

# Errata and Corrections to *Elliptic Diophantine Equations*

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**Acknowledgements.** I would like to thank the following people for sending me comments and corrections: Andrew Bremner, Nikolaos Katsipis.

- Errata (highlighted) due to jaundiced Ralph<sup>1</sup>:

page vii, line 1: “This book **170** is about...”. Take out “170”.

page 53, line 5: “**137** Computing the elliptic logarithm”. Take out “137”.

page 68, table 6.1, first line below table-label: “**010**  $Q(u) = au^4 + bu^3 + cu^2 + du + e^2$ ”. Take out “010”.

page 82, line 15: “Thesecond class consists of the series **105**”. Take out “105”.

page 82, line -8: “where **100**  $\omega$  is ...”. Take out “100”.

page 83, line -16: “**106** Computing the finite extension  $\mathbb{Q}(\alpha_\mu, \dots, \alpha_{\mu+k_0})$ ”. Take out “106”.

page 124, relation (10.7): “ $2^{k(k+1)/2}(k + \frac{1}{2})^{k+1}B_1(N)^{k+1}$ , **013**”. Take out “013”.

page 127, table 10.1, right most column in the first line below table-label: “**125**  $PC = (u, v)$ ”. Take out “125”.

page 129, table 10.2, left most column in the first line below table-label: “ $n_1, n_2, n_3$  **125**”. Take out “125”.

page 133, table 10.3, left most column in the first line below table-label: “ $n_1, n_2, T^E$  **123**”. Take out “123”.

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<sup>1</sup>Ralph is the imp of mischief in a printing house, according to Chambers English Dictionary. Admittedly, this time Ralph turned out very inventive!

page 135, table 10.4, left most column in the first line below table-label: “ **123**  $n_1, n_2$ ”. Take out “12 3”.

• Misprints and oversights due to the author:

✓ page viii, line 14: Replace  $y = 136/27$  by  $y = 17/9$ .

✓ page 11, three lines below relation (2.9) and second line of footnote <sup>2</sup>: Instead of “strict inequality holds” write “equality holds”.

✓ page 17, relation (2.21): Instead of  $H(1 : a/b) = H(b : a)$  write  $\log H(1 : a/b) = \log H(b : a)$ .

✓ page 18, relation (2.24): Replace  $\prod_{\mathfrak{p} \in \mathcal{P}_K} |\alpha|_{\mathfrak{p}}$  by  $\prod_{\mathfrak{p} \in \mathcal{P}_K} |\alpha|_{\mathfrak{p}}^{d_{\mathfrak{p}}}$ .

✓ page 20, two lines above relation (2.28): Instead of  $E^P$  write  $P^E$ .

✓ page 22, first line of relation (2.32): Replace  $h(x, y)$  by  $h(1 : x : y)$ .

✓ page 24, relation (2.37): In the right-hand side, replace  $2^N P$  by  $2^N P^D$ .

✓ page 26, line 5: Replace  $c_1$  by  $\rho$ .

✓ page 28, Proposition 2.6.4: The correct upper bound for  $x_1(\hat{P})$  (displayed inequality) is  $\frac{1}{2}h(x_1(P)) + \frac{1}{12} \log |\Delta_1| + \frac{1}{12} \log^+ |j_1| + \frac{1}{2} \log^+ |b_2/12| + \frac{1}{2} \log 2^* + 1.07$ .

✓ page 34, line 3: Replace  $\wp'(r)$  by  $\wp'(z_1)$ .

✓ page 63, line 4: After  $(e_1, +\infty)$  take out full stop and add “if  $\sigma = -1$  and  $x(u) \in (e_1, x_0) \subset (e_1, +\infty)$  if  $\sigma = +1$ .”

✓ page 63, line 9: Replace “decreasing” by “increasing”.

✓ page 63, line 17: Replace  $(6ez + 3td + ct^26e^2)/(3t^2)$  by  $(6ez + 3dw + cw^2 + 6e^2)/(3w^2)$ .

✓ page 66, last line: Replace “Proposition 5.4” by “Theorem 5.2”.

✓ page 70, line -4: Left-hand side of (7.13) should be  $(a, b, c, d, e)$ .

✓ page 79, relation (8.6), last line: Insert a coefficient 2 to  $u(u - 1)^3$ .

✓ page 82, lines -6 and -7: In both lines replace  $m_i$  by  $\mu_i$ .

✓ page 85, relation (8.11): Replace  $u^{-k/\nu_s}$  by  $t^{-k/\nu_s}$ .

✓ pages 89-91. [Section 8.5 modified](#). Lemma 8.5.1 was corrected; as a consequence Section 8.5 was modified and simplified at various points.

✓ pages 91-92: Throughout Section 8.6, replace all occurrences of  $v_s$  by  $v_s$ . More specifically,  $v_s$  appear in page 91, lines 2, 3 (two occurrences), -1 (two occurrences) and in page 92, line 1 (two occurrences), 3 (three occurrences).

✓ page 96: Replace 22730 by 22763 in lines 8, 9, 16, 18.

- ✓ page 97, relation (8.26): In the right-hand side replace  $h(P)$  by  $h(x(P))$ .
- ✓ page 103, relation (8.7): Replace  $\mathcal{V}(u, v)$  by  $\mathcal{V}(x, y)$ .
- ✓ page 106, line -3 up to page 107, line 2: Replace text “Analogously for the *height pairing matrix* ... that one may adopt” by the following: “Analogously for the *height-pairing matrix*, if either of the routines `HeightPairingMatrix` or `ellheightmatrix` of, respectively, MAGMA or PARI is used. As a consequence, when we use these routines for computing a height-pairing matrix and then its minimum eigenvalue, we have to divide it by 2 in order to compute the constant  $\rho$  of Proposition 2.6.2.
- ✓ page 109, line 4: Replace  $\rho \approx 0.6106414163983847$  by  $\rho \approx 0.3053207081991923$ .
- ✓ page 109, line 7: Replace “take 1 and 0” by “take 1 and  $-5/12$ ”.
- ✓ page 109, line -8: Replace  $-0.61064M^2$  by  $-0.30532M^2$ .
- ✓ page 109, line -7: Replace  $M > 1.1 \cdot 10^{41}$  by  $M > 1.6 \cdot 10^{41}$ .
- ✓ page 109, relation (9.10): Replace  $1.1 \cdot 10^{41}$  (two occurrences) by  $1.6 \cdot 10^{41}$ .
- ✓ page 109, line -4: Replace  $\mathcal{B}(1.07 \cdot 10^{41})$  by  $\mathcal{B}(1.5 \cdot 10^{41})$ .
- ✓ page 111, line 4: Replace  $\rho \approx 0.8549017536692952$  by  $\rho \approx 0.4274508768346476$ .
- ✓ page 113, line 13: Replace  $-0.8549M^2$  by  $-0.42745M^2$ .
- ✓ page 113, line 14: Replace  $M \geq 2.3 \cdot 10^{88}$  by  $M \geq 3.2 \cdot 10^{88}$ .
- ✓ page 113, relation (9.11) : Replace  $2.3 \cdot 10^{88}$  (two occurrences) by  $3.2 \cdot 10^{88}$ .
- ✓ page 113, line 17: Replace  $\mathcal{B}(2.2 \cdot 10^{88})$  by  $\mathcal{B}(3.1 \cdot 10^{88})$
- ✓ page 112, line -3: Replace 4.12 by 3.544.
- ✓ page 116, line -3: Replace (128, 0) by (128/3, 0).
- ✓ page 117, line -1: Replace  $\rho \approx 0.9301430899748535$  by  $\rho \approx 0.4650715449874268$ .
- ✓ page 118, line 5: Replace  $\gamma \approx 5.4867659523958290$  by  $\gamma \approx 6.1799131329557745$ .
- ✓ page 118, line -1: Insert minus sign before  $P_0$ .
- ✓ page 119, lines 6 and 10: Insert minus sign before  $P_0^E$ .
- ✓ page 119, line -12: Replace  $(d, d_1, d_2, d_0) = (1, 1, 1, 0)$  by  $(\delta, d_1, d_2, \delta_0) = (-1, 1, 1, 0)$ .
- ✓ page 119, line -4: Replace  $\rho \approx 0.4645951770663837$  by  $\rho \approx 0.2322975885331918$ .
- ✓ page 120, line 5: Replace  $\gamma \approx 5.4600863256138232$  by  $\gamma \approx 6.5586986142819330$ .
- ✓ page 120, line 7: Replace  $c_{13} \approx 1.042 \cdot 10^{74}$  by  $c_{13} \approx 2.46557 \cdot 10^{74}$ .
- ✓ page 120, line 12: Replace  $-0.46459M^2$  by  $-0.23229M^2$ .
- ✓ page 120, line 13: Replace  $M \geq 3.1 \cdot 10^{41}$  by  $M \geq 6.6 \cdot 10^{41}$

- ✓ page 120, relation (9.16): Replace  $3.1 \cdot 10^{41}$  (two occurrences) by  $6.6 \cdot 10^{41}$ .
- ✓ page 125, line -9: Replace  $\kappa_1 = 6.73841$  by  $\kappa_1 = 6.7384$ .
- ✓ page 125, line -4: Replace  $1.1 \cdot 10^{41}$  by  $1.6 \cdot 10^{41}$ .
- ✓ page 125, line -3: Replace  $1.664 \cdot 10^{125}$  by  $5.12 \cdot 10^{125}$ .
- ✓ page 126, line 3: Replace  $\dots 0929152369$  by  $\dots 6408365911$ , and  $\dots 9728533696$  by  $\dots 4828212017$ .
- ✓ [page 128 corrected](#), As I redo my computations, I realize that the numerical values must be corrected in quite a number of places. Therefore I think preferable to upload the whole updated page 128. All corrections have to do with numerical computations.
- ✓ page 129, table 10.2, first entry below  $n_1, n_2, n_3$ : In “ $-1, -1, O$ ”, replace  $O$  by 0.
- ✓ page 134, line 11: Replace  $49/400$  by  $81/100$ .
- ✓ page 136, last three lines: Replace “We have, thus, proved  $\dots (u, v) = (0, 0), (243, -3)$  as its only integer solutions.” by the following:  
 “Taking into account Table 10.4 and the “small” points which we computed on page 90, after Example<sup>Step 4</sup>, we have the following proposition:  
**Propositon 10.2.4** *The equation  $3v^5 + 3uv^3 - 271uv - 3u^2 = 0$  has  $(u, v) = (0, 0), (1, -3), (243, -3)$  as its only integer solutions.”*
- ✓ page 137, relation (11.1): Replace  $a_3$  by  $a_3v$ .
- ✓ page 138, line 14: Replace  $\tilde{O}$  by  $\tilde{O}$ .
- ✓ page 138, line 16: Replace  $C_1(\mathbb{F}_p), C_0(\mathbb{F}_p)$  by  $C_1(\mathbb{Q}_p), C_0(\mathbb{Q}_p)$ , respectively.
- ✓ page 141, line 2: In “ $\dots$  is at least  $\tau$ ” replace  $\tau$  by  $k$ .
- ✓ page 141, line 2: Replace  $t$  (two occurrences) by  $z$ .
- ✓ page 142, line 1: Replace  $\leq -4$  by  $\leq -2$ .
- ✓ page 142, beginning of line 4: In  $-\nu_p(u(Q)) > N/2$  insert a factor  $1/2$  in the left-hand side.
- ✓ page 146, line -6: Replace  $2\nu_q(\kappa) - b_q$  by  $-2\nu_q(\kappa) + b_q$ .
- ✓ page 150, lines 15-19: Replace all occurrences of  $P$  by  $t_q P_i$ .
- ✓ page 150, line 18: Replace  $P_i^C$  by  $t_q P_i^C$ .
- ✓ page 150, line 18: Replace  $r$  by  $s - 1$ .
- ✓ page 150, line 19: Replace  $P_i^E$  by  $t_q P_i^E$ .
- ✓ page 150, lines -5 and -3: Both occurrences of  $\nu_q(t_q P_i^C)$  should be replaced by  $\nu_q(u(t_q P_i^C))$ .
- ✓ page 151, line 6: The right-hand side of the inequality should be  $\frac{1}{2}rM + 1$ .
- ✓ page 152, line -12: Replace (5.1) by (2.42).

✓ page 153, 2nd line below (11.23), in the definition of  $c_{23}$  take out the minus sign.

✓ [Example 11.4 corrected](#), pages 158-161. As I redo my computations of this section, I realize that the numerical values must be corrected in many places. Therefore I think preferable to upload the whole updated text except for Table 11.1. In this corrected version all changes have to do with numerical computations or they are stylistic.

✓ page 173, line -3: Replace *Acta Arith.* **67** by *Acta Arith.* **68**.