

$$L := \lim_{u\to\infty} \frac{\tau(u)}{u} = 0. \tag{H3}$$

Proof. We only give the main idea of the proof, for details see [4]. Note that y is a weak solution of (5) on any open subset of $\mathbb{R}^2 \backslash \{(x,t) : t > 0, x = 0\}$. Moreover, the sequences $(y^n)_{tt}$ and $\tau(y^n_x)_x$ restricted to $\{(x,t) : |x| > \frac{1}{n} \vee t < 0\}$ are uniformly bounded and converge pointwise. Thus, it is sufficient to check whether

$$\lim_{n\to\infty} \int_0^\infty \int_{-\frac{1}{n}}^{\frac{1}{n}} \tau(y^n_x(x,t))\psi_x(x,t)\, dx\, dt = 0$$

for all $\psi \in C_c^1(\mathbb{R} \times \mathbb{R}, \mathbb{R})$. We compute

$$\lim_{n\to\infty} \int_{-\frac{1}{n}}^{\frac{1}{n}} \tau(y_x^n(x,t))\psi_x(x,t)\,dx = 2LtY(0)\psi_x(0,t),$$

which proves the lemma. □

This shows that there only exist discontinuous slic-solutions of the form (11) to (5), (6) in case the material exhibits softening elastic response. In the sequel we always assume (H3) to be satisfied.

3. **The energy needed to open a crack.** It seems reasonable to consider the energies of the approximate solutions y^n as an approximation of the energy of the slic-solution y. As the approximate solutions are smooth they satisfy the energy identity

$$\left(\frac{1}{2}(v_n)^2 + W(u_n)\right)_t - (v_n\tau(u_n))_x = ((v_n)_t - \tau(u_n)_x)\, v_n, \tag{14}$$

where $v_n := y_t^n$ and $u_n = y_x^n$. Let us define the residual of y^n in the wave equation (5) as f_n, i.e.

$$f_n := (v_n)_t - \tau(u_n)_x. \tag{15}$$

This quantity may be interpreted as an exterior force which (if it were applied) would act on a neighborhood of the origin and make y^n the exact solution. Let $B = (-r, r)$ some interval containing a neighborhood of the whole wave fan at time t. Then (for sufficiently large n) $v_n|_{\partial B} = 0$. The energy of the solution inside B

$$E_B[y^n](t) := \int_B \frac{1}{2}(v_n(x,t))^2 + W(u_n(x,t))\,dx \tag{16}$$

evolves according to

$$\frac{d}{dt}E_B[y^n] = \int_B ((v_n)_t - \tau(u_n)_x)\, v_n\,dx. \tag{17}$$

Proposition 2. *Let $v_n|_{\partial B} = 0$ and $n > \frac{2}{\sigma t}$ then the energy change rate is given by*

$$\begin{aligned}\frac{d}{dt}E_B[y^n](t) = Y(0)^2\sigma + 2\sigma(W(\alpha) - W(\lambda))\\ + 2\int_0^{\frac{1}{n}} \tau(\alpha + 2\phi_n(x)tY(0))2\phi_n(x)Y(0)\,dx.\end{aligned} \tag{18}$$

Proof. We have

$$\frac{d}{dt}E_B[y^n] = \int_{-r}^{r} v_n(v_n)_t + \tau(u_n)(u_n)_t\,dx = 2\int_0^r v_n(v_n)_t + \tau(u_n)(u_n)_t\,dx.$$

For $n\sigma t > 2$ and $x > 0$, we calculate

$$v_n(x,t) = -Y(0)\int_{-\sigma t}^{0} \phi_n(x-z)\,dz + Y(0)\int_0^{\sigma t} \phi_n(x-z)\,dz$$

$$u_n(x,t) = \begin{cases} \alpha + 2tY(0)\phi_n(x) & : \quad x < \frac{1}{n} \\ \alpha & : \quad \frac{1}{n} < x < \sigma t - \frac{1}{n} \\ \alpha \int_0^{\sigma t} \phi_n(x-z)\,dz + \lambda \int_{\sigma t}^{\infty} \phi_n(x-z)\,dz & : \quad \sigma t - \frac{1}{n} < x < \sigma t + \frac{1}{n} \\ \lambda & : \quad x > \sigma t + \frac{1}{n} \end{cases} \tag{19}$$

such that

$$\begin{aligned}(v_n)_t(x,t) &= -Y(0)\sigma\phi_n(x+\sigma t) + Y(0)\sigma\phi_n(x-\sigma t)\\ (u_n)_t(x,t) &= 2Y(0)\phi_n(x) + (\alpha-\lambda)\sigma\phi_n(x-\sigma t)\end{aligned} \tag{20}$$

and thus, as $n > \frac{2}{\sigma t}$,

$$\begin{aligned}\frac{d}{dt}E_B[y^n] = {}& 2\int_{\sigma t-\frac{1}{n}}^{\sigma t+\frac{1}{n}} Y(0)^2\left(\int_0^{\sigma t}\phi_n(x-z)\,dz\right)\sigma\phi_n(x-\sigma t)\,dx\\ &+ 2\int_0^{\frac{1}{n}}\tau(\alpha+2tY(0)\phi_n(x))2Y(0)\phi_n(x)\,dx\\ &+ 2\int_{\sigma t-\frac{1}{n}}^{\sigma t+\frac{1}{n}}\tau\left(\alpha+(\lambda-\alpha)\int_{\sigma t}^{\infty}\phi_n(x-z)\,dz\right)(\alpha-\lambda)\,\sigma\phi_n(x-\sigma t)\,dx\\ &=: I_1^n + I_2^n + I_3^n\end{aligned} \tag{21}$$

We find

$$I_1^n = -\int_{\sigma t-\frac{1}{n}}^{\sigma t+\frac{1}{n}} Y(0)^2\sigma\frac{d}{dx}\left(\int_0^{\sigma t}\phi_n(x-z)\,dz\right)^2 dx = \frac{1}{2}Y(0)^2\sigma \tag{22}$$

and

$$I_3^n = -\sigma\int_{\sigma t-\frac{1}{n}}^{\sigma t+\frac{1}{n}}\frac{d}{dx}W\left(\alpha+(\lambda-\alpha)\int_{\sigma t}^{\infty}\phi_n(x-z)\,dz\right)dx = \sigma(W(\alpha)-W(\lambda)). \tag{23}$$

Inserting (22) and (23) into (21) we obtain the assertion of the Lemma. □

The solution contains three waves, all of which contribute to the energy rate. There are the two shocks located at $\xi = \pm\sigma$ and the discontinuity at 0. Both shocks satisfy the Lax criterion and therefore there is energy dissipation along them. Moreover, the energy dissipation at the shocks $\mu_\sigma, \mu_{-\sigma}$ can be calculated by classical Riemann problem theory, see [3, Sec. 8.5]. This gives

$$\begin{aligned}\mu_\sigma &= -\sigma\left(-\frac{1}{2}Y(0)^2 + W(\lambda) - W(\alpha)\right) + Y(0)\tau(\alpha)\\ &= Y(0)\left(-\frac{W(\lambda)-W(\alpha)}{\lambda-\alpha} + \frac{1}{2}(\tau(\lambda)+\tau(\alpha))\right) < 0,\end{aligned} \tag{24}$$

because $\tau'' < 0$, and

$$\mu_{-\sigma} = \sigma\left(\frac{1}{2}Y(0)^2 + W(\alpha) - W(\lambda)\right) + Y(0)\tau(\alpha) < 0. \tag{25}$$

As there is no energy dissipation/creation away from the three waves the energy balance can be expressed as

$$\frac{d}{dt}E_B[y^n] = \mu_{-\sigma} + p_c^n + \mu_\sigma \tag{26}$$

where

$$p_c^n = 2\left(\int_0^{\frac{1}{n}}\tau(\alpha+2\phi_n(x)tY(0))2\phi_n(x)Y(0)\,dx - Y(0)\tau(\alpha)\right) \tag{27}$$

can be interpreted as the work performed by the force f_n at the crack. Its limiting contribution is given by

$$\begin{aligned}\lim_{n\to\infty} p_c^n &= \lim_{n\to\infty} 2\left(\int_0^{\frac{1}{n}} \tau(\alpha+2\phi_n(x)tY(0))2\phi_n(x)Y(0)\,dx - Y(0)\tau(\alpha)\right)\\ &= \lim_{n\to\infty} 2\left(\int_0^{1} \tau(\alpha+2n\phi(z)tY(0))2\phi(z)Y(0)\,dz - Y(0)\tau(\alpha)\right)\\ &= \begin{cases} \infty & \text{if } \lim_{u\to\infty}\tau(u)=\infty\\ 2(\tau_\infty-\tau(\alpha))Y(0) & \text{if } \lim_{u\to\infty}\tau(u)=:\tau_\infty<\infty.\end{cases}\end{aligned} \tag{28}$$

Thus, the energy rate needed to open a crack is infinite in case $\lim_{u\to\infty}\tau(u)=\infty$ while it is finite in case $\lim_{u\to\infty}\tau(u)<\infty$. In that case the energy needed to create a crack can be computed as follows:

Proposition 3. *Let $\tau_\infty := \lim_{u\to\infty}\tau(u)<\infty$ then the total energy of the wave fan satisfies*

$$\frac{d}{dt}E_B[y^n] = \mu_{-\sigma} + p_c^n + \mu_\sigma \overset{n\to\infty}{\longrightarrow} \mu_{-\sigma} + 2(\tau_\infty-\tau(\alpha))Y(0) + \mu_\sigma =: T > 0, \tag{29}$$

where $\mu_{\pm\sigma}$ denotes the energy dissipation at the shocks and $2(\tau_\infty-\tau(\alpha))Y(0)$ is the cost for opening the crack. The total energy rate T is positive, i.e. more energy is needed to open the crack then is dissipated at the shocks.

Proof. Given Proposition 2 and (28) it only remains to show $T>0$. Inserting (24) and (25) into the definition of T we get

$$T \;=\; \sigma Y(0)^2 \;+\; 2Y(0)\left(\tau_\infty - \frac{W(\lambda)-W(\alpha)}{\lambda-\alpha}\right) \;>\; 2Y(0)(\tau_\infty \;-\; \tau(u^*)) \;>\; 0$$

for some $u^*\in(\alpha,\lambda)$. □

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